The Origins of Velocity Functions

Thomas M. Humphrey

Like any practical, policy-oriented discipline, monetary economics employs useful concepts long after their prototypes and originators are forgotten. A case in point is the notion of a velocity function relating money's rate of turnover to its independent determining variables.

Most economists recognize Milton Friedman's influential 1956 version of the function. Written $v = Y/M = v(r_b, r_e, 1/PdP/dt, w, Y/P, u)$, it expresses income velocity as a function of bond interest rates, equity yields, expected inflation, wealth, real income, and a catch-all taste-and-technology variable that captures the impact of a myriad of influences on velocity, including degree of monetization, spread of banking, proliferation of money substitutes, development of cash management practices, confidence in the future stability of the economy and the like.

Many also are aware of Irving Fisher's 1911 transactions velocity function, although few realize that it incorporates most of the same variables as Friedman's.[1] On velocity's interest rate determinant, Fisher writes: "Each person regulates his turnover" to avoid "waste of interest" (1963, p. 152). When rates rise, cashholders "will avoid carrying too much" money thus prompting a rise in velocity. On expected inflation, he says: "When . . . depreciation is anticipated, there is a tendency among owners of money to spend it speedily . . . the result being to raise prices by increasing the velocity of circulation" (p. 263). And on real income: "The rich have a higher rate of turnover than the poor. They spend money faster, not only absolutely but relatively to the money they keep on hand. . . . We may therefore infer that, if a nation grows richer per capita, the velocity of circulation of money will increase" (p. 167). Finally, with respect to the catch-all variable, Fisher cites all of the following

[1] Among the few is Boris Pesek (1976, pp. 857–58) who notes that Fisher's velocity function contains more variables than Friedman's.

as affecting velocity: "habits as to thrift and hoarding," "book credit," "use of checks," "frequency and regularity of receipts and payments," "density of population," and "extent and speed of transportation" (p. 79). A comprehensive list indeed.

The purpose of this article, however, is not to evaluate Friedman's and Fisher's velocity functions. Rather it is to correct the impression that such functions begin with Fisher. Thus J. S. Cramer (1992), in his authoritative article "Velocity of Circulation" in Volume 3 of the *New Palgrave Dictionary of Money and Finance*, traces the concept to the equation of exchange "which is due to Irving Fisher" (p. 757). Countless macro and money and banking textbooks echo this view. Even Merton Miller and Charles Upton's 1975 classic *Macroeconomics: A Neoclassical Introduction* categorically asserts that the term "velocity of circulation" is "associated with Irving Fisher" (1986, p. 231). Such statements overlook 250 years of monetary theorizing. For, as demonstrated below, the notion of a functional relationship between velocity and its determinants dates from the middle of the seventeenth century and received frequent restatement throughout the eighteenth and nineteenth centuries before being bequeathed to Fisher and his successors in the twentieth. Seen in this perspective, Fisher emerges not as the originator of velocity functions but rather as a particularly innovative recipient of them.

Before documenting the latter assertion, however, it should be noted that one era's velocity determinants become another's money-stock components. Changes in the definition of money ensure as much. Thus modern analysts define money to include coin, paper currency, and deposits subject to check. By contrast, most of the pre-Fisher velocity theorists discussed below defined money as consisting solely of gold and silver coin. They excluded bank notes and deposits on the ground that such instruments lack the unconditional power of specie to settle final transactions and thus are not money per se but rather devices to accelerate money's velocity. Consequently, they saw note and deposit expansions and contractions as velocity shifts rather than as money-stock shifts. Their view may seem strange to the modern reader accustomed to regarding notes and deposits as cash, but it was entirely consistent with their metallist conception of money.

1. THE FIRST VELOCITY FUNCTION

Sir William Petty (1623–1687) enunciated the first velocity function, albeit in verbal rather than algebraic form, in his *A Treatise of Taxes and Contributions* (1662) and *Verbum Sapienti* (1664). He did so in an effort to estimate the amount of money—defined by him as consisting solely of gold coin—necessary to support the commercial activity of a nation. This amount he saw as depending on velocity or its inverse, the ratio of money to trade. Unlike writers such as John Briscoe (1696), who identified the requisite money stock with national income and so assumed a ratio of unity, Petty treated it as a fractional magnitude.[2] His pathbreaking statistical studies of the economies of Ireland and England had convinced him that the money stock was but a small part of national expenditure, which meant that a velocity coefficient greater than 1 existed to adjust money to the needs of trade. Here is the origin of the notion of velocity as the multiplier that equates the stock of money with the flow of income.[3]

Petty's statistical studies also suggested certain institutional characteristics that determine velocity. His function embodies these characteristics in the form of five independent variables: (1) frequency of payments, (2) size of payments, (3) income, (4) its distribution among socioeconomic classes, and (5) banking.

Of these variables, the first enters the function with a positive sign, reflecting Petty's belief that the more frequently income recipients are paid, i.e., the shorter the pay period, the less cash per unit of income they need to hold between paydays and so the higher is velocity. Illustrating this point, Petty claimed that workers receiving wages once per week would spend a unit of money an average of 52 times a year whereas landlords receiving rents quarterly would spend the same monetary unit only 4 times per year.[4]

Unlike the payment-interval variable, Petty's size-of-payments variable bears a negative sign. He believed larger payments require a greater accumulation of cash in advance relative to income than do smaller payments. To him, many small payments at short intervals spelled a higher velocity than did a few large payments at long intervals.[5]

Like Fisher, Petty saw income entering the function with a positive sign. "The most thriving men," he said, "keep little . . . money by them, but turn and wind it into various commodities to their great profit" (quoted in Marshall [1923], p. 41). Evidently he believed that scale economies in cash holding permit the rich to hold smaller balances in relation to their incomes than do the poor so that velocity rises with incomes. Only such economies can explain why

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[2] On Briscoe's formula *money stock = national income*, see Heckscher (1983), p. 224, Schumpeter (1954), pp. 314–15, and Viner (1937), p. 42.

[3] On Petty's contribution see Holtrop (1929), Roncaglia (1985), Schumpeter (1954), and Wu (1939).

[4] Petty's frequency-of-payments analysis launched a line of research leading to modern endogenous payment period models. See Grossman and Policano (1975) and the references cited there. In such models, rises in the cost of holding cash induce agents to shorten the pay interval and increase velocity. Such was the case in the German hyperinflation of 1923 when employers, to avoid the astronomical depreciation cost of holding marks to meet the wage bill, started paying workers daily rather than weekly.

[5] Recent theorizing on this point tends to support Petty. Thus Grossman and Policano (1975) model the case where households purchase some goods more frequently, and other goods less frequently, than they receive income. The model predicts that velocity will rise with purchasers' holdings of the first class of goods and fall with holdings of the second.

his "thriving men" hold so little money and spend it so fast. Certainly he did not see money as a luxury good whose velocity varies inversely with income. Pierre Boisguilbert (1704) enunciated the luxury-good hypothesis when, in his *Dissertation de la nature des richesses,* he declared that a coin spent by the poor has a velocity "a hundred times more" than one spent by the rich "in whose coffers large sums of money may remain useless for months and whole years at a time" (quoted in Hutchison [1988], p. 110).

The fourth velocity-determining variable in Petty's function is the distribution of income across socioeconomic classes. Because workers, landlords, and other income recipients have different pay periods, their transaction needs for cash per unit of income and thus their velocities differ. The economywide aggregate velocity figure, being a weighted average of the various velocities of the income groups, obviously depends on relative shares and the fraction of the money stock each group commands. Indeed, as mentioned below, Petty employed such weights to estimate aggregate velocity.

As for banks, Petty saw them as speeding up velocity. "Where there are banks," he wrote, "less money is necessary to drive a trade" (quoted in Wu [1939], p. 37). In his view, banks economize on the use of money—that is, gold coin—by issuing money substitutes in the form of notes. The notes effectuate transactions formerly mediated by gold, thus freeing the latter for other uses. With less money required to circulate trade, the velocity of the remaining stock increases. It is easy to understand why Petty regarded the spread of banks as a form of technological progress. Banks saved on scarce metallic reserves, thus enabling a given volume of transactions to be supported by a smaller gold stock or a larger volume of transactions by a given stock. "A bank," he wrote, "doth almost double the effect of our coined money" (quoted in Spengler [1954], p. 415). In doing so, banks helped reduce the real resource cost of effecting the nation's business.

Having specified velocity's determinants, Petty used the velocity concept, together with the exchange identity $MV = Y$, to estimate the minimum amount of money required to finance a given volume of income and trade. Assuming a national income of £40 million, he reckoned that, if money traveled a weekly circuit from employers to workers and back, annual velocity would be 52, thus rendering a money stock of £40/52 million sufficient to meet the needs of trade. If, instead, the income circuit involved quarterly rent and tax payments alone, then velocity would be 4. In this case, a money stock of £10 million would be required to accommodate trade. Finally, if money had to traverse both circuits at once, aggregate velocity V, the average of the individual circuit velocities V_1 and V_2 weighted by their circuit money shares $M_1/(M_1 + M_2)$ and $M_2/(M_1 + M_2)$ would be roughly 7.5 and the corresponding required money stock M would total approximately £5.5 million. That is, $V = V_1[M_1/(M_1 + M_2)] + V_2[M_2/(M_1 + M_2)] = 52\{[40\text{m}/52]/[(40\text{m}/52) + 10\text{m}]\} + 4\{10\text{m}/[(40\text{m}/52) + 10\text{m}]\} = 7.429 \approx 7.5$ and $M = Y/V = 40\text{m}/7.5 =$

£5.333 million ≈ £5.5 million.[6] In still another calculation, Petty, using agricultural income as a proxy for national income, estimated money's annual income velocity to be 10.

2. JOHN LOCKE'S FUNCTION

John Locke's (1632–1704) place in the history of velocity theory is secured by three contributions.[7] He was the first to explicitly relate velocity functions to the underlying money demand functions of cashholders, a point barely hinted at by Petty. In his *Some Considerations of the Consequences of the Lowering of Interest and Raising the Value of Money* (1691) he sought to "consider how much money [defined by him as gold coin] it is necessary to suppose must rest constantly in each man's hand, as requisite to the carrying on of trade" (quoted in Vickers [1959], p. 58). For laborers and their employers, he estimated this amount to be a fiftieth part of wages, for brokers (i.e., merchants and tradesmen) a twentieth part of their annual returns, and for landlords and their tenants one-fourth of the yearly revenue of land. Elsewhere, however, he halved these requisite amounts, presumably on the grounds that credit could substitute for money in driving trade.

Second, while retaining Petty's income, pay period, and distributional arguments, he introduced a new variable, the interest rate, into the velocity function. He viewed the interest rate as measuring the opportunity cost of holding money, a noninterest-earning asset, instead of assets yielding an explicit rate of return. A fall in the rate, he argued, lowers the cost of holding idle balances. In so doing it increases the quantity of such balances demanded. As a result, bankers and other monied men are, in his words, "content to have more money lie dead by them" when rates fall (quoted in Holtrop [1929], p. 506). The consequent rise in the quantity of money held per unit of income lowers velocity.

Motivating Locke's analysis of velocity's interest rate determinant was his strong opposition to contemporary English proposals for a legal 4 percent interest rate ceiling. As noted by Leigh (1974), he feared that the imposition of a below-equilibrium rate would depress output and employment in two ways. First, it would deprive the country of the money needed to drive trade. By precipitating capital outflows financed by corresponding drains of gold, the artificially low rate would create a shortage of money as investors moved their funds abroad to realize higher foreign yields. Second, it would lower velocity by reducing the cost of holding idle balances in the manner described above. Together, the velocity and money-stock reductions would constitute a contraction

[6] Petty actually expressed the required money stock M as half the sum of the individual circuit stocks M_1 and M_2. That his expression is equivalent to the ratio of income to aggregate velocity $M = Y/V$ can be seen by substituting into the latter equation his assumptions $V = (M_1V_1 + M_2V_2)/(M_1 + M_2)$ and $M_1V_1 = M_2V_2 = Y$ to obtain $M = (M_1 + M_2)/2$.

[7] On Locke, see Holtrop (1929), Leigh (1974), and Vickers (1959).

of aggregate demand. With English prices imperfectly flexible or exogenously given from world markets by purchasing power parity considerations, the aggregate demand contractions would cause corresponding contractions in real activity. For this reason, Locke advocated removal of rate ceilings so that money, velocity, spending, output, and employment could return to their equilibrium levels.

Third, Locke said that velocity could be speeded up if there were fewer middlemen standing between producers and consumers. Here is the origin of the notion that velocity varies inversely with the number of stages of production separating raw materials from finished product and so increases with the degree of vertical integration.

Like Petty, Locke regarded velocity increases as beneficial. Such increases either reduced the quantity of money required to support a given volume of trade or raised the volume of trade that could be supported by a given stock of money. To this end, he recommended a shortening of pay periods. By enhancing velocity, such shortening would be "better for trade, and consequently for everybody (for more money would be stirring and less would be necessary to do the business)" (quoted in Hutchison [1988], p. 65). He failed, however, to note the equivalence of velocity increases and money-stock increases in raising the price level in a closed economy. Not until 1755 was this equivalence articulated in published form. And the first economist to do so was Richard Cantillon (1680–1734), the foremost velocity theorist of the eighteenth century.

3. RICHARD CANTILLON

The prize for introducing the largest number of variables into an eighteenth-century velocity function goes to Cantillon.[8] Certainly his function, as presented in his 1755 *Essai sur la nature du commerce en général,* was the most elaborate to be found in the literature of that era. As the premier economist of his day, he possessed a profound understanding of the real forces shaping velocity. And as a banker and foreign exchange specialist who amassed two fortunes speculating on the South Sea Bubble and Mississippi System schemes, he also had a keen appreciation of the monetary and financial forces involved. Some of these forces—urbanization, monetization, growing financial sophistication, advent of new credit facilities and the like—pertained to France's emerging transition from a predominantly agricultural economy to a mercantile and manufacturing one. Others were stressed by his predecessors, Petty and Locke, whose metallist conception of money he also shared. All were assimilated into Cantillon's velocity analysis. Thus his velocity function contains the following arguments: (1) income, (2) frequency of payments, (3) size of payments, (4) stages of production, (5) interest rates, (6) distribution among social classes,

[8] On Cantillon, see Bordo (1983) and Murphy (1986).

(7) banking, (8) trade credit, (9) extent of barter, (10) urbanization (monetization), (11) hoarding, (12) uncertain expectations of the future, and (13) minimum denomination restrictions on asset purchases. Of these, the first seven he took from Petty and Locke. The last six, however, were original with him.

According to Cantillon, urbanization, hoarding, uncertainty, and minimum denomination restrictions all tend to reduce velocity. Trade credit and barter, on the other hand, enlarge it. Urbanization—the growth of cities and towns—expands the sphere of money transactions relative to barter transactions and production for one's own use. It does so because "all country produce is furnished by labour which may ... be carried on with little or no actual money" whereas "all merchandise is made in cities or market towns by the labour of men who must be paid in actual money" (1964, p. 143). The resulting monetization of economic activity boosts the demand for cash per unit of income so that velocity falls. Hoarding likewise slows velocity as "many miserly and timid people bury and hoard cash for considerable periods" (p. 147). Similarly, uncertainty induces people to "keep some cash in their pockets or safes against unforeseen emergencies and not to be run out of money" (p. 147). The consequent rise in the precautionary demand for cash lowers velocity. Finally, minimum denomination restrictions, which establish lower limits or floors to the size of asset purchases, retard velocity by compelling agents to "keep out of circulation small amounts of cash until they have enough to invest at interest or profit" (p. 147).

Working in the opposite direction is the use of trade credit, clearing arrangements, and other substitutes for money.[9] These items, by allowing businessmen to dispense with money in financing ongoing commercial transactions and by permitting them to cancel claims against each other so that only net balances need be paid, "seem to economize much cash in circulation, or at least to accelerate its movement" (p. 141). Thus it "is not without reason that it is commonly said that commercial credit makes money less scarce." The same is true of barter which likewise reduces the need for cash and so raises velocity.

Taking these factors into account, Cantillon estimated income velocity to be 9. With all determinants of money demand considered, he calculated that a country's money stock M should be one-third of landowners' annual rent R. Since he reckoned rent to constitute one-third of the value of annual produce Y, he obtained velocity V as $V = (R/M)(Y/R) = Y/M = 3 \times 3 = 9$. His estimate, which like Petty's used farm income as a proxy for national income, was close to Petty's estimate of 10.

[9] Cantillon's analysis thus implies a U-shaped pattern over time for velocity in developing economies. At first, increasing monetization causes velocity to fall. Thereafter, increasing financial sophistication and the growth of money substitutes cause velocity to rise. Recent work in the Cantillon tradition offers strong empirical support for this hypothesis. See the studies of Ireland (1991) and Bordo and Jonung (1987).

4. INTRODUCTION OF INFLATIONARY EXPECTATIONS INTO THE FUNCTION

Cantillon, in his list of velocity determinants, had neglected to include inflationary expectations. This step was taken in the first three decades of the nineteenth century by economists who had witnessed the debacle of the French assignats (1794–1796). This episode, Western Europe's first hyperinflation, left a lasting impression. It revealed that excessive monetary expansion, by generating anticipations of future inflation, could precipitate a flight from cash and a corresponding rise in velocity such that prices would rise even faster than the money stock. It was an easy task to incorporate this lesson into velocity theory.

Henry Thornton (1760–1815), in his 1802 *Paper Credit of Great Britain,* was the first to do so. He said that when cashholders extrapolate observed current falls in the purchasing power of the currency into expected future falls, the expectations themselves will speed up velocity and quicken the currency's depreciation (p. 108). Using this insight, he explained how the excessive issue of French assignats had "operated on their credit, and became a very powerful cause of their depreciation" (p. 233).

J. B. Say (1776–1832), in his 1803 *Traité d'économie politique,* likewise attributed the assignats' "prodigious" rate of turnover to cashholders' attempts to rid themselves of a depreciating currency as fast as possible.[10] The same point was made by Simonde de Sismondi (1773–1842) in his 1827 *Nouveaux principes*. He contrasted (1) distrust of the future stability of the real economy with (2) distrust of the future value of the currency. The first type of distrust, he said, tends to lower velocity whereas the second type tends to raise it.[11]

But the most precise account of the impact of inflationary expectations on velocity and thus on the inflation rate itself came from Nassau Senior (1790–1864) in his 1830 *Three Lectures on the Cost of Obtaining Money*. Referring to the depreciation of the assignats stemming from the loss of confidence in their future value, he wrote: "The prices of commodities rose in proportion, not merely to the existing depreciation [true of course by definition], but to the well-founded apprehension of a still further depreciation" (quoted in Eshag [1963], p. 16). The result of such perceptions of the likely future depreciation of the currency was exactly what one would expect: "Everybody taxed his ingenuity to find employment for a currency of which the value evaporated from hour to hour. It was passed on as it was received, as if it burned everyone's hands who touched it" (quoted in Eshag, p. 16). After Senior's exposition, it would be hard indeed to claim that anticipated inflation had been left out of the velocity function.

[10] See Holtrop (1929), p. 519.

[11] See Holtrop (1929), p. 520.

5. OTHER NINETEENTH-CENTURY CONTRIBUTIONS

The preceding hardly begins to exhaust the wealth of pre-twentieth century writing on velocity functions. Holtrop (1929, pp. 518–20) notes that in the nineteenth century alone, writers John Stuart Mill, Thomas Tooke, Christian von Schlozer, Heinrich Storch, Karl Heinrich Rau, and Johann Karl Rodbertus all discussed velocity functions. By far the most important contributions, however, came from Henry Thornton and Knut Wicksell. Towering above the rest, their pathbreaking work constitutes the peak achievement of velocity-function analysis prior to Irving Fisher.

We have already met Thornton, the pioneer of inflation-expectations analysis. This contribution alone would warrant his mention in any survey of velocity theory. But he contributed much more to the theory than merely introducing an expectations argument into the velocity function. Advancing beyond his predecessors, he defined the relevant monetary aggregate as the total stock of circulating media rather than its narrow specie component. Moreover he was the first to specify how two variables, namely (1) the composition of the payments media and (2) the state of business confidence, influenced velocity. He had observed how these variables operated to produce the velocity swings of the turbulent 1790s and sought to correct the tendency of his predecessors to neglect them. In addition, as a banker and financial expert who had connections with correspondent banks throughout the country, he was particularly alert to the fundamental changes occurring in the English credit mechanism (Hayek [1939], p. 38). These changes, which included rapid growth in the number of country banks, the increasing use of checks, the establishment of the London Clearing House, and the emergence of the Bank of England as the central bank and lender of last resort, induced him to extend Cantillon's analysis of the velocity-enhancing effects of financial innovation.

His first task was to show how the composition of the payments media enters the velocity function. He argued that the total means of payment consists of coin, banknotes, and bills of exchange. Each circulates with a speed that varies inversely with the opportunity cost of holding it. This cost is measured as the differential between the instrument's own rate of return and the prevailing market rate. The lower the own rate relative to the prevailing rate, the greater the cost of holding the instrument and the stronger the incentive to spend it instead. Thus coin and banknotes, which yield no interest, circulate faster than interest-bearing bills of exchange. Add to this the fact that gold coins are hoarded more than notes and so circulate more slowly than the latter in times of panic and it becomes apparent that different instruments possess different velocities. It follows that aggregate velocity, the weighted average of the component velocities, depends on the composition of the payments media. When that composition changes, so does aggregate velocity.

Thornton next identified as a determinant of velocity the state of mercantile confidence arising from general business and financial conditions. Confidence refers to the certainty of agents' beliefs that receipts will match expenditures, thus obviating the need to hold emergency reserves. A high state of confidence produces a low demand for precautionary balances and a rapid velocity. Conversely, a low state of confidence stemming from distrust and alarm produces a high demand for precautionary balances and a slow velocity. Thornton summarizes:

> A high state of confidence serves to quicken [money's] circulation. . . . [It] contributes to make men provide less amply against contingencies. At such a time, they trust, that if the demand upon them for a payment, which is now doubtful and contingent, should actually be made, they shall be able to provide for it at the moment; and they are loth to . . . make the provision much before the period at which it shall be wanted. When, on the contrary, a season of distrust arises, prudence suggests, that the loss of interest arising from a detention of notes for a few additional days should not be regarded. . . . Every one fearing lest he should not have his notes ready when the day of payment should come, would endeavor to provide himself with them beforehand. (1939, pp. 96–98)

Thornton concluded that no single money stock always supports the same level of nominal activity. Since velocity fluctuates with the state of confidence, more money is required to effect a given volume of transactions when confidence is low than when it is high.

As for financial innovations, Thornton saw them as boosting money's turnover rate. He explained how the invention of the clearinghouse, with its mutual cancellation of claims, economized on the amount of money required to settle transactions. And he cited still other developments—correspondent banking arrangements, improved communications, and the like—that had the same effect. Like Cantillon, he drew the conclusion that such devices economize on the use of money and speed up velocity.

Thornton's analysis of financial innovations influenced his contemporaries. Classical quantity theorists, notably David Ricardo (1772–1823) and the authors of the 1810 *Bullion Report,* endorsed it. But so too did anti-quantity theorists. Thornton's work initiated the notion that monetary contraction stimulates the very financial innovation and compensating rise in velocity that offsets the initial monetary contraction. Indeed, nobody stated this idea better than Thornton himself. Let such a contraction occur, he said, and the resulting "great limitation of the number of bank notes would, therefore, lead . . . to some new modes of economy in the use of the existing notes: the effect of which economy on prices would be the same, in all respects, as that of the restoration of the usual quantity of bank notes" (p. 119). Coming from a leading classical quantity theorist, this was a startling admission indeed.

6. KNUT WICKSELL

Thornton's work illustrates the flourishing of velocity analysis at the century's beginning. Knut Wicksell's work illustrates its vitality at the century's end. Thus Wicksell (1851–1926) devoted the entire 30-page Chapter 6 of his 1898 volume *Interest and Prices* to a discussion of the determinants of "The Velocity of Circulation of Money."[12]

He began by defining money as consisting solely of gold coin. His definition rules out notes and deposits, which he treated as credit instruments that raise the "virtual velocity" of money. He explained that such instruments, when used in payment, free an equivalent amount of coin to facilitate purchases elsewhere. In so doing, they effect a virtual turnover of coin and thus raise the velocity of money. Having made this point, he next defined velocity as the inverse of the "average period of idleness" or "interval of rest" of coin. In so doing, he evoked the notion of money demand as velocity's reciprocal. Finally, he identified at least five determinants of velocity.

The first is a transactions demand for cash to bridge the gap caused by the lack of synchronization between receipts and expected payments. The second consists of a precautionary demand to meet unexpected payments. Although Cantillon and Thornton had incorporated these demands into the velocity function before Wicksell, they had not derived them from probability theory. Wicksell, however, did so. Inspired by Francis Edgeworth's (1888) application of probability theory to banking, he argued that the frequency with which cash shortfalls of various amounts are likely to occur could be described by a probability distribution whose mean represents expected shortfalls and whose dispersion or spread measures the risk that actual shortfalls will be larger than expected.

For his dispersion parameter, Wicksell used a statistic called the probable deviation. Equaling 0.6745 times the standard deviation, this statistic has the following property. When positioned on both sides of the mean, it includes half of the elements of the distribution. That is, half the elements lie within, and half without, one probable deviation of the mean. It follows that cashholders wishing to secure themselves against a 50–50 chance of an unexpected shortage of cash will hold precautionary balances equivalent to one probable deviation. And cashholders with still greater degrees of risk aversion will hold even more. Wicksell explained:

> Suppose that experience has shown that . . . the excess of payments over simultaneous receipts . . . tends to oscillate from year to year about a certain mean value, *a*. Let the "probable deviation" be *b*: this means that the odds are even . . . in favor of the payments over the period in question lying between

[12] On Wicksell's velocity analysis, see Laidler (1991), pp. 123–29, and Uhr (1960), pp. 220–24.

> $a + b$ and $a - b$. If the business man is satisfied with this so-called simple margin of safety, he must have by him a cash holding of $a + b$. But if he demands a greater degree of security against the possible exhaustion of his till, his cash holding must of course be somewhat larger. With a cash holding of as little as $a + 2b$, the betting on the total exhaustion of his till . . . would, according to the laws of probability, be more than 9 to 1; with a cash holding of $a + 3b$ it would be more than 44 to 1; and with one of $a + 5b$ it would be more than 2600 to 1, *i.e.* the till would be exhausted only about *once in three thousand five hundred years.* (1936, pp. 57–58)

From this analysis it follows that the distribution's mean and probable deviation parameters a and b constitute arguments of the velocity function.

Wicksell also entered into his velocity function what he called simple trade credit between businessmen. This variable bears a positive sign since the availability of trade credit reduces the amount of cash businessmen need to hold relative to income to finance regular recurring transactions. By far the most important determinant, however, is "organized credit" involving the operations of commercial banks. Banks, Wicksell argued, boost velocity by multiplying the volume of credit instruments—notes and deposits—erected on a given money base. Once created, the notes and deposits mediate additional exchanges. In so doing, they raise the volume of transactions per unit of money (gold) and so enhance velocity.

To illustrate how banks evolved to raise money's efficiency in supporting more transactions, Wicksell sketched the following hypothetical sequence of events. First, the emergence of banks allows agents to dispense with money (gold) holdings by converting them into credit instruments instead. The resulting flow of gold into banks continues until those institutions eventually hold the entire stock of the precious metal as reserves.

At the same time, bankers discover that three considerations—(1) the regularity of chance or law of large numbers, (2) the interdependence of firms such that payments of one set of bank customers are the receipts of another, and (3) the practice of settling offsetting claims of different customers of the same bank through bookkeeping transfers from one account to another rather than through the use of money—permit them to operate with fractional reserves. These same inducements spur banks to form clearinghouse associations. Scale and settlement economies also provide incentives to consolidate the banking system's reserve holdings in a centrally located bank.

Together, these developments tend to reduce the fractional reserve ratio to negligible proportions. The ensuing potentially unlimited expansion of the stock of credit instruments mediates a much larger volume of trade than would the gold itself if it were used directly in making payments. Here is the essence of Wicksell's doctrine that bank notes and deposits raise the "virtual" velocity of gold reserves resting in bank vaults with an actual physical velocity of zero.

Finally, Wicksell saw velocity as a function of the difference between the market (loan) and natural (equilibrium) rates of interest. In his famous cumulative process analysis of price-level movements, he argued that excesses of the natural rate over the market rate produce corresponding excesses of desired investment spending over desired saving. As a result, the demand for loanable funds to finance investment exceeds the amount of such funds voluntarily supplied by savers. Banks supply the remainder through credit (i.e., note and deposit) creation. The consequent rise in the volume of bank credit erected on a given money stock constitutes a rise in the virtual velocity of that stock. This rise in turn puts upward pressure on prices. Thus price-level movements emanate from rate differentials—more precisely from natural rate movements given the market rate—operating through the velocity function.

This conclusion—that velocity rises with the natural rate-market rate differential—is entirely the result of Wicksell's definition of money to exclude notes and deposits. Had he included those items in his definition, he would have seen the rate differential as boosting the money stock rather than its velocity. This point notwithstanding, he provided the most complete analysis of velocity and its determinants since Thornton. His work is proof positive that a sophisticated literature on the subject existed before Fisher.

7. CONCLUSION

The preceding discussion has concentrated exclusively on major landmarks in the evolution of velocity functions. In so doing, it has no doubt neglected other milestones. For example, nothing was said about Alfred Marshall's work on velocity in the 1870s and 1880s. D. P. O'Brien (1981, pp. 58–59) notes that Marshall (1824–1924) followed Thornton and the *Bullion Report* in attributing velocity's movements to fluctuations in the state of confidence and economic activity, to financial innovation and the growth of money substitutes, to technical progress in production, and to changes in transportation, communications and the like.[13] Much like Wicksell, Marshall viewed bank deposits not as money but rather as a device for economizing on its use and speeding up velocity.

Nor was anything said about Thomas Attwood's 1817 distinction between income velocity and transaction velocity. The distinction between the two velocity concepts is often traced to Arthur Cecil Pigou, who discussed it in his 1927 book *Industrial Fluctuations.* It originates, however, with Attwood, who estimated income velocity at 4 and transaction velocity at 50 per annum.[14]

Nor was mention made of the pathbreaking 1895 statistical work of Pierre des Essars. His cross-country time-series estimates of the deposit turnover rates

[13] On Marshall, see also Eshag (1963), pp. 2–18, and Whitaker (1975), pp. 172–73.

[14] On Attwood, see Marget (1938), p. 358.

of continental European banks for the period 1884–1894 anticipated all later empirical work on velocity. In essence, he computed deposit velocity as the ratio of bank debits to average balances in deposit accounts.[15] Irving Fisher (1963, pp. 63 and 87) cited his findings as evidence that population density and anticipated inflation act to raise velocity.

Also unmentioned was E. W. Kemmerer's 1907 attempt to verify the Thornton-Marshall hypothesis that velocity varies directly with the state of business confidence. Not the least of Kemmerer's achievements was his construction, from data on business failure rates and the dollar liabilities of failed firms, of an index of business distrust. Movements of the index, he thought, accounted for corresponding movements in velocity.

Finally, nothing was said about early versions of the $MV = Py$ equation of exchange. The pre-Fisher literature boasts at least 14 such equations.[16] All contain at least one velocity variable and two contain separate velocity terms for each component of the payments media.

Nevertheless, enough has been said to document the main contention of the article, namely that velocity functions long predate Irving Fisher and his recent counterparts. This is not to say, however, that older and modern versions of the function are identical. On the contrary, modern versions tend to be stated mathematically, often in the form of least-squares regression equations yielding numerical estimates of the equation's coefficients.[17] By contrast, older versions of the function tended to be expressed verbally rather than algebraically.

Still, the basic notion of a stable functional relationship between velocity and its independent determining variables has remained unchanged since the time of Petty. So too has the practice of specifying the function's arguments. Thus Petty's successors in the eighteenth and nineteenth centuries completed his list of velocity determinants and bequeathed it to twentieth-century writers. Seen in this perspective, the work of Fisher, Friedman, and other modern velocity theorists constitutes the culmination of a long tradition rather than the beginning of a new one.

[15] On Des Essars' estimates, see Kemmerer (1907), pp. 115–16.

[16] On pre-Fisher versions of the equation of exchange, see Humphrey (1984) and the references cited there.

[17] For examples, see Bordo and Jonung (1987), pp. 32–39, and Goldfeld (1973), p. 633.

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