When Geometry Emerged: Some Neglected Early Contributions to Offer-Curve Analysis

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I n his 1952 *A Geometry of International Trade,* Nobel Laureate James Meade presented the definitive modern version of the celebrated reciprocal-demand, or offer-curve, diagram of the trade theorist. The diagram features curves depicting alternative quantities of exports and imports countries are willing to trade at all possible prices (see Figure 1).

Let two countries, home and foreign, trade two goods, x and y. Measure quantities of these goods along the horizontal and vertical axes, respectively. Suppose the home country exports good x and imports good y while the foreign country does the converse. The slope of any ray from the origin expresses the relative price of x in terms of y. That is, it expresses the quantity of y exchanged per unit of x, or y price of x. Curve H is the home country's offer curve. Curve F is the foreigner's. Each curve shows alternative quantities of imports demanded and exports supplied at all price ratios or terms of trade.

As drawn, the curves display declining *elasticity,* or price responsiveness, throughout their length. They slope upward from left to right when the demand for imports in terms of exports is elastic—that is, when more exports are offered for imports at successively lower import prices. They cease to slope upward when import demand becomes unit-elastic. At such points, the quantity of exports offered for total imports remains unchanged as import prices fall. They bend backward (or downward) when import demand is inelastic. Along such segments, fewer exports are offered for total imports when import prices fall.

Figure 1   Offer-Curve Diagram

Foreign Exports
Home Imports
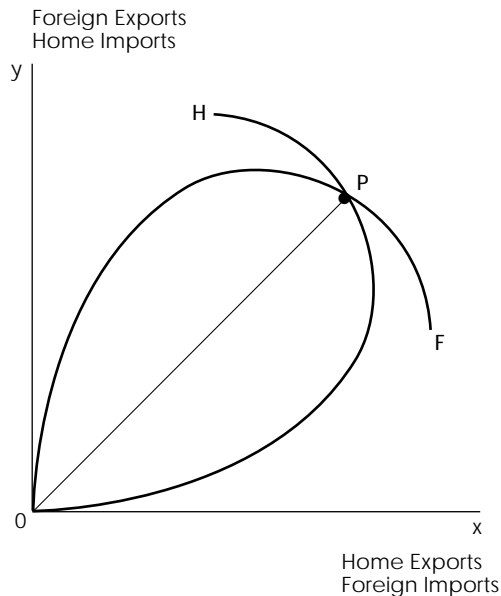
Curves *H* and *F* show alternative quantities of exports offered and imports demanded by each country at all different price ratios. Equilibrium occurs at intersection point *P*. The slope of ray 0*P* is the equilibrium price ratio or terms of trade. The coordinates of point *P* depict the equilibrium quantities of goods traded.

Home Exports
Foreign Imports

World trade equilibrium occurs at point *P*, where the offer curves intersect. At that point, the market-clearing price ratio, or terms of trade, given by the slope of the ray 0*P* equates each nation's import demand with the other's export supply. The supply of both commodities equals the demand for them, and the coordinates of point *P* show the resulting equilibrium volume of world trade.

The foregoing diagram has proved indispensable in illuminating the central ideas of trade theory. Generations of professors and their students have employed it to demonstrate how the strength and elasticity of each country's demand for the other's product determine the equilibrium volume and terms of world trade. Likewise, scores of textbooks use it to illustrate how tariffs, technological advances, resource discoveries, taste changes, and other such disturbances shift the offer curves and thereby alter world trade equilibrium.

That a simple geometrical diagram would prove so useful is hardly surprising. Other economic diagrams, including the Keynesian cross, Marshallian scissors, Hicksian IS-LM, Knightian circular flow, Vinerian cost envelope, Fisher-Haberler production possibility frontier, and expectations-augmented Phillips Curve, or zero long-run trade-off between inflation and unemployment, have proved equally indispensable. Indeed, as long ago as 1879, Alfred Marshall insisted that diagrams are absolutely essential to exact reasoning in economics

because they yield many of the same results as higher mathematics while being accessible to the mathematically untrained.

What is surprising is how little has been written on the doctrinal history of offer curves. Few systematic surveys of that topic exist. Textbooks scarcely do it justice. Even history-of-thought treatises spotlight at best only a handful of the chief contributions. Meade himself was largely silent on the diagram's history even though it was more than 100 years old when he published his *Geometry*.

The development of offer-curve analysis involves some of the leading names in classical and neoclassical economics. John Stuart Mill, Robert Torrens, Alfred Marshall, Francis Ysidro Edgeworth, and Abba Lerner all contributed to the diagram's development and policy applications. Mill invented reciprocal-demand schedules. He used them to determine precisely where, within the limits set by comparative-cost ratios, the terms of trade, or quantity of imports bought by a unit of exports, must fall. He used them also to estimate the impact of tariffs and technology shocks on the terms of trade. Torrens likewise employed such schedules to argue the merits of a policy of reciprocity in tariff erection and removal. Marshall translated Mill into geometry and examined the stability of offer-curve equilibria. Edgeworth combined offer curves with indifference maps to derive the theory of the optimum tariff. Lerner corrected Edgeworth's error of alleging an asymmetry between export and import taxes and also showed how the government's disposal of the tax receipts influences the position of the curves. Finally, Meade completed the analysis by deriving offer curves from price vectors and trade indifference maps—themselves derived from underlying production possibility frontiers and consumption indifference curves. His demonstration was crucial. It proved once and for all that domestic production as well as consumption conditions influence offer curves.

The paragraphs below attempt to trace this evolution and to identify specific contributions to it. Besides exhuming lost or forgotten insights, such an exercise serves as a partial corrective to the tendency of modern trade theory textbooks to overemphasize Meade's contribution at the expense of those of his predecessors. By resurrecting pathbreaking earlier work, the exercise dispels misconceptions concerning the origins of offer-curve analysis. It establishes that the diagram is not a twentieth-century innovation. In this connection, it reveals that at least two of Meade's predecessors instinctively grasped the concept of offer curves long before Marshall invented the diagram. It nevertheless indicates that the diagram played a crucial role in advancing the analysis. By crystallizing, condensing, and generalizing earlier insights into a powerful yet simple visual image, the diagram at once rendered them transparent and easy to comprehend. For evidence of the diagram's power to illuminate and enhance earlier work, one need only refer to Mill's and Torrens' laborious verbal and numerical examples. Those examples convey their full meaning only when translated into geometry. For this reason, the following paragraphs take the liberty of interpreting Mill and

Torrens with the aid of offer-curve diagrams unavailable to them when they wrote. Such anachronisms involve little distortion when, as is the case here, they correspond faithfully to the original work.

1. JOHN STUART MILL

Reciprocal-demand, or offer-curve, analysis originated to fill a gap in David Ricardo's theory of comparative advantage. Ricardo, in his 1817 volume *On the Principles of Political Economy and Taxation,* demonstrated (1) that comparative-cost ratios in each country determine pre-trade relative prices, (2) that international differences in such prices render trade advantageous, and (3) that countries therefore trade when their comparative-cost ratios differ, exporting their relatively cheap-to-produce goods and importing their relatively dear-to-produce ones. Ricardo also indicated that the post-trade terms of trade must fall somewhere between these limiting cost ratios. He did not, however, explain what determines the terms of trade or where it would tend to settle. He merely assumed it would fall roughly halfway between the cost ratios without explaining why.

Priority for identifying the relative strength of each country's demand for the other's product as the determinant of the terms of trade goes to James Pennington, Robert Torrens, and, above all, John Stuart Mill. Pennington in 1840 was the first to state the point in print. His account, however, was marred by the notion that volatile reciprocal demands cause the terms of trade to oscillate ceaselessly within the limiting cost ratios rather than to achieve a stable determinate value. Torrens was the first to coin the phrase "reciprocal demand" (see Viner [1937], p. 536). Because he used the concept to argue against unilateral tariff reduction, however, his analysis was condemned by his classical contemporaries, most of whom were free-traders. Of the three originators of the reciprocal-demand idea, John Stuart Mill exerted by far the greatest influence. His conception of reciprocal demand as a schedule or function of price enabled him to convey its importance more clearly, systematically, and convincingly than had Pennington and Torrens. In any case, it was from Mill rather than from the latter two writers that later economists took the idea.

Reciprocal-Demand Theory

Mill stated the idea first in his essay "On the Laws of Interchange Between Nations," which he wrote in 1829–30 but did not publish until 1844 in response to Torrens' (1841–42) *The Budget*. He presented it again in Chapter 18 of Book III of his 1848 *Principles of Political Economy*. His statement is as modern as the latest trade textbook.

First comes the notion of comparative-cost ratios as limiting values for the terms of trade.[1] Next comes the argument that reciprocal-demand schedules express an inverse relationship between import relative price and quantity demanded.[2] There follows the idea that to each quantity of imports demanded along a reciprocal-demand schedule there corresponds an associated amount of exports supplied. This amount equals the product of import price and quantity. It expresses the condition that a trading country's export receipts constitute its means of purchasing imports of the same value.[3] In other words, reciprocal-demand schedules are at once demand-and-supply curves expressing import demand in terms of export supply.[4]

Finally comes Mill's requirement that reciprocal-demand schedules intersect at the equilibrium volume and terms of trade. The latter variable, the equilibrium price ratio, clears the world goods market such that each country's demand for imports equals the other's export supply. Mill referred to this equilibrium condition as the Law of International Values.[5]

Having shown how reciprocal-demand schedules intersect to determine world trade equilibrium, Mill examined the stability of that equilibrium. In language familiar to any modern economist, he argued that a displacement of the terms of trade from its equilibrium value would invoke an excess world demand for one good and corresponding world excess supply of the other. These excess demands and supplies would exert corrective pressure on the terms of trade until it returned to equilibrium.

Clearly, Mill had put his stamp on the diagram just as surely as if he had drawn it himself. That much is evident from how easily his statements map into offer-curve space (see Figure 2). Closest to the axes lie the comparative-cost (CC) lines. Their slopes represent the production substitution, opportunity cost, and domestic price ratios of the two goods in each country in the absence of trade. The lines are drawn straight to correspond to the Ricardian or classical

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[1] "The limits within which the [relative price or exchange ratio between importables and exportables] is confined, are the ratio between their costs of production in the one country, and the ratio between their costs of production in the other" ([1844] 1968, p. 12).

[2] "The higher the price, the fewer will be the purchasers, and the smaller the quantity sold. The lower the price, the greater will in general be the number of purchasers, and the greater the quantity disposed of" ([1844] 1968, p. 9).

[3] Let X and M denote export and import quantities and p_x and p_m their money prices. Then, at each point on a reciprocal-demand schedule, the value of exports supplied $p_x X$ equals the value of imports demanded $p_m M$, or $p_x X = p_m M$. Dividing both sides of this equation by p_x yields $X = (p_m/p_x)M$, which says that the amount of exports offered equals the product of import relative price and quantity.

[4] "The supply brought by the one constitutes his demand for what is brought by the other. So that supply and demand are but another expression for reciprocal demand" ([1848] 1909, p. 593).

[5] "The produce of a country exchanges for the produce of other countries, at such values as are required in order that the whole of her exports may exactly pay for the whole of her imports. This law of International Values is but an extension of the more general law of Value, which we called the Equation of Supply and Demand" ([1848] 1909, p. 592).

Figure 2   Mill's Reciprocal-Demand Schedules

Foreign Exports
Home Imports
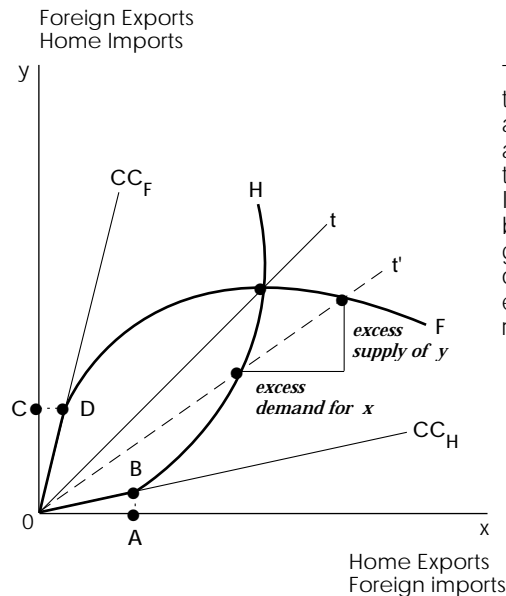
The schedules *H* and *F* follow the comparative-cost lines over a range in which the countries are indifferent to trade. Then they bend toward equilibrium. If the terms-of-trade ray lies below the market-clearing ray going through the point of offer-curve intersection, the resulting excess demands and supplies restore it to equilibrium.

excess supply of y

excess demand for x

Home Exports
Foreign imports

assumption of constant marginal and average costs. That the home country's line is the flatter of the two indicates that it (the home country) possesses a comparative-cost advantage in producing the good measured along the horizontal axis. Conversely, the steep slope of the foreigner's cost line signifies his comparative-cost advantage in producing the good measured along the vertical axis.

As for the offer curves, they follow the cost lines over a range in which the countries are indifferent to trade. Thus if the home country faces world terms of trade equal to its domestic opportunity cost ratio *AB*/0*A*, it cares not whether it obtains *AB* units of good *y* through domestic production or through foreign trade. Either way, the cost is the same, namely, 0*A* units of good *x*. Likewise, when the terms-of-trade ray coincides with the foreigner's cost line, he is equally willing to obtain *CD* units of good *x* through trade or domestic production. In each case, he sacrifices 0*C* units of good *y*.

At a certain point, however, the offer curves depart from the cost lines. Thus, at point *B*, the home country's curve begins to bend away from its cost schedule. Precisely at this point, the home country specializes completely in the production of its exportable, the excess of which it trades for importables to reach its desired consumption bundle—the same bundle it would consume under

self-sufficiency. Beyond this point, however, the offer curve bends upward in response to better terms of trade. The resulting fall in the export price of imports has a twofold effect. It increases the quantity of imports demanded. And, by inducing the country to consume fewer exportables, it makes more of those goods available for sale abroad and so raises the quantity of exports offered. This latter step is necessary since the country already is at its specialization point and can produce no more goods for export. A similar analysis holds for the foreign offer curve, which at point D bends away from its cost line toward equilibrium.

Equilibrium occurs where the curves intersect. Running through that intersection point is a ray from the origin whose slope represents the market-clearing price ratio or terms of trade. A self-correcting mechanism ensures this equilibrium price ratio will prevail. Should a disequilibrium terms of trade such as that indicated by the slope of the lower dashed ray occur, the result would be an excess demand for the home country's exports and an excess supply of its imports. The resulting rise in export prices and fall in import prices would restore the equilibrium terms of trade.
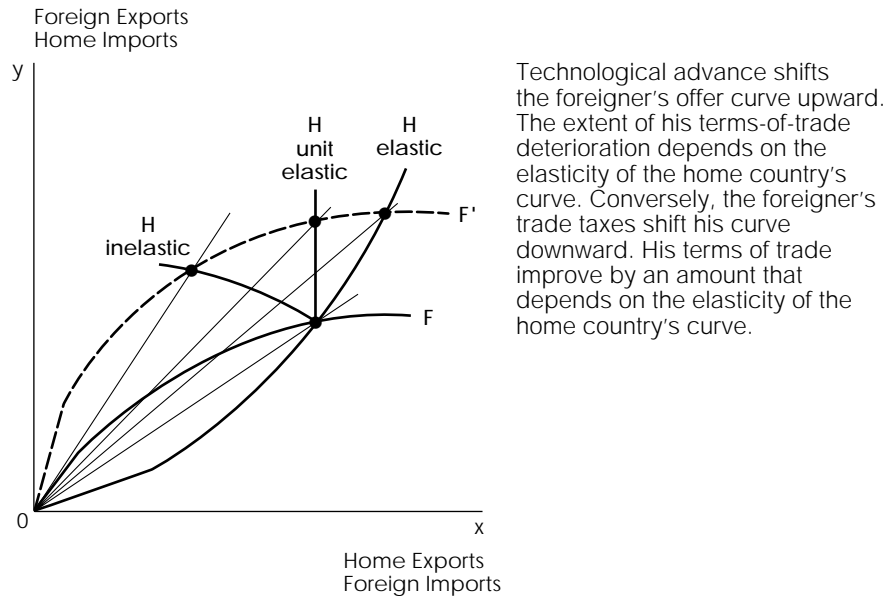
Applications

Having derived his reciprocal-demand apparatus, Mill put it to work in analyzing a variety of cases. He showed that where the terms of trade settle between the autarkic cost lines depends on the relative strength and elasticities of the reciprocal demands. The greater and more elastic one country's demand relative to the other's, the more the terms of trade would move against the first country and in favor of the second. And in the case of a large country trading with a small one, he showed that the latter's offer curve might cut the former's in its linear segment. If so, the terms of trade would coincide with the large country's cost ratio. All gains from trade would go to the small country, and the large one would be incompletely specialized in production.

Technological Improvements and the Terms of Trade

He likewise applied his reciprocal-demand technique to predict the terms-of-trade effects of a cost-reducing improvement in the foreign country's export sector (see Figure 3). To do so, he distinguished between elastic, unit-elastic, and inelastic home import demands. Such elasticities result in greater, unchanged, and smaller outlays of exports as import prices fall. Accordingly, they give rise to upward-sloping, vertical, and backward-bending home offer curves, respectively.

Mill concluded that the improvement would, by raising the supply of exports relative to the demand for them, turn the terms of trade against the foreign country by an amount that depended on the home country's import-demand elasticity. The improvement lowers the foreigner's opportunity cost of

Figure 3   Technological Advance and Trade Taxes

Foreign Exports
Home Imports

Technological advance shifts
the foreigner's offer curve upward.
The extent of his terms-of-trade
deterioration depends on the
elasticity of the home country's
curve. Conversely, the foreigner's
trade taxes shift his curve
downward. His terms of trade
improve by an amount that
depends on the elasticity of the
home country's curve.

Home Exports
Foreign Imports

producing exports. It thus enables him to offer more for any given quantity of
imports. In so doing, it shifts up his offer curve equiproportionally to the cost
reduction.[6] The resulting counterclockwise rotation of the equilibrium price ray
constitutes the terms-of-trade deterioration. The deterioration is proportionally
greater than, equal to, or less than the cost reduction as the home offer curve
is inelastic (backward-bending), unitary-elastic (vertical), or elastic (upward-
sloping). Mill's inelastic case anticipated the modern concept of *immiserising
growth* in which the adverse terms-of-trade effects of improved productivity
swamp the beneficial output effects and so make the country worse off than
before.

Trade Taxes and the Terms of Trade

Mill also employed his reciprocal-demand apparatus to examine the terms-of-
trade effects of a tax on exports or imports. Despite his aversion to all forms
of trade restriction, he demonstrated that such taxes could improve the levying

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[6] Mill admitted that this result might not hold exactly if the cost reduction exerted an income
effect on the foreigner's own demand for his exportable good.

country's terms of trade in proportion to the elasticity of the other's reciprocal demand.

Let the foreign government levy the tax and consume the proceeds (see Figure 3 again). If the tax is on imports, it reduces the demand for them. If it is on exports, it reduces their supply. Either tax, therefore, causes an equiproportionate downward shift in the foreigner's offer curve and thus improves his terms of trade. The improvement is in greater, equal, or lesser proportion than the tax depending on whether the home country's offer curve is backward-bending (inelastic), vertical (unit-elastic), or upward-sloping (elastic).

Theoretically, then, trade taxes could improve the terms of trade and thus national welfare. Nevertheless, Mill opposed them on practical and moral grounds. In his view, they invite retaliatory duties abroad that nullify the initial terms-of-trade improvement. Worse still, they bring costly reductions in the volume of world trade. Even in the absence of retaliation, they are unjust because they benefit the levying country at the expense of other countries. Since the rest of the world's loss exceeds the dutying country's gain, such taxes are inimical to global welfare and cannot be justified from a cosmopolitan standpoint.
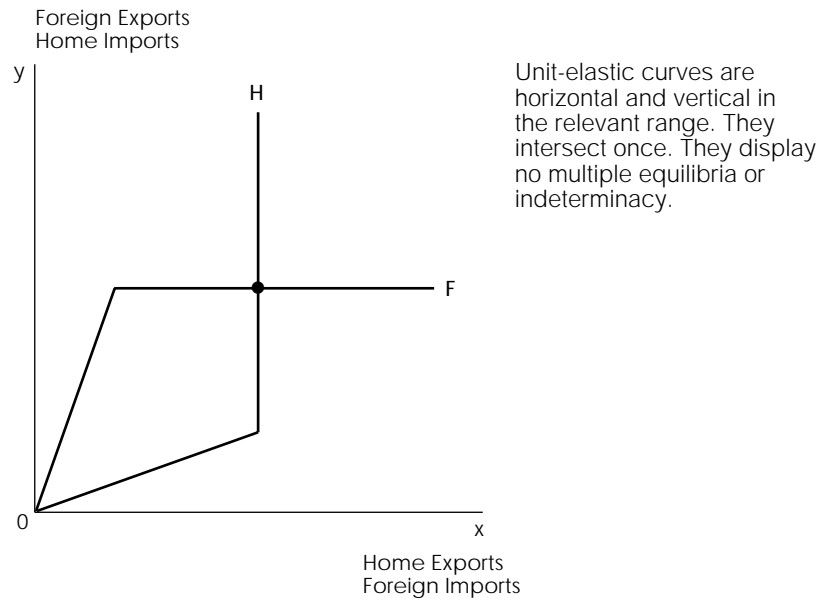
Mill's Failure

The bulk of Mill's analysis remains as valid today as when he wrote it. Still, he was not completely successful. He failed to resolve the problems that arise when offer curves exhibit (1) multiple equilibria and (2) indeterminacy of equilibrium. The first problem arises when the curves intersect more than once; the second when they coincide over certain ranges. Both phenomena require for their occurrence inelastic offer curves. Unfortunately, however, Mill chose to analyze them under the special assumption that the curves are unit-elastic. Neither Alfred Marshall ([1879] 1975, pp. 148–49) nor Francis Edgeworth (1894a, pp. 609–14) let this slip pass unnoticed. They pointed out that unit-elastic curves intersect only once and cannot coincide (see Figure 4). Accordingly, they concluded that Mill's choice of unit-elastic curves was useless in resolving questions of indeterminacy and multiple equilibria.

Mill's Paradox

Mill was successful, however, in using his unit-elastic schedules to demonstrate what Akira Takayama (1972, pp. 144–45) calls "Mill's paradox." That paradox states that a country's gains from trade decline as its resource endowment expands.

Let the offer curves be unit-elastic beyond the production-specialization points on the comparative-cost lines (see Figure 5). Suppose that prior to resource expansion the foreign curve initially cuts the home curve at its kink. The result is that the terms of trade coincide with the home country's cost ratio and the foreigner reaps all the gains from trade.

Figure 4 Unit-Elastic Reciprocal-Demand Curves

Foreign Exports
Home Imports

Unit-elastic curves are
horizontal and vertical in
the relevant range. They
intersect once. They display
no multiple equilibria or
indeterminacy.

Home Exports
Foreign Imports

Mill then assumes that resource expansion occurs in the foreign country. Such expansion, provided it raises the output of exportables more than it raises the foreigner's own demand for them, shifts upward his production (export-capacity) point and with it his offer curve. The resulting growth-augmented curve cuts the unit-elastic segment of the home country's curve, thus yielding a terms-of-trade deterioration for the foreigner. In the limit, growth continues until the terms of trade coincide with the foreigner's cost ratio and all trade gains accrue to the home country. Here is the rationale for Mill's statement that "the richest countries, ceteris paribus, gain the least by a given amount of foreign commerce" ([1848] 1909, p. 604).

Assessment

Overall, Mill's analysis must be judged one of the greatest contributions in the history of economics. It generalized classical value theory by shifting the emphasis from cost of production to equilibrium of demand and supply. True, Mill's predecessors occasionally acknowledged demand as a determinant of price. But they did so only for the singular case of nonreproducible goods in absolutely fixed supply. Mill now extended that analysis to cover labor-produced goods as well. He showed that even if cost determines the autarkic
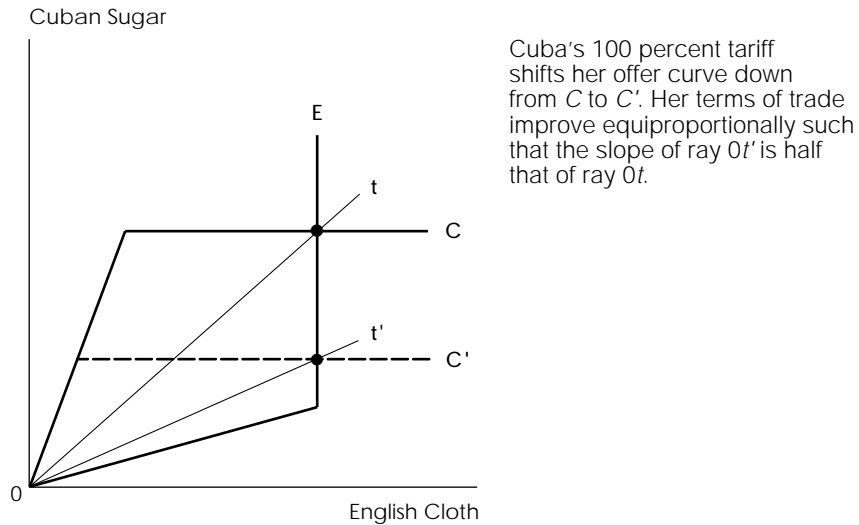
Figure 5   Mill's Paradox
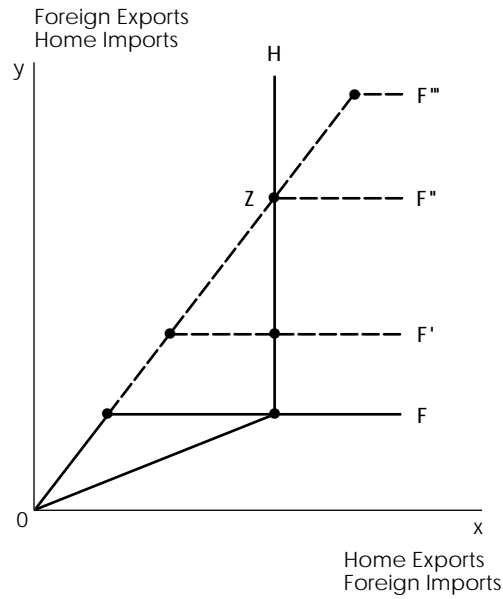
Growth of the foreigner's resource endowment shifts his offer curve upward. His terms of trade deteriorate until they eventually coincide with his cost ratio. Beyond point Z he ceases to specialize.

value of such goods, as Ricardo claimed, another principle, namely that of reciprocal demand, determines their international value. By distinguishing between cost-determined domestic prices and international prices determined jointly by supply and demand, he identified both blades of the Marshallian scissors. True, it remained for the neoclassical school to elaborate his insight into a full microeconomic theory of price determination. But Mill clearly pointed the way.

2.   ROBERT TORRENS

Mill had shown that with unit-elastic home demand for imports, the foreigner's trade tax improves his terms of trade equiproportionally with the tax. Two years before Mill published his analysis, Robert Torrens (1841–42) independently reached this same conclusion in a numerical example presented in his Postscript to Letter IX of *The Budget*. There he showed that a 100 percent import tariff, through its effect on reciprocal demands, improves the levying country's terms of trade by the same 100 percent. His example has the foreign country, Cuba, imposing the tariff on imports of English cloth. His assumption of unit-elastic reciprocal demands implies that Cuba's offer curve is horizontal and England's curve is vertical in the relevant range (see Figure 6).

Figure 6   Torrens' Tariff Model

Cuban Sugar

Cuba's 100 percent tariff shifts her offer curve down from *C* to *C'*. Her terms of trade improve equiproportionally such that the slope of ray 0*t'* is half that of ray 0*t*.

E

t

C

t'

C'

0

English Cloth

Start from the free-trade equilibrium. Cuba's imposition of the 100 percent tariff shifts her effective offer curve down to half its initial level. At the original terms of trade, there occurs an excess world demand for Cuba's export good, sugar, and a corresponding excess world supply of her import good, English cloth. To eliminate these excess demands and supplies, Cuba's terms of trade must improve—and England's deteriorate—by 100 percent. In the new, tariff-ridden equilibrium, Cuba imports the same initial amount of cloth at the cost of only half the initial amount of sugar given up. England, on the other hand, receives only half the initial amount of sugar at the cost of the same amount of cloth sacrificed. Torrens' conclusion: Foreign governments can, by means of tariffs, manipulate reciprocal demands to their advantage and thereby worsen the other country's terms of trade.

Having shown how the home country might lose from the foreigner's tariff, Torrens next used his reciprocal-demand schedules to argue for reciprocity in tariff removal (see Figure 7). He pointed out that the home country's unilateral abolition of tariffs would, in the face of their existence abroad, only worsen her terms of trade. He likewise noted that the home country's retaliatory duties would cancel the unfavorable terms-of-trade effects of foreign levies. Finally, he observed that the simultaneous imposition or removal of duties by all countries tends to leave the terms of trade unchanged. Like today's proponents of "a level playing field," he proposed that England counter foreign tariffs with

Figure 7   Tariff-Policy Reciprocity

Cuban Sugar

E

E'

t

C

1

t'

C'

3   2

0

English Cloth
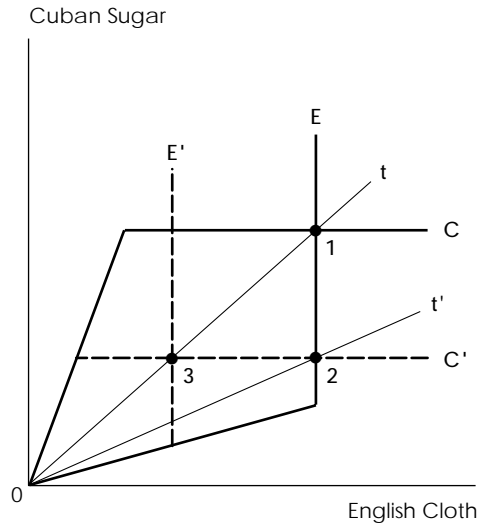
Cuba's unilateral tariff improves her terms of trade. Equilibrium moves from point 1 to point 2. England's reciprocal duty shifts her offer curve leftward and corrects her terms-of-trade deterioration (point 2 to point 3). Simultaneous tariff removal by both parties restores initial equilibrium without affecting the terms of trade (point 3 to point 1).

equal duties of her own, that she trade freely only with countries admitting her goods duty-free, and that she drop her tariffs only insofar as her trading partners abolish theirs.

Torrens' analysis was unsympathetically received by his classical contemporaries who feared it would undermine the case for free trade (see O'Brien [1975], pp. 194–97). They noted that tariff removal would hardly worsen England's terms of trade to the extent Torrens claimed if reciprocal-demand elasticities were, as they believed, greater than one.[7] Furthermore, they contended that any adverse terms-of-trade effects of moving toward freer trade would be more than offset by gains in productivity and competitiveness due to enhanced international specialization and division of labor. Finally, they noted that the gist of Torrens' analysis implied that England should levy not equal but higher retaliatory duties than those levied abroad to improve her terms of trade. They saw such action as intensifying the danger of a trade war with all parties

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[7] To Torrens' critics, high elasticities stemmed from the availability of numerous alternative goods and markets in the world economy. Such availability meant that a country could avoid tariffs levied by its trade partner. The country could divert production from taxed to nontaxed exports. Or it could export to third, tariff-free countries, competition from which would limit the tariff-imposing country's power to manipulate the terms of trade. In short, access to multiple export outlets and import sources rendered reciprocal demands extremely elastic with respect to price changes emanating from any single source.

losing. In response to the criticism that his analysis aided protectionists, Torrens rather disingenuously protested that he was a free trader merely applying the logic of the classical model.

3.   ALFRED MARSHALL

Without actually drawing the offer-curve diagram, Mill and Torrens had described the workings of its principal components. Indeed, Mill's account, in the words of Joseph A. Schumpeter (1954), "reads almost like a somewhat clumsy instruction for choosing these curves rather than others" (p. 609). It was Alfred Marshall who took the crucial step of translating Mill's instructions into geometry and thus invented the diagram that bears his name.

That Marshall was the first to draw the diagram is beyond dispute. What is disputed is the originality of his contribution. Did his trade diagrams do no more than merely "polish and develop Mill's meaning," as Schumpeter (1954, p. 609) claimed? Or were they "of such a character in their grasp, comprehensiveness and scientific accuracy" as to put them "far beyond the 'bright ideas' of his predecessors," as John Maynard Keynes ([1925] 1956, p. 24) thought? Marshall himself disclaimed originality by stressing the Millian pedigree of his diagrams. "As to International Trade curves," he wrote, "mine were set to a definite tune, that called by Mill" (Pigou [1925] 1956, p. 451). He dismissed his curves as nothing more than "a diagrammatic treatment of Mill's problem of international values" (Pigou [1925] 1956, p. 416).

Of the fourteen diagrams Marshall presented in his *Pure Theory of Foreign Trade* (1879), at least five appear to confirm Schumpeter's and Marshall's judgments. For they merely elaborate in elegant and compact geometry what Mill had already expressed in words and numerical examples. Certainly Mill would have found unexceptional the curves in Figure 8a just as he would Marshall's explanation of their convex (bowed in toward their respective axes) shapes and their positive slopes. Their convexity, Marshall held, captures the inverse demand relationship between import price and quantity. And their positive slopes indicate the normal case of elastic demands in which import-sales proceeds— and thus the quantity of exports produced with the aid of those proceeds—rise with import-quantity demanded.[8]

Nor would Mill have been surprised by Figure 8b. There Marshall depicts a case of inelastic import demand as manifested in a backward-bending offer curve. Mill would have agreed with Marshall that beyond point *B*, import-sales proceeds, and so the export volume they finance, must fall as import-quantity demanded rises. He had said much the same thing himself.

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[8] Marshall always assumed that the price-times-quantity sales receipts of importers pay for the cost of exports. Import receipts finance export production.

Figure 8   Marshall's Normal Class and Exceptional Class I Curves
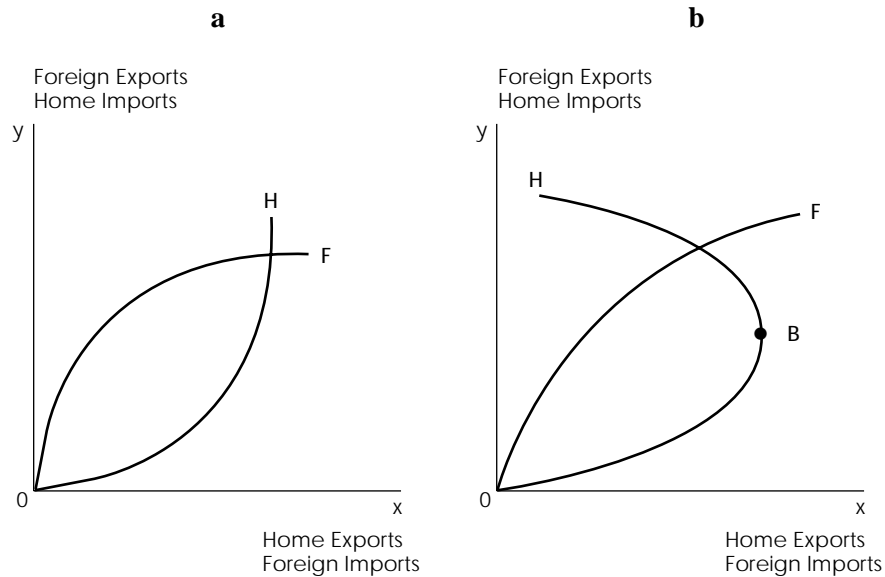
In panel a, both curves are elastic. Import and export quantities rise as the relative price of imports falls. In panel b, the home curve becomes inelastic beyond point *B*. Import quantity rises but export quantity falls with decreases in the relative price of imports.

Finally, Mill would have found Marshall's diagrammatic treatment of trade taxes totally unsurprising. Marshall showed that when both offer curves are elastic (provided the foreigner's is not infinitely so), tariffs and export taxes always improve the levying country's terms of trade. He also showed that when the levying country's curve cuts the foreigner's curve in its inelastic range, a trade tax yields a twofold gain (see Figure 9). The taxer's terms of trade improve. And, by obtaining a larger quantity of imports at the sacrifice of a smaller quantity of exports, the taxer has more of both goods to consume at home. A country lucky enough to face an inelastic foreign offer curve, said Marshall, has nothing to lose and everything to gain by exploiting it.[9] But Mill had already arrived at these conclusions. Thus Marshall's diagrammatic tax analysis goes little beyond Mill's work on that subject.

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[9] Marshall assigned a low probability to this case. He thought that (1) international competition, (2) countries' ability to shift production from taxed to nontaxed exports, and (3) the option of trading with third, free-trade nations rendered offer curves highly elastic. Levying countries were left with little scope for tariff-induced improvements in the terms of trade.

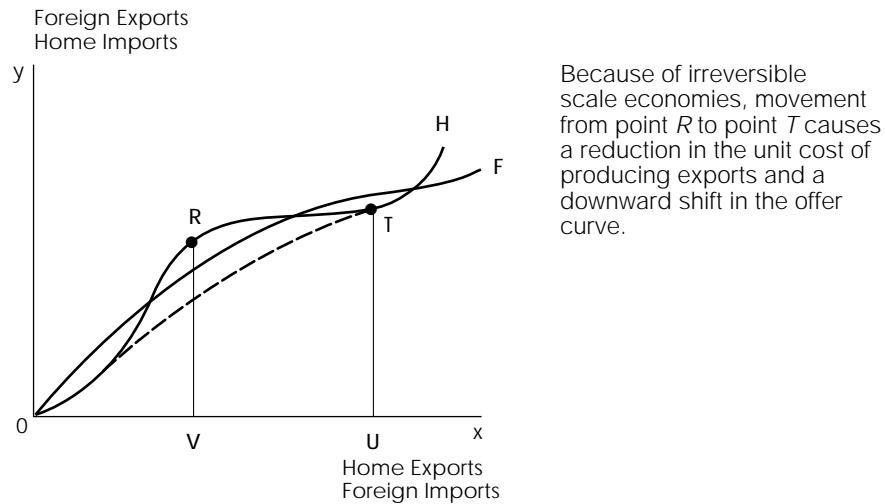**Figure 9   Exploiting an Inelastic Foreign Offer Curve**
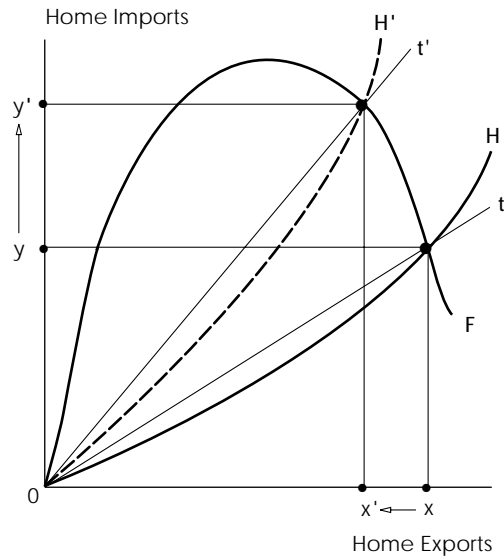
The home country imposes a trade tax that shifts her offer curve counterclockwise. Her terms of trade improve. And she obtains more imports at the cost of fewer exports given up and so has more of both goods to consume at home.

Scale Economies and Offer Curves

Figures 10 through 13, on the other hand, go far beyond anything Mill or Torrens had to offer. Figure 10 constitutes what John Chipman (1965) calls "the first fairly rigorous approach to the treatment of scale economies in international trade" (p. 738). It depicts Marshall's Exceptional Class II, in which economies in the production of exportables render the offer curve nonconvex and subject to irreversible downward shifts.

Let trade expansion move the home country from point R to point T on its offer curve. The resulting increased export production invokes scale economies associated with enhanced specialization and division of labor, with improved know-how (learning by doing), and with use of advanced technology and large machines. These economies in turn enable the larger quantity of exports to be produced at lower unit cost than the original quantity. Since unit-cost reductions pass through into product prices, it follows that scale economies cause export prices to fall from $RV/0V$ to $TU/0U$. Such economies account for the inflection points on the offer curve.

Moreover, once the scale economies are put in place, they and their associated cost reductions cannot be reversed even if output drops back to its original level. To capture such irreversible path-dependent effects, the offer curve shifts downward toward the export axis (see dashed line). In short, scale economies

Figure 10   Marshall on Scale Economies

Foreign Exports
Home Imports

Because of irreversible
scale economies, movement
from point *R* to point *T* causes
a reduction in the unit cost of
producing exports and a
downward shift in the offer
curve.

Home Exports
Foreign Imports

constitute a form of technical progress that shifts the offer curve simultaneously with movements along it. Here was a novel element in offer-curve analysis.

Uniqueness and Stability of Equilibrium

Further confirming Marshall's originality was his analysis of uniqueness and stability of offer-curve equilibrium. Indeed, Murray Kemp (1964) calls this analysis one of "the most remarkable contributions ever made to theoretical economics" (p. 66). Regarding uniqueness, Marshall noted that there can be but one equilibrium when both curves are positively sloped and possess no inflexion points (see Figure 11a). Equilibrium is likewise singular when one curve is elastic and the other inelastic in a certain range (see Figure 11b). If, however, both curves are inelastic (Figure 11c), or if at least one contains inflexion points (Figure 11d), multiple equilibria may result. Such equilibria, according to Marshall, are always odd in number. Moreover, they are alternately stable and unstable, with the stable equilibria flanking the unstable ones (see Figures 11c and d in which stable equilibrium points *A* and *C* flank the unstable point *B*).

   As for stability of equilibrium, Marshall analyzed it with phase diagrams superimposed on his offer curves. His phase diagrams—the first ever used in print by an economist—treat points off the curves as disequilibrium phenomena produced by random real shocks such as wars, harvest failures, and the like. For any given disequilibrium trading point, Marshall sketched the dynamic

Figure 11   Single and Multiple Equilibria
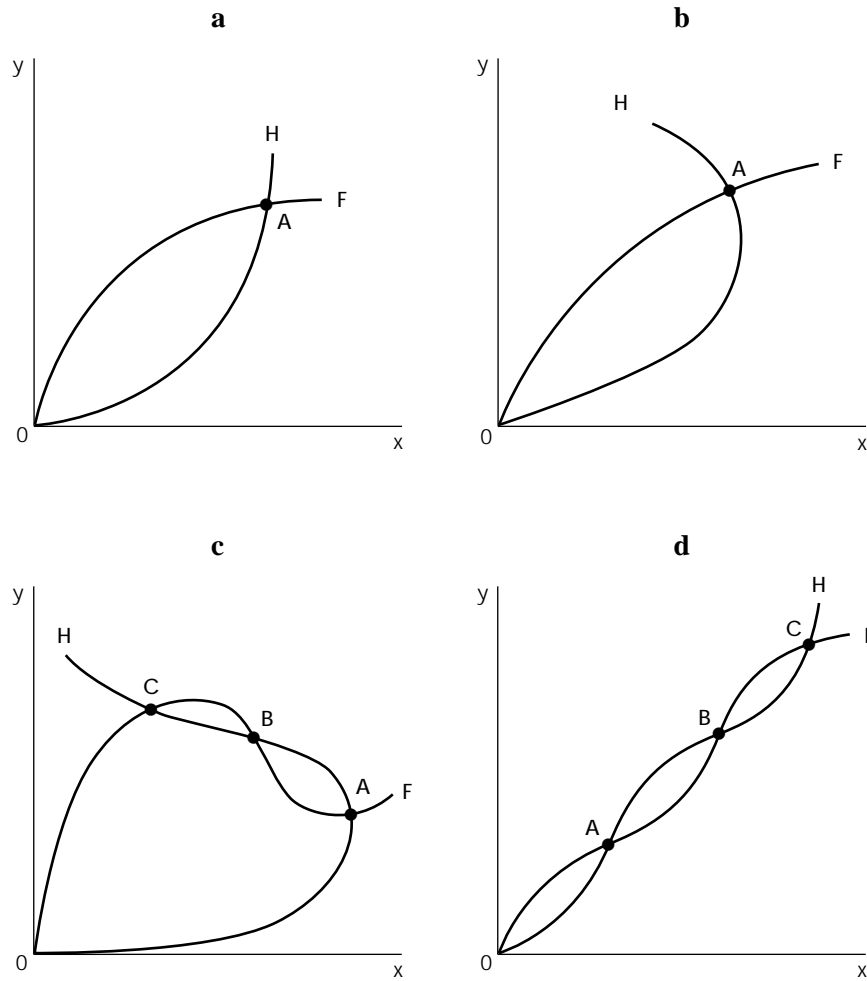
Panels a and b display cases of unique (singular) equilibrium. Panels c and d display multiple equilibria, with the stable equilibria (*A* and *C*) bounding or bracketing the unstable one (*B*).

adjustment mechanism that moves the point toward equilibrium. To him, the propelling force consisted of the profitability of expanding production of exports when they are in short supply.

Consider trading points to the left of the home country's curve and below the foreigner's curve. Such points represent shortfalls of actual quantities of

Figure 12   Marshall's Disequilibrium Mechanism

Foreign Exports
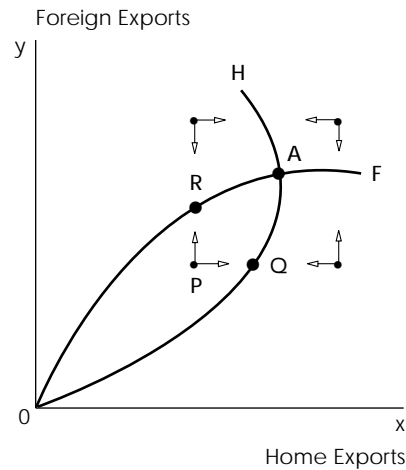
The shortfall of actual below desired exports at trading point *P* stimulates production of exports and so moves that point toward equilibrium.
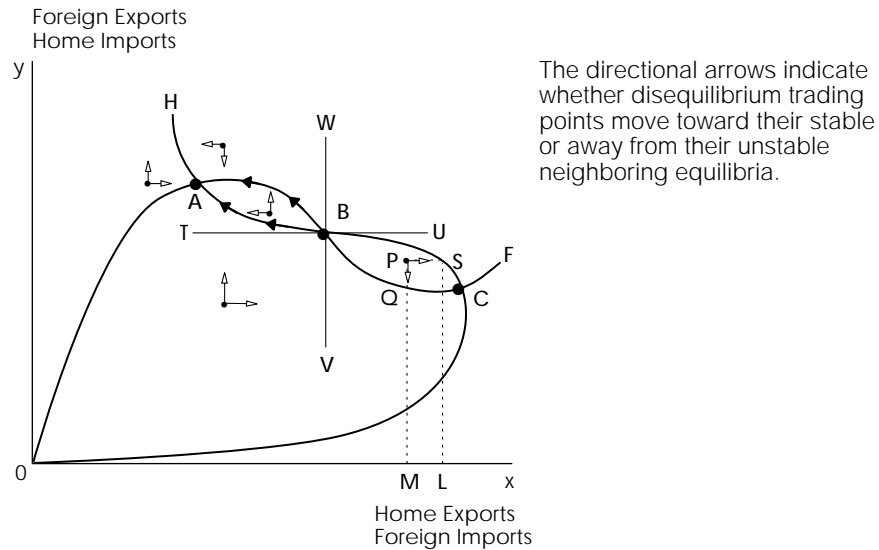
Home Exports

exports below quantities the countries are willing to offer (see Figure 12). These shortfalls render exports extraordinarily profitable and induce competitive producers to produce more. The resulting export expansion moves the trading point toward the curves just as—to use Marshall's analogy—the force of magnetic attraction moves metal filings toward a rigid wire. The arrows point rightward and upward to indicate the trading point's movement.

Conversely, disequilibrium trading points to the right of the home country's curve and above the foreigner's curve spell surpluses of actual over desired exports. The resulting losses bring declines in export production as shown by the leftward and downward direction of the arrows.

In all cases, the directional arrows indicate whether trading points will move away from or toward their neighboring equilibria (see Figure 13). On the basis of such analysis, Marshall concluded that every equilibrium intersection is stable except those in which (1) both curves slope in the same direction and (2) the foreign curve is more nearly vertical than the domestic one (see Amano [1968], pp. 327–28).

Surplus Analysis of Gains from Trade

Further proof of Marshall's originality is his diagrammatic treatment of the gains from trade. In a straightforward application of his concept of consumer surplus, he expressed such gains as the excess of the maximum prices a country would be willing to pay for successive units of imports over the market price, or terms of trade, it actually pays. Accordingly, he devised a technique

Figure 13   Marshall's Phase Diagram

Foreign Exports
Home Imports

The directional arrows indicate whether disequilibrium trading points move toward their stable or away from their unstable neighboring equilibria.

Home Exports
Foreign Imports

for projecting from the offer curve a series of unit surpluses into a triangle resembling the area lying between an ordinary Marshallian demand curve and the price line (see Figure 14). Expressed in terms of export quantities, the resulting triangular area UHA' sums the excesses of the maximum unit prices over the prevailing terms of trade shown by the slope of the ray through the trade equilibrium point A. This was his measure of the net benefit a country derives from trade.[10]

Assessment

It should be obvious by now that Keynes was right. Marshall's diagrams were more than a mere refinement of Mill's analysis. They were a major innovation and a powerful aid to theorizing. The mystery is why Marshall himself refused to acknowledge as much. Perhaps a desire to stress the intellectual continuity of trade theory led him to disguise his contribution modestly as part of the accumulated wisdom of his classical predecessors. Or perhaps his reluctance to claim originality for his diagrams stemmed from a puritanical sense of guilt over the pleasure he derived from them. Jacob Viner (1941) writes that mathematics,

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[10] Actually, Marshall divided the surplus triangle by the distance $0D$ to correct for the arbitrary choice of point D. That point fixes the location of the vertical line used in projecting unit price surpluses onto the quasi-demand curve $UP'A'$.

Figure 14   Surplus Measure of the Gain from Trade

The maximum price the country is willing to pay for successive units of imports ranges from the slope of the top ray to the slope of the bottom one. A mapping of the excess of these unit prices over the equilibrium terms of trade forms the surplus triangle *UHA'*. The net gain from trade is measured by the triangle *UHA'* divided by the distance 0*D*, which corrects for the arbitrary choice of the location of the projection line *DR*.

especially geometry, yielded Marshall "so much intellectual and aesthetic delight that it for that reason alone became somewhat suspect to him as a worthwhile occupation. Mathematics, and especially graphs, were Marshall's fleshpots, and if he frequently succumbed to their lure it was not without struggle with his conscience. . . . [W]hen he did succumb he . . . warned his readers not to take his mathematical adventures too seriously" (p. 231). Keynes agreed. He pointed out that when Marshall's "intellect chased diagrams and Foreign Trade and Money, there was an evangelical moralizer of an imp somewhere inside him, that was so ill-advised as to disapprove" ([1925] 1956, p. 37). But disapprove Marshall did and in so doing disclaimed originality for his invention.

4.   FRANCIS YSIDRO EDGEWORTH

As sophisticated as they were, Marshall's offer curves lacked clear grounding in the underlying utility functions. Credit for establishing these foundations and for introducing utility considerations into the diagram goes to F. Y. Edgeworth, Marshall's colleague and the inventor of the indifference map and the contract curve.

Edgeworth's earliest work on the diagram appears on pages 113–14 of his *Mathematical Psychics* (1881). There he derived offer curves for two representative price-taking individuals. Each possesses (1) a fixed endowment of

goods and (2) a utility function described by a consumption indifference map. Like trade theorists today, Edgeworth defined each individual's offer curve as the locus of points of tangency of indifference curves and the price ray as it pivots about the origin (see Figure 15). Each point represents an outcome of constrained utility maximization in which the price ratio, or slope of the price ray, equals the ratio of marginal utilities, or slope of the indifference curves.

In the same diagram, he demonstrated that the offer curves must intersect on the contract curve, or locus of points at which one trader's indifference curves are tangent to the other's. Along the contract curve, neither trader's utility can be increased without reducing the other's. In demonstrating that offer curves intersect on the contract curve, Edgeworth proved that price-taking equilibrium is efficient in the sense that both traders together cannot be made better off by another outcome. Moreover, since the equilibrium outcome lies between the two indifference curves passing through the origin, or endowment point, where no trade occurs, he also proved that free trade leaves each party at least as well off as no trade.

He next extended this latter insight to measure a single country's gain from trade. In so doing he provided an alternative to Marshall's measure of the gain. In his 1889 *Journal of the Royal Statistical Society* article, "On the

Figure 15   Traders' Price-Taking Equilibrium

Offer curves intersect on the contract curve *CC'*. At free-trade equilibrium, both traders occupy indifference curves superior to those going through the origin or endowment point where no trade occurs.
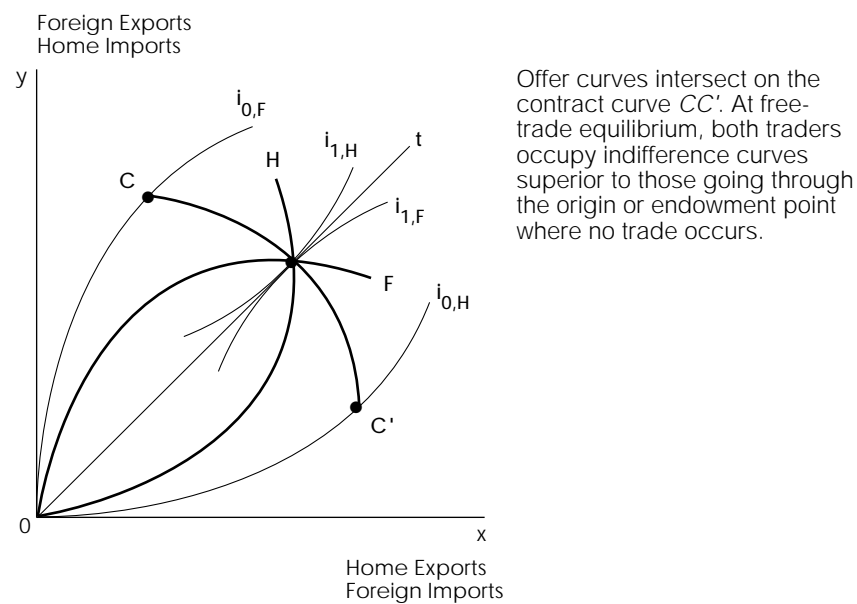
Application of Mathematics to Political Economy," he drew a community trade indifference curve passing through the origin (see Figure 16). This curve, of course, shows all combinations of exports and imports that leave the country no better off than if it refrained from trade. The vertical distance between this "no gain from trade" curve and the country's offer curve—or rather the indifference contour going through it—at free-trade equilibrium measures the utility gain from trade.

Figure 16 Utility Gain from Trade
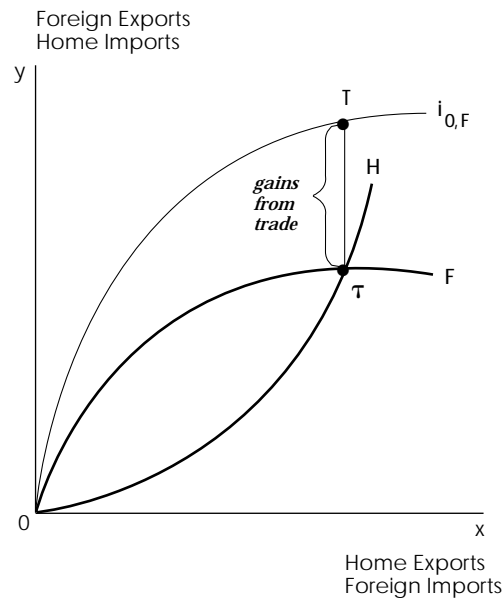
The vertical distance between the trade equilibrium point and the foreigner's "no gain from trade" indifference curve–the particular curve passing through the origin–constitutes the foreigner's utility gain from trade.

Optimum-Tariff Analysis

The foregoing pathbreaking innovations were but a prelude to Edgeworth's crowning achievement—his geometrical demonstration of the optimum-tariff argument. An optimum tariff, of course, maximizes the excess of the gains from terms-of-trade improvement over the loss from lower trade volume and reduced international specialization and division of labor. The idea itself goes back to Mill and Torrens. They had argued that a large country caring only for its own welfare and facing an imperfectly elastic foreign demand schedule could exploit its monopoly power in world markets through such a tariff. But rigorous diagrammatic illustration of the argument was lacking until Edgeworth provided it in his 1894 *Economic Journal* article, "The Theory of International Values, II."

Figure 17   The Optimum Tariff

Foreign Exports
Home Imports
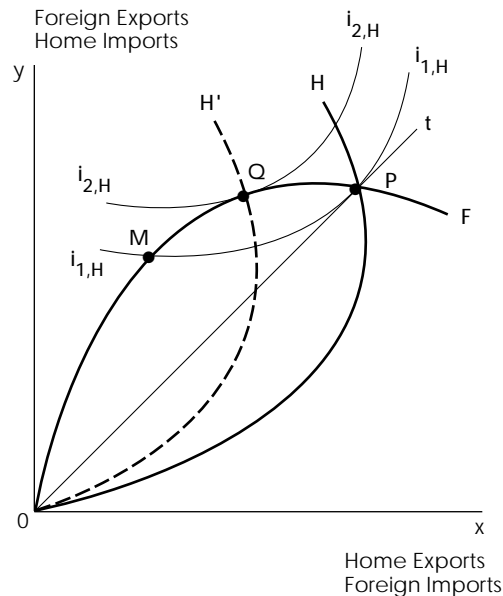
The optimum tariff puts the
levying country on the point
where its highest attainable
trade indifference curve is
tangent to the foreigner's
offer curve.

Home Exports
Foreign Imports

His demonstration begins with the home country's trade indifference curve
passing through the free-trade point where the offer curves intersect (see Figure
17). That particular indifference curve indicates the level of welfare or satisfac-
tion the home country enjoys under free trade. It provides a benchmark against
which to compare alternative welfare levels produced by different degrees of
trade restriction.

It also, together with the foreign offer curve, specifies the range of tariff
rates beneficial to the home country. In this connection, Edgeworth noted that
the same indifference curve that passes through the free-trade point P also
cuts the foreign offer curve at point M. That latter point therefore yields the
same level of welfare as free trade. Since all points on the foreign offer curve
between these two extremes lie on higher indifference curves, it follows that
any movement to a position between points P and M will result in the home
country being better off than under free trade. In other words, points P and M
mark the range of tariff-induced terms-of-trade improvement beneficial to the
home country. Somewhere within this range, benefit is at a maximum.

Edgeworth identified this maximum with point Q, the point where the home
country reaches its highest possible trade indifference curve given the foreign
offer curve. The optimum tariff, he argued, is that which distorts the home

country's offer curve such that it intersects the foreign offer curve at this point of tangency with the highest attainable indifference curve. Here is the famous tangency solution to the determination of the optimum tariff.

Edgeworth then showed that if the tariff is too high, it reduces rather than increases welfare. Suppose the country progressively raises its tariff from the zero rate corresponding to point *P* to positive rates corresponding to points *Q* and *M*. As it does so, it finds that its welfare first rises, then reaches a maximum, and finally starts to fall. If it persists in raising the rate beyond that corresponding to point *M*, it will discover that its welfare has fallen below the level attained at the free-trade position *P*. It follows that the tariff must not be too large if the nation is to benefit.

Finally, Edgeworth noted some pitfalls to the practical application of his diagram. First, the optimum tariff, though precisely identified in theory, cannot be ascertained with any accuracy in practice. Second, protectionists will exert strong political pressure on policymakers to raise tariffs far beyond the optimum point, thereby reducing welfare. Third, retaliation by foreign countries may erase any gains generated by the tariff. Fourth, viewed from a global standpoint, tariffs are harmful since other countries lose more than the levying country gains. For these reasons, free trade remains the best and most practical policy for a country to pursue.

Alleged Asymmetry of Export and Import Taxes

No scholar is infallible, not even one of Edgeworth's stature. In the very same *Economic Journal* article containing his optimum-tariff demonstration, Edgeworth (1894b) committed a celebrated error. He rejected the standard proposition that export and import taxes are equivalent in the sense of having identical real effects. Other leading classical and neoclassical theorists, including Mill, Marshall, A. C. Pigou, and C. F. Bastable, took such equivalence for granted. But Edgeworth alleged that the two taxes shift the dutying country's offer curve differently and therefore have disparate real effects.

According to Edgeworth, export taxes shift the curve horizontally to the left. But import taxes shift it vertically upward such that it lies everywhere above the original curve. The result is that the tax-ridden curves intersect the foreign offer curve at different points, especially when both foreign and domestic curves are in their inelastic ranges (see Figure 18). In such cases, the export tax-ridden equilibrium lies to the northwest of the free-trade point so that the levying country is on a higher indifference curve with better terms of trade. By contrast, the import tax-ridden equilibrium lies to the southeast of the free-trade point, putting the country on a lower indifference curve with worsened terms of trade. Such was Edgeworth's allegation.

It took 42 years to identify and correct Edgeworth's error. Abba Lerner finally did so in his classic 1936 paper, "The Symmetry Between Import and

**Figure 18 Alleged Asymmetry of Export and Import Taxes**
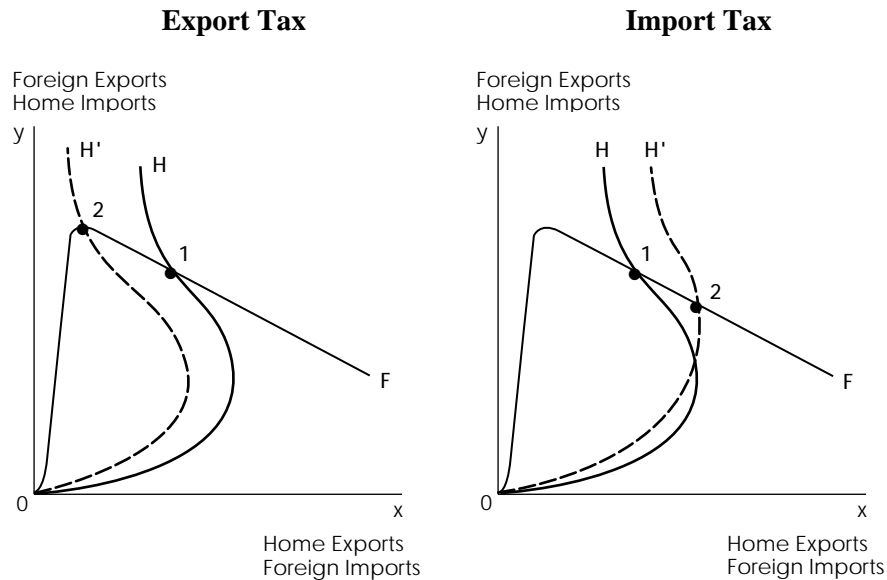
Edgeworth alleged that an export tax shifts the home country's offer curve horizontally to the left whereas an import tax shifts it vertically. As a result, the export tax moves the home country to a superior position (better terms of trade and a higher indifference curve), whereas the import tax moves her to an inferior position.

Export Taxes." There Lerner argued, contrary to Edgeworth, that export and import taxes indeed affect the offer curve identically. They thus have symmetrical effects on the volume and the terms of trade. Differential effects stem not from the taxes per se. Rather they stem from how the government disposes of the revenue. The greater the proportion spent on the levying country's export good, the greater the improvement in its welfare and terms of trade. Conversely, the greater the proportion spent on imports, the smaller the improvement. Edgeworth's first result obtains when all the proceeds are spent on export goods; his second when all are spent on imports. His error lay in confusing these expenditure effects for tax effects. What he saw as differential results of trade taxes were really outcomes of how the government spent the revenue.

5. ABBA LERNER

Lerner (1936) established the foregoing results by means of the ingenious device of a geometrical *pencil,* or wedge, superimposed on the offer curves (see Figure 19). Consisting of two price radiants, the pencil expresses the tax-induced divergence between world and domestic relative prices. Its width shows the rate of the tax. Its location on the offer curves depicts the government's apportionment of the proceeds between exportable goods and imports. And its position around the free-trade price ray shows how tax imposition and disposal affects the terms of trade and domestic relative prices. Finally, the pencil embodies the symmetry notion that export taxes are equivalent to import taxes of

Figure 19   Trade-Tax Pencil

Foreign Exports
Home Imports
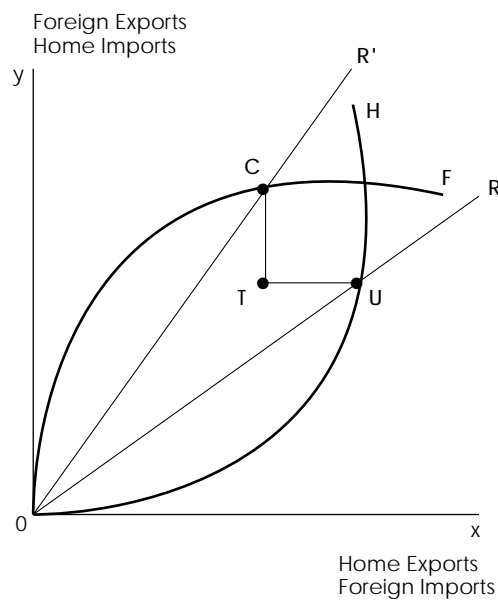
The pencil *R′* 0*R* represents the tax wedge between world and domestic relative prices. The pencil's dimensions are the same whether the tax is levied on imports or exports. The arms of right angle *CTU* correspond to the proportions of the tax revenue that the government spends on the two goods.

Home Exports
Foreign Imports

the same percentage rate. Since both taxes produce the same divergence between world and domestic prices in a two-good model, the pencil's dimensions are the same measured in either tax.[11] What matters is not which good is taxed but how the government disposes of the tax proceeds.

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[11] A tax on imports renders them dearer at home than abroad. By contrast, a tax on exports raises their foreign price, thus making them cheaper at home than abroad. But a fall in the relative price of exports is equivalent to a rise in the relative price of imports in Lerner's two-good model. Hence, an export tax raises the domestic real price of imports above the world price just as does an import tax. The two taxes are symmetrical.

Lerner's demonstration of this point was at once seminal and definitive. Inserting the pencil into the offer curves, he obtained a right angle *CTU* connecting the points of entry of the pencil's radiants. This right angle has a special meaning. Its vertical and horizontal arms measure the world excess supplies of the two goods resulting from the tax. Market-clearing equilibrium requires that the government eliminate these excess supplies by consuming them in the proportion in which they occur. That is, world market equilibrium obtains when the ratio of the lengths of the right angle's arms matches the ratio in which the government consumes the two goods.

Lerner took this latter ratio as given and known. Then he found equilibrium by pivoting the pencil about the origin until the matching right angle appeared. For example, suppose the government spends the tax proceeds equally on exports and imports. Then, following Lerner, swing the pencil until it yields a right angle whose arms are of equal length (see lines *CT* and *TU* in all panels of Figure 20). Alternatively, suppose the government spends all the proceeds on exportables such that the right angle reduces to a horizontal line. Then rotate the pencil counterclockwise until it yields a flat line traversing the pencil and meeting the offer curves (see lines $C_1 U_1$ in all panels). Finally, suppose the government spends all the proceeds on imports so that the right angle reduces to a vertical line. Then swivel the pencil clockwise until it yields a vertical line between the offer curves (see lines $C_2 U_2$ in all panels). In each case, Lerner examined the resulting location of the pencil's radiants relative to the free-trade price ray passing through the point of offer-curve intersection. These indicate how the disposition of the tax affects the terms of trade and domestic price ratio, respectively. Radiants to the left of the free-trade ray represent a fall and those to the right a rise in post-tax relative prices.

Standard Tariff Propositions

Employing this technique, Lerner derived four key propositions of standard tariff theory. His derivation marks a turning point in the diagram's history. Before him, the diagram was largely regarded as an esoteric tool employed by a select circle of economists. After him, it was seen as a conventional instrument and widely used. His work, more than any other, convinced the economics profession of the diagram's power and versatility as an analytical tool.

His standard propositions are as follows. First, provided both offer curves are elastic but not infinitely so, a trade tax, no matter how spent, improves the levying country's terms of trade and raises the domestic relative price of its imports (see Figure 20a). In other words, the radiants of the pencil encompass the free-trade price ray.

Second, regardless of elasticities, a tariff improves the terms of trade more (or worsens it less) the larger the fraction of the tax spent on the country's exportable good. Thus, the pencil's upper radiant lies more to the left (or less

Figure 20   Lerner's Trade-Tax Analysis
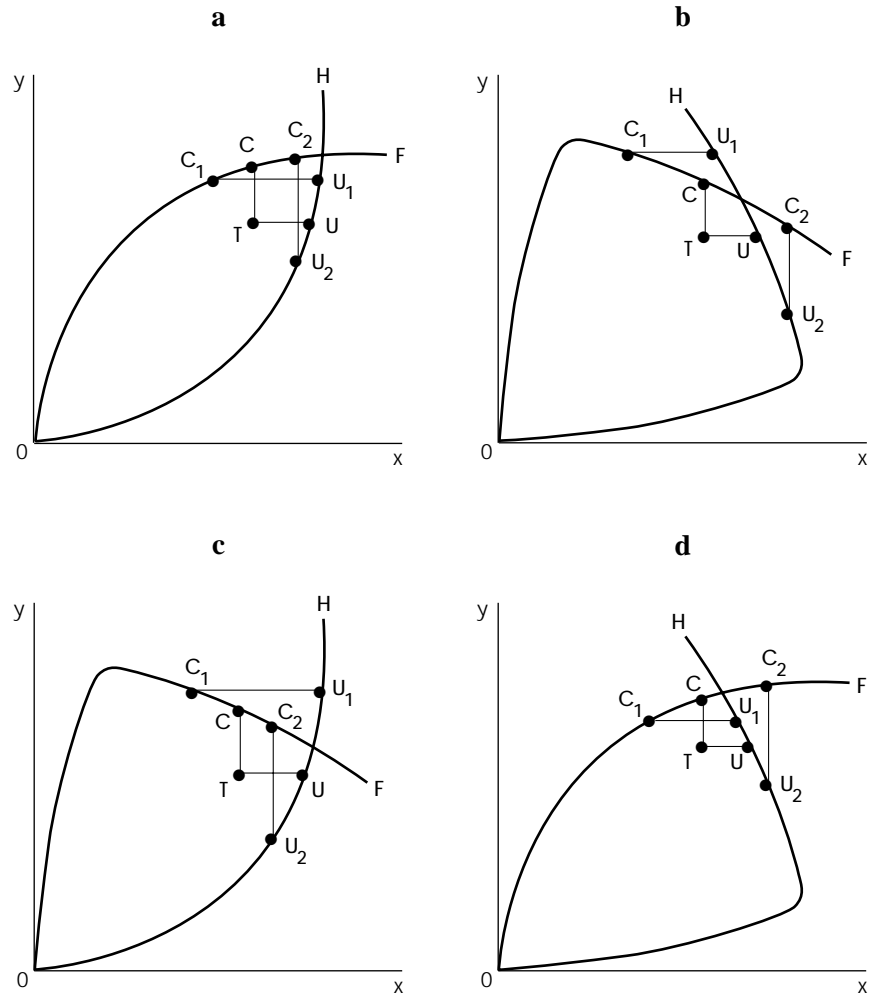
The pairs of *C-U* points show where the upper and lower radiants of the pencil enter the offer curves. Pairs C_1-U_1, *C-U*, and C_2-U_2 correspond to tax proceeds allocated all, some, and none, respectively, to exportables. Taxes and their disposal improve the terms of trade when radiants through the *C* points lie to the left of the free-trade ray (not shown) that passes through the point of offer-curve intersection. Taxes and their disposal raise the domestic relative price of imports when radiants through the *U* points lie to the right of the free-trade ray.

to the right) of the free-trade ray as it passes through points C_1, C, and C_2, representing export expenditure shares of one, half, and none, respectively (see Figure 20, all panels).

Third, suppose the taxing country spends all the tax on imports and possesses an inelastic import demand, or backward-bending offer curve. In this case, a trade tax actually worsens the terms of trade. Geometrically, the radiant passing through point C_2 lies to the right of the free-trade ray (see Figure 20b and d). Indeed, one can relax the assumption that all the tax is allocated to imports. Lerner's result holds as long as the taxer's import-demand elasticity is less than the fraction of the proceeds spent on imports.[12]

Fourth, assume the home country faces an inelastic foreign offer curve and spends its tariff proceeds largely on its exportable good. In this case, a tariff may improve the terms of trade by more than the tariff such that the domestic price of imports falls below its free-trade level. Geometrically, the radiant passing through point U_1 lies to the left of the free-trade ray (see Figure 20b and c). If so, the tariff achieves the opposite of its intended purpose. By lowering the domestic price of imports, it harms rather than protects domestic import-competing industries and the relatively scarce factors they employ intensively. Today, textbooks attribute this paradoxical result to Lloyd Metzler. He proved, in 1949, that it holds when the dutying country's marginal propensity to spend the tariff proceeds on its own export good exceeds the foreigner's elasticity of demand for that good.[13] But it was Lerner, not Metzler, who first established this proposition.

6.   CONTROVERSIES IN THE 1920s AND 1930s

Offer curves also constituted the focus of Frank Graham's (1923, 1932) celebrated critique of Marshall's work. Graham's critique raised issues not fully resolved until the 1950s.

The first issue concerned the effects of demand-induced shifts in the home country's curve. In Appendix J of his 1923 *Money, Credit and Commerce*, Marshall analyzed such shifts stemming from autonomous increases in import demand. He argued that the resulting extent of terms-of-trade deterioration would vary directly with the home import-demand elasticity and inversely with

[12] A high import-expenditure fraction augments the demand for imports and tends to raise their relative price. But a low elasticity spells little offsetting fall in import-quantity demanded in response to the higher price. The resulting excess demand for imports raises their price and causes the terms of trade to deteriorate.

[13] The government's high propensity to spend on its exportable puts upward pressure on the domestic (and world) price of that good. But the low foreign demand elasticity militates against offsetting falls in quantity demanded abroad in response to the higher price. The net result is an excess demand for exportables. This excess demand raises the relative price of exports and lowers its inverse, the relative price of imports.

the foreign one. Similarly, he thought the accompanying degree of expansion in trade volume would vary directly with both elasticities.

Graham, however, disagreed. He thought that the extent of terms-of-trade deterioration would vary inversely with both elasticities. He also thought trade-volume expansion would vary directly with the foreign elasticity but inversely with the home elasticity.

How could two leading economists differ over something as elementary as the effects of shifts of offer curves? Murray Kemp supplied the answer in 1956. The disagreement stemmed from ambiguity of the phrase "increase in reciprocal demand." More precisely, it stemmed from Marshall's failure to state explicitly the type of shift postulated. It turns out that he implicitly posited horizontal shifts due to increases in the quantity of exports offered against a given quantity of imports. By contrast, Graham posited equiproportional or radial shifts due to increases in the quantity of exports offered at given terms of trade. Both were right in terms of their own implicit definitions. Still, the controversy taught the economics profession a lesson. Elasticity affects the extent to which demand shifts alter price and quantity. Exactly how it does so depends on the precise definition of such shifts (see Bhagwati and Johnson [1960], p. 78).

The second issue concerned the link between offer curves and the underlying production conditions. Graham accused Marshall, Mill, and their followers of neglecting these conditions and overemphasizing demand. But this accusation was hardly fair since Marshall and the others always viewed the offer curve as embodying an exhaustive classification of all its determinants, supply as well as demand. True, Edgeworth initially derived Marshallian curves for a pure exchange economy involving no production. But he later explicitly acknowledged underlying changes in production in his famous analogy comparing Marshall's offer curves to the hands of a clock driven by the workings of a complex but hidden machinery.

The full revelation of the machinery, however, had to wait for the famous demonstrations of Leontief (1933) and Meade (1952). Both derived offer curves from production transformation frontiers (expressing supply conditions) and consumption indifference curves (expressing demand conditions).

Leontief's derivation was the simplest. He superimposed a trading country's consumption indifference curves directly on its transformation curve. He then assumed alternative international price ratios represented by negatively sloped lines. Tangency of such lines with the transformation and indifference curves gave him the quantities of the two goods produced and consumed at each price ratio. The excess of production over consumption of the one good and of consumption over production of the other at each price ratio constituted export-import bundles lying on the offer curve.

Meade, on the other hand, derived offer curves in two stages (see Figure 21). First, he slid a transformation curve, or production block, along a succession of consumption indifference curves. The origin of the block traced out a set

Figure 21   Meade's Derivation of the Offer Curve
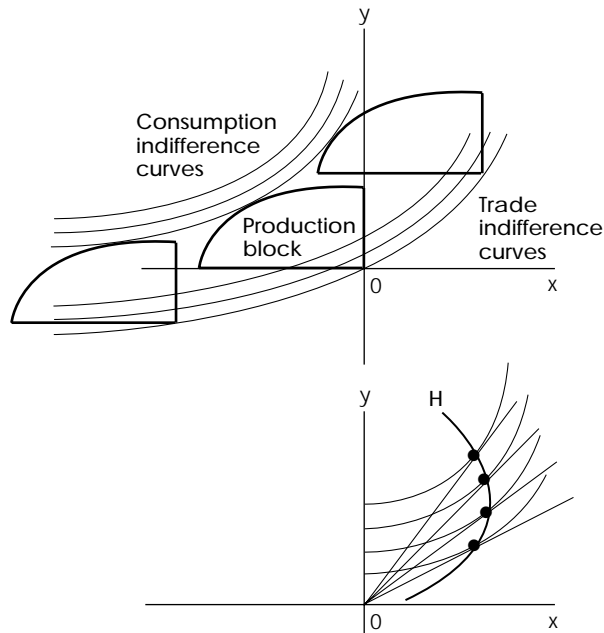
Meade slides the production block along each consumption indifference curve. The block's origin traces out a set of trade indifference curves. He then derives the offer curve as the locus of the points of tangency of the trade indifference curves and the price line as it swivels about the origin.

of trade indifference curves, each curve showing alternative export-import bundles that yield the same level of collective satisfaction. From these trade indifference curves he derived offer curves just as Edgeworth had done. He found the locus of points of tangency of trade indifference curves and alternative price rays emanating from the origin. This locus constituted the offer curve. In deriving the curves from the underlying production conditions, Leontief and Meade vindicated Marshall and exonerated him from Graham's charge.

7.   CONCLUSION

Historically, the offer-curve apparatus has been put to two uses. Modern analysts employ it as a pedagogical or expository device to illustrate established truths. By contrast, the concept's originators applied it as an analytical tool to derive new propositions and postulates. They used it to generate key theorems on the gains from trade, on the efficiency of free-trade equilibrium, on the effects of tariffs and technological change on the terms of trade, and on the specification of the optimum rate of a tariff. That they were able to do so using nothing more sophisticated than numerical examples and geometrical diagrams shows what keen minds can accomplish with the simplest of analytical tools.

Their successive accomplishments typify the workings of normal science wherein the drive to perfect an existing paradigm propels advances in theory. In their case, the paradigm consisted of the Mill-Marshall model of terms-of-trade determination. Perfecting it meant (1) making it more precise, (2) generalizing it to cover the widest possible range of cases, and (3) purging it of errors and inconsistencies.

Offer-curve pioneers were more than up to these tasks. Thus Mill's and Torrens' concept of reciprocal demand expunged terms-of-trade indeterminacy from Ricardo's analysis. Mill generalized Torrens' unit-elastic demand schedules to include elasticities ranging from zero to infinity. Marshall generalized Mill's model to cover cases of (1) multiple as well as singular equilibrium and (2) nonconstant as well as constant costs. Edgeworth's invention of indifference maps and the contract curve lent precision to Marshall's concept of the gains from trade. Lerner's innovation, the tax pencil, helped correct Edgeworth's error regarding the symmetry of trade taxes. Finally, Leontief and Meade extended the entire apparatus to include production as well as preference functions.

The result was that offer curves became a fixture of trade theory and a commonplace of textbooks. The survival of the concept testifies to its continued usefulness. Modern students owe the originators of this tool a debt of gratitude. Even today, if one understands the diagram, one comprehends how various disturbances—technology shocks, resource discoveries, taste shifts, erection and removal of trade barriers, and the like—affect the volume and terms of world trade.

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