Stark Optimal Fiscal Policies and Sovereign Lending

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Capital income taxes are a salient feature in the taxation schemes of many modern countries, though countries make distinctions between capital gains and income earned by capital. Most countries include rents received on capital in the income calculation for each taxpayer. When the rates are averaged over the 1965–1996 period, average capital income tax rates can be moderately high in industrially advanced nations, ranging from 24.1 percent in France to 54.1 percent in the United Kingdom. When averaged across the first six years of the 1990s, average capital income tax rates for the same countries are 25 percent and 47.7 percent, respectively. Over the same periods, the United States had average capital income taxes of 40.1 percent (1965–1996) and 39.7 percent (1990–1996) (see Domeij and Heathcote 2004).

Following the work of Judd (1985), and Chamley (1986), much of the literature on optimal taxation has argued that it is not efficient to tax capital in the long run. As shown in Atkeson, Chari, and Kehoe (1999), this policy prescription is relatively robust in the sense that it holds whether agents are heterogenous or identical, the economy's growth rate is endogenous or exogenous, and the economy is open or closed.[1] At the same time, however, the notion that long-run capital taxation is inefficient arises in settings where

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[1] Exceptions to this recommendation include Correia (1996), and Jones, Manuelli, and Rossi (1997), who show that the optimal long-run tax on capital differs from zero when other factors of production are either untaxed or not taxed optimally. As pointed out in Erosa and Gervais (2001), capital income taxes in an overlapping generations environment are not just distortionary, they involve some redistribution among agents. Hence, optimal steady state capital income taxes need not be zero in such a framework, as shown in the early work of Atkinson and Sandmo (1980), and later Garriga (1999), as well as in Erosa and Gervais (2002) with age-dependent taxes. Finally,

optimal policies are often extreme at shorter horizons. For instance, when the capital stock is sufficiently large and no restrictions are placed on the capital income tax, it is optimal for the government to raise all revenues through a single capital levy at date 0 and never again tax either capital or labor. For that reason, Chamley (1986) imposes a 100 percent exogenous upper bound on the capital income tax, which Chari, Christiano, and Kehoe (1994) show can be motivated by assuming that households have the option of holding onto their capital, subject to depreciation, rather than renting it to firms. With an upper bound imposed on capital income tax rates, optimal fiscal policy continues to be surprisingly stark, with the optimal capital tax rate set at confiscatory levels for a finite number of periods, after which the tax takes on an intermediate value between 0 and 100 percent for one period, and is zero, thereafter. This article points out that government lending to households, which is seldom observed in practice, plays a crucial role in generating extreme optimal fiscal policies. Absent a domestic debt instrument, more moderate capital and labor tax rates emerge as optimal, although the capital tax rate does converge to zero, asymptotically.

Without an upper bound imposed on capital income taxes, it is easy to see why a single capital levy at date 0 is optimal in the environment studied by Chamley (1986). Since the capital stock is fixed at date 0, the initial capital levy amounts to a single lump sum tax and no distortions are ever imposed on resource allocations over time. The economy, therefore, achieves a first-best optimum. That said, the presence of a debt instrument plays a key role in the implementation of a single initial capital levy. In particular, such an instrument allows the government to front-load all taxes in the initial period (equal to the net present discounted value of future government expenses), lend the proceeds to the private sector, and finance government expenditures from interest revenue on the loans. Thus, Chamley's model never generates government debt but generates government surpluses which are lent back to households.

Although Chamley's single initial capital levy allows for first-best allocations to be achieved, such a taxation scheme has evidently little to do with the kinds of policies one observes in practice. In particular, almost all countries continue to rely on distortional tax systems to finance their public expenditures. This article highlights the fact that environments that limit ex ante government lending are more apt to generate nonzero optimal distortional taxes at all dates. In particular, we provide an analysis of Chamley's taxation problem without a policy instrument that allows for sovereign lending. In that case, the government cannot carry the proceeds from a large initial tax to future

see Aiyagari (1995) for an environment with idiosyncratic uninsurable shocks where optimal capital income taxes are not zero in the long run.

periods, and the single capital levy prescribed in Chamley (1986) cannot be implemented.[2]

For the purposes of this article, we think of restrictions on sovereign lending as a (rudimentary or ad hoc) way of getting rid of extreme taxation policies in the short run.[3] More generally, gaining a better understanding of institutional or agency constraints that endogenously limit the kinds of contracts the government can write with households seems central in generating optimal fiscal policies that more closely resemble those observed in practice. It may be helpful, for instance, to consider in greater depth the kinds of frictions that may limit or impede government lending. At first glance, such frictions are not necessarily obvious. In particular, any potential commitment problems on the part of households (to repay their loans) should easily be overcome by suitable punishment such as the garnishment of wages. There have also been times in U.S. history, (such as during World War II), where the government directly owned privately operated capital.[4]

Formally, the taxation problem we study, introduced by Kydland and Prescott (1980), discusses the time inconsistency of optimal policy. While matters related to time inconsistency lie beyond the scope of this article, we use the insights of Kydland and Prescott (1980), as well as more recent work by Marcet and Marimon (1999), to present the solution to this problem in terms of stationary linear difference equations that can be solved using standard numerical methods. While Kydland and Prescott (1980) show how the taxation problem they consider can be written (and solved) as a recursive dynamic program, the article does not ultimately present properties of the solution either in the short run or long run.

What do optimal tax rates look like when one exogenously prohibits sovereign lending? Numerical simulations carried out in this article indicate that capital income tax rates are never set either at 100 percent or zero at any point during the transition to the long-run equilibrium. Furthermore, we find that labor income is subsidized in the first few periods. This feature of optimal fiscal policy drives up labor supply and allows household consumption to be everywhere above its long-run equilibrium along its transition path. Finally, we provide straightforward proof of the optimality of zero long-run capital

[2] In this case, what matters is a government's ability to bequeath revenue-generating assets to its successor that, potentially, render the use of future taxes unnecessary. Thus, optimal confiscatory short-run capital taxes would behave as stated in Chamley (1986) in environments in which the government can lend directly to firms. Alternatively, one can also imagine optimal allocations emerging in an environment in which the government directly owned the capital stock and had no disadvantage in operating production directly.

[3] With this restriction in place, an exogenously imposed upper bound on capital income tax rates is no longer necessary. Moreover, the upper limit of 100 percent imposed by Chamley (1986) is not helpful in creating moderate optimal tax rates since this limit turns out to be binding in the short run.

[4] See McGrattan and Ohanian (1999).

taxes that does not rely on the primal approach used in Chamley (1986) and summarized in Ljungqvist and Sargent (2000, chapter 12).

This article is organized as follows. Section 1 provides a brief summary of findings in the literature on optimal fiscal policy in the presence of domestic lending and borrowing. In Section 2, we describe the economic environment under consideration. Section 3 presents the Ramsey problem associated with the analysis of optimal fiscal policy. Salient properties of the long-run Ramsey equilibrium are discussed in Section 4. Section 5 gives a numerical characterization of the transitional dynamics of optimal tax rates. Section 6 offers concluding remarks.

1. A BRIEF DESCRIPTION OF STARK FISCAL POLICIES WITH SOVEREIGN LENDING

This section describes the kinds of extreme optimal fiscal policies that have been described in the optimal taxation literature. Beginning with Chamley (1986), we have already seen in the introduction that a single capital levy at date 0, if feasible, allows the economy to achieve first-best allocations. The problem, of course, is that the associated capital income tax rate might well exceed 100 percent, in which case one might interpret the tax as not only applying to capital income but more directly to the capital stock. Whether this policy is feasible ultimately depends on if the initial capital stock is large enough to finance the net present discounted value of future government expenditures. If so, the necessary revenue is raised entirely through the levy at date 0, and lent back to households with the proceeds from the loans used to finance the stream of government expenditures over time. In that sense, a need for strictly positive distortional taxes never arises.

When capital income tax rates are restricted to be at most 100 percent, Chari, Christiano, and Kehoe (1994) show that it is optimal to set tax rates at their upper limit for a finite number of periods, after which the capital tax rate takes on an intermediate value and is zero, thereafter. The intuition underlying this result relates directly to the distortional nature of capital income tax rates. In particular, having the capital tax rate positive in some period $t > 0$ distorts savings decisions, and thus, private capital allocations, in all prior periods. Hence, front-loading capital income taxes, by having the associated tax rates set at their upper limit from date 0 to some finite date $\bar{t} > 0$, distorts the least number of investment periods. This intuition is only partially complete in that household preferences also play an important role in determining the horizon over which capital income is initially taxed. When preferences are separable in consumption and leisure, it is not optimal to tax capital after the initial period, although labor taxes may be positive at all dates (see Chari, Christiano, and Kehoe 1994). Xie (1997) shows that when preferences are logarithmic in consumption less leisure, it is optimal never to tax labor while capital income tax rates (Chari, Christiano, and Kehoe 1994) hit their upper bound for a finite number of periods and are zero, thereafter.

All of the above policies have in common a radical character and a lack of resemblance to more moderate capital tax rates in practice (i.e., capital tax rates that are neither set at confiscatory rates nor zero). However, a key part underlying the mechanics of these policies relates to the fact that the government is able to build large negative debt holdings by having the capital income tax rate hit its upper limit over some initial period of time, date 0 to date $\bar{t} > 0$. In Xie (1997), it is apparent that once these negative debt holdings are large enough to finance the remaining net present discounted value of government expenditures, then no distortional taxes need ever be set again. In essence, date $\bar{t}$ is then analogous to date 0 in Chamley (1986).

The following sections examine the problem of optimal taxation initially posed by Chamley (1986), but without the policy instrument that allows the government to accumulate large negative debt holdings. Absent this instrument, numerical simulations suggest that it is possible to have more moderate taxes on capital and labor emerge as optimal at every date, without any bounds necessarily imposed on either capital or labor income tax rates. Since the restriction on sovereign lending takes away the usefulness of building a large negative debt position on the part of the government, setting capital tax rates at confiscatory levels is no longer warranted. More importantly, this observation suggests further consideration of the role of institutional or agency constraints that prevent the government from confiscating capital income for an extended period of time, frictions that limit sovereign lending in practice, and how these constraints help shape optimal fiscal policy more generally.

2. ECONOMIC ENVIRONMENT

Consider an economy populated by infinitely many households whose preferences are given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \nu \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right], \sigma > 0, \gamma > 0, \tag{1}$$

where c_t and n_t denote household consumption and labor effort at date t, respectively, and $\beta \in (0, 1)$ is a subjective discount rate.

A single consumption good, y_t, is produced using the technology

$$y_t = k_t^{\alpha} n_t^{1-a}, 0 < \alpha < 1, \tag{2}$$

where k_t denotes the date t stock of private capital. Capital can be accumulated over time and evolves according to

$$k_{t+1} = i_t + (1-\delta)k_t, \tag{3}$$

where $\delta \in (0, 1)$ denotes the depreciation rate and i_t represents household investment. Production can be used for either private or government consumption, or to increase the capital stock,

$$c_t + i_t + g_t = k_t^{\alpha} n_{t,}^{1-\alpha} \tag{4}$$

where $\{g_t\}_{t=0}^{\infty}$ is an exogenously given sequence of public expenditures.

As in Chamley (1986), the government finances its purchases using time-varying linear taxes on labor income and capital income. We denote these tax rates by τ_t^n and τ_t^k, respectively. At each date, the government's budget constraint is given by

$$\tau_t^k r_t k_t + \tau_t^n w_t n_t = g_t, \tag{5}$$

where r_t and w_t are the market rates of return to capital and labor. The left- and right-hand sides of (5) represent sources and uses of government revenue, respectively.

There exists a large number of homogenous small size firms that act competitively. Taking the sequences of prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ as given, each firm maximizes profits and solves

$$\max_{k_t,\, n_t} \; k_t^{\alpha} n_t^{1-\alpha} - r_t k_t - w_t n_t. \tag{6}$$

The implied first-order conditions equate prices to their corresponding marginal products, $r_t = \alpha k_t^{\alpha-1} n_t^{1-\alpha} = \alpha \frac{y_t}{k_t}$ and $w_t = (1-\alpha) k_t^{\alpha} n_t^{-\alpha} = (1-\alpha)\frac{y_t}{n_t}$.

At each date, households decide how much to consume and save in the form of private capital investment, as well as how much labor effort to provide. Taking the sequences of government expenditures, $\{g_t\}_{t=0}^{\infty}$, and tax rates, $\{\tau_t^n, \tau_t^k\}_{t=0}^{\infty}$, as given, these households maximize lifetime utility subject to their budget constraint,

$$\max_{c_t,\, n_t,\, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \nu \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right] \tag{P$^{\text{H}}$}$$

subject to

$$\begin{aligned} c_t + k_{t+1} &= (1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t n_t + (1-\delta) k_t, \qquad (7) \\ k_0 &> 0 \text{ given.} \end{aligned}$$

The first-order necessary conditions implied by problem (7) yield a static equation describing households' optimal labor-leisure choice,

$$\nu n_t^{\frac{1}{\gamma}} = c_t^{-\sigma} (1-\tau_t^n) w_t, \tag{8}$$

as well as a standard Euler equation describing optimal consumption allocations over time,

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta \right]. \tag{9}$$

The constraints (5) and (7), together with the optimality conditions (8) and (9) and the expression for prices given above, describe our economy's decentralized allocations over time.

3. THE RAMSEY PROBLEM

Having described the decentralized behavior of households and firms, we now tackle the problem of choosing policy optimally. Thus, consider a benevolent government that, at date 0, is concerned with choosing a sequence of tax rates that maximize household welfare given the exogenous sequence of government spending. In choosing policy, this government takes as given the behavior of households and firms. We further assume that, at date 0, the government can credibly commit to any sequence of policy actions. The problem faced by this benevolent planner is to maximize (1) subject to the constraints (5) and (7), and households' optimality conditions (8) and (9), where prices are given by marginal products.[5]

We can address the policy problem described at the start of this section by solving the following Lagrangian,

$$\max_{c_t,\, n_t,\, \tau_t^k,\, \tau_t^n,\, k_{t+1}} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \nu \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right] \qquad (\mathrm{P^R})$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_{1t} \left[\beta c_{t+1}^{-\sigma} \left[(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta \right] - c_t^{-\sigma} \right]$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_{2t} \left[\tau_t^k r_t k_t + \tau_t^n w_t n_t - g_t \right]$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_{3t} \left[(1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t n_t + (1-\delta) k_t - c_t - k_{t+1} \right]$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_{4t} \left[c_t^{-\sigma} (1-\tau_t^n) w_t - \nu n_t^{\frac{1}{\gamma}} \right],$$

where the Lagrange multipliers μ_{jt}, $j = 1, ..., 4$, are all nonnegative at the optimum.

The first-order necessary conditions associated with problem (10) that are related to the optimal choices of c_t, n_t, and τ_t^k are as follows:

[5] It is tempting at this point to simply solve a Lagrangian corresponding to the policy problem we have just described. The exact way in which to write this Lagrangian, however, is not immediately clear. To apply Lagrangian methods to this constrained maximization problem, and in particular, to interpret the Lagrange multipliers associated with constraints (5), (7), (8), and (9) as nonnegative, one must first write these constraints as inequalities that define convex sets. See the Appendix for details.

$$c_t : c_t^{-\sigma} - \sigma\mu_{1t-1}c_t^{-\sigma-1}[(1-\tau_t^k)r_t + 1 - \delta] + \sigma\mu_{1t}c_t^{-\sigma-1} \tag{10}$$

$$-\mu_{3t} - \sigma\mu_{4t}c_t^{-\sigma-1}(1-\tau_t^n)w_t = 0, t > 0,$$

with

$$c_0^{-\sigma} - \sigma\mu_{10}c_0^{-\sigma-1} - \mu_{30} - \sigma\mu_{40}c_0^{-\sigma-1}(1-\tau_0^n)w_0 = 0 \text{ at } t = 0, \tag{11}$$

$$\begin{aligned} n_t \quad : \quad & -\nu n_t^{\frac{1}{\gamma}} + \mu_{2t}\left[\tau_t^k k_t \frac{\partial r_t}{\partial n_t} + \tau_t^n (w_t + n_t \frac{\partial w_t}{\partial n_t})\right] \\ & +\mu_{3t}\left[(1-\tau_t^k)k_t \frac{\partial r_t}{\partial n_t} + (1-\tau_t^n)(w_t + n_t \frac{\partial w_t}{\partial n_t})\right] \\ & +\mu_{4t}\left[c_t^{-\sigma}(1-\tau_t^n)\frac{\partial w_t}{\partial n_t} - \frac{\nu}{\gamma}n_t^{\frac{1-\gamma}{\gamma}}\right] + \mu_{1t-1}c_t^{-\sigma}(1-\tau_t^k)\frac{\partial r_t}{\partial n_t} \\ = \quad & 0, \ t > 0, \end{aligned} \tag{12}$$

with

$$\begin{aligned} & -\nu n_0^{\frac{1}{\gamma}} + \mu_{20}\left[\tau_0^k k_0 \frac{\partial r_0}{\partial n_0} + \tau_0^n (w_0 + n_0 \frac{\partial w_0}{\partial n_0})\right] \\ & +\mu_{30}\left[(1-\tau_0^k)k_0 \frac{\partial r_0}{\partial n_0} + (1-\tau_0^n)(w_0 + n_0 \frac{\partial w_0}{\partial n_0})\right] \\ & +\mu_{40}\left[c_0^{-\sigma}(1-\tau_0^n)\frac{\partial w_0}{\partial n_0} - \frac{\nu}{\gamma}n_0^{\frac{1-\gamma}{\gamma}}\right] \\ = \ & 0, \ \text{at } t = 0, \end{aligned} \tag{13}$$

and

$$\tau_t^k : -\mu_{1t-1}c_t^{-\sigma} + (\mu_{2t} - \mu_{3t})k_t = 0, \ t > 0, \tag{14}$$

with

$$\mu_{20} - \mu_{30} = 0, \ \text{at } t = 0. \tag{15}$$

The fact that the above first-order conditions differ at $t = 0$ and $t > 0$ suggests an incentive to take advantage of initial conditions in the first period only, with the promise never to do so in the future. It is exactly in this sense that the optimal policy is not time consistent. Once date 0 has passed, a planner at date $t > 0$ who re-optimizes would want to start with choices for consumption, labor effort, and capital taxes that differ from what was chosen for that date at time 0.

It should be clear that the incentives identified by Chamley (1986) continue to be present in our model economy. Consider that the difference between equations (14) and (15), which governs the optimal choice of τ_t at dates $t = 0$ and $t > 0$, and involves an additional term in (14),

$$-\mu_{1t-1}u_c(c_t) < 0. \tag{16}$$

This term originates from the Euler constraint in problem (10), $\beta u_c(c_t)\,[(1-\tau_t)r_t + 1 - \delta] = u_c(c_{t-1})$, and corresponds to the reduction in the after-tax real return to investment made at date $t-1$ which is created by an increase in the tax rate at time t. Consequently, in committing to a tax rate in a given period $t > 0$, the government takes into account the implied substitution effect on investment decisions undertaken in the preceding period. Of course, at date $t = 0$, no such distortion exists since history commences on that date with a predetermined capital stock, k_0. In choosing τ_0, therefore, the government is free to ignore its effects on previous investment decisions that can be thought of as "sunk"; and there exists some incentive for the optimal sequence of tax rates to begin with a high tax in period 0 relative to all other dates.

A central insight in Kydland and Prescott (1980) is that despite the time inconsistency problem we have just mentioned, it is actually possible to collapse equations (10) through (15) into a set of stationary difference equations $\forall t \geq 0$. This requires interpreting the lagged Lagrange multiplier μ_{1t-1} as a predetermined variable with initial condition $\mu_{1t-1} = 0$ at $t = 0$.

The remaining first-order conditions associated with problem (10) determining the optimal choice of labor income taxes and private capital are, respectively,

$$\tau_t^n : (\mu_{2t} - \mu_{3t})n_t - \mu_{4t}c_t^{-\sigma} = 0,\ t \geq 0, \tag{17}$$

and

$$k_{t+1} : \mu_{1t}\beta c_{t+1}^{-\sigma}(1-\tau_{t+1}^k)\frac{\partial r_{t+1}}{\partial k_{t+1}} + \beta\mu_{2t+1}$$

$$\left[\tau_{t+1}^k(r_{t+1} + k_{t+1}\frac{\partial r_{t+1}}{\partial k_{t+1}}) + \tau_{t+1}^n n_{t+1}\frac{\partial w_{t+1}}{\partial k_{t+1}}\right] - \mu_{3t} + \beta\mu_{3t+1}$$

$$\left[(1-\tau_{t+1}^k)[r_{t+1} + k_{t+1}\frac{\partial r_{t+1}}{\partial k_{t+1}}] + (1-\tau_{t+1}^n)n_{t+1}\frac{\partial w_{t+1}}{\partial k_{t+1}} + 1 - \delta\right] \tag{18}$$

$$+\beta\mu_{4t+1}c_{t+1}^{-\sigma}(1-\tau_{t+1}^n)\frac{\partial w_{t+1}}{\partial k_{t+1}} = 0,\ t \geq 0.$$

4. THE STATIONARY RAMSEY EQUILIBRIUM

With the optimality conditions (10) through (18) in hand, we first turn to long-run properties of optimal taxes and revisit the notion that it is not efficient to tax capital in the long run. We do so, however, without any reference to the primal approach that is standard in the literature, but rely instead on the simple first-order conditions we have just derived. To this end, we define the long-run equilibrium of the Ramsey problem as follows:

Definition: *A stationary Ramsey equilibrium is a ninetuple* $(c, n, k, \tau^n, \tau^k, \mu_1, \mu_2, \mu_3, \mu_4)$ *that solves the government budget constraint (5), households' budget constraint (7), the optimality condition for labor effort (8), and the Euler equation (9), as well as the first-order conditions associated with problem (10), equations (10), (12), (14), (17), and (18), all without time subscripts.*

It is straightforward to show that in a stationary Ramsey equilibrium, equations (14), (17), and (18) imply that

$$\tau^k \mu_2 \beta r - \mu_3 \left[1 - \beta\left((1-\tau^k)r + 1 - \delta\right)\right] = 0. \tag{18}$$

From the Euler equation in the stationary equilibrium, it follows that $1 - \beta[\alpha(1-\tau^k)\frac{y}{k} + 1 - \delta] = 0$. Hence equation (18) above reduces to

$$\tau^k \mu_2 \beta r = 0. \tag{19}$$

Now, we have that either $\tau^k > 0$ or $\tau^k = 0$. Suppose first that $\tau^k > 0$. Then, it must be the case that $\mu_2 = 0$. From equation (14), this would mean that

$$\mu_1 = -\mu_3 k c^{\sigma},$$

which implies that $\mu_1 = \mu_3 = 0$ since μ_1 and μ_3 are both nonnegative. However, in that case, all Lagrange multipliers are zero in the steady state and $c^{-\sigma} = 0$ from equation (10), which cannot be a solution because household utility would be unbounded. Hence, $\tau^k > 0$ cannot be a solution, and therefore, $\tau^k = 0$. As in Chamley (1986), it is optimal not to tax capital in the long run. From the budget constraint, this implies that the steady state tax on labor is essentially determined by the extent of government expenditures. For instance, if government spending was a constant fraction, ϕ of output in the long run, we would have the optimal tax on labor income in the long run to be simply $\tau^n = \frac{\phi}{1-\alpha}$.

The notion that it is optimal to set capital income tax rates to zero in the long run is independent of whether government lending takes place. This is relatively easy to see within our framework when no upper bound is imposed on the capital income tax rate. In that case, the government budget constraint (5) reads as

$$\tau_t^k r_t k_t + \tau_t^n w_t n_t + b_{t+1} = g_t + (1 + r_t^b) b_t, \tag{20}$$

where b_t denotes one-period government bonds that are perfectly substitutable with capital, and r_t^b is the return on bonds from period $t-1$ to t. Government lending takes place when $b_t < 0$. Moreover, the household budget constraint becomes

$$c_t + k_{t+1} + b_{t+1} = (1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t n_t + (1-\delta) k_t + (1+r_t^b) b_t. \tag{21}$$

Substituting these modified constraints in problem (10), the planner now also has to decide how much sovereign lending will take place. A simple arbitrage equation (obtained from the modified household problem) dictates that in the decentralized equilibrium, $1 + r_t^b = (1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta$. Hence, the first-order condition associated with the optimal choice of b_{t+1} in the Ramsey problem is

$$(\mu_{2t} - \mu_{3t}) - \beta[(1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta](\mu_{2t+1} - \mu_{3t+1}) = 0 \; \forall t \geq 0. \quad (22)$$

It is now easy for us to show that capital income taxes are zero in the long run. In fact, with no upper bound imposed on the capital income rate, $\tau_t^k = 0 \; \forall t > 0$. To see this, observe that equations (15) and (22) imply that $\mu_{2t} - \mu_{3t} = 0$ $\forall t \geq 0$. It follows from (14) that $\mu_{1t-1} = 0 \; \forall t \geq 0$ and from (17) that $\mu_{4t} = 0$ $\forall t \geq 0$. Substituting these results into equation (18) gives

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[r_{t+1} + 1 - \delta \right] \; \forall t \geq 0,$$

which is simply the household's Euler equation (9) when $\tau_t^k = 0 \; \forall t > 0$. When an upper bound is imposed on the capital income tax rate, $\tau_t^k \leq 1$ $\forall t \geq 0$, it is still the case that $\tau_t^k = 0 \; \forall t > 0$ when preferences are separable in consumption and leisure, and that $\lim_{t \to \infty} \tau_t^k = 0$, otherwise. Proof of the latter results is more difficult to see using our Lagrangian formulation, but is nicely presented in Erosa and Gervais (2001).

5. TRANSITIONS TO THE STEADY STATE

Even in the absence of an instrument that allows government lending to households, we saw in the previous section that the optimal fiscal policy with commitment prescribes zero capital taxes in the long run. In the short and medium run, however, capital income tax rates are not as extreme as predicted in a model with a debt instrument. Compared to the environment studied by Chari, Christiano, and Kehoe (1994) for instance, where capital income tax rates are set at their upper bound up to some date $\bar{t}$ and are zero thereafter, capital income tax rates in our framework approach confiscatory rates only in the initial period and then decline monotonically over time. Labor income tax rates are also moderate at every point along the transition.

To illustrate these points, we carry out a numerical simulation of our economy when fiscal policy is determined optimally. The parameters we use are standard and selected along the lines of other studies in quantitative general equilibrium theory. A time period represents a quarter and we assume a 6.5 percent annual real interest rate, $\beta = 0.984$, and a 10 percent capital depreciation rate, $\delta = 0.025$. We set the intertemporal elasticity of substitution, $1/\sigma$, to 1/2 and the Frisch elasticity of labor supply, γ, to 1.25. The share of private capital in output in the United States is approximately 33 percent so we assign a value of $\alpha = 1/3$. Finally, we fix the share of government expenditures in output at 0.20.

To compute the transitional dynamics associated with optimal capital and labor income tax rates, we replace the optimality conditions in Section 3 with log-linear approximations around the stationary Ramsey equilibrium. The solution paths for the state and co-state variables are then computed using techniques described in Blanchard and Kahn (1980) or in King, Plosser, Rebelo (1988). The resulting system of linearized equations possesses a continuum of solutions, but only one of these is consistent with the transversality condition associated with the household problem.

Figure 1 depicts transitions to the stationary Ramsey equilibrium when the initial capital stock is set at its long-run level. In other words, Figure 1 shows transitions to the steady state when restarting the problem. In Panel A, we can see that capital income tax rates start near confiscatory rates in the initial period but quickly fall within 10 quarters to a more moderate range, at less than 35 percent.[6] Thus, the notion that capital income tax rates are optimally higher in the initial periods remain, but these rates are within a moderate range for the greater part of the transition. More specifically, in contrast to Chari, Christiano, and Kehoe (1994), capital income tax rates are never set at either 100 percent or zero at any point during the transition.[7] Interestingly, the optimal fiscal policy suggests subsidizing labor income in the first few periods, after which labor income taxes monotonically rise to their steady state. Because labor income represents $2/3$ of total output in our calibrated economy, and because government expenditures account for 20 percent of output, the labor income tax rate approaches 30 percent asymptotically as capital income tax rates approach zero.

The initial subsidization of labor income generates an increase in labor input, shown in Figure 1, Panel D, along the transition to the steady state. As a result, household consumption is everywhere above its long-run level on its way to the steady state. Moreover, the optimal fiscal policy is such that households are able to front-load consumption.

6. CONCLUSION

In this article, we highlighted that environments in which ex ante government lending is limited are more apt to generate optimal distortional taxes at all dates that do not share the stark character of those typically presented in the literature. Absent an instrument that allows households to borrow from the government, the government cannot carry the proceeds from a large initial tax to future periods, and the single capital levy prescribed in Chamley (1986) cannot be implemented.

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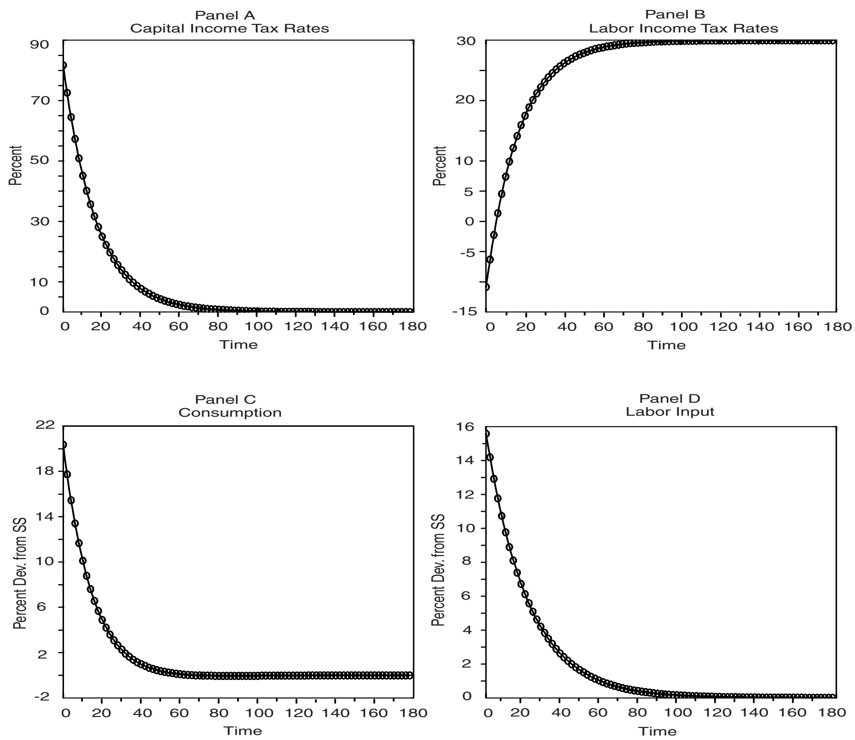
[6] The captial stock is fixed in period 0. It then decreases slowly while capital income tax rates are relatively high and converges back to its long-run level.

[7] Chari, Christiano, and Kehoe (1994) present an actual numerical solution to the problem without linearizing. The linearization in our case involves an approximation, but the fact that optimal taxes do not involve corner solutions in our framework does not depend on the approximation.

Figure 1 Transitions to the Steady State, $k_0 k_{ss}$



For an economy whose capital stock is initially below its long-run level, we have shown that capital income tax rates are never set at either 100 percent or zero at any point during the transition to the long-run equilibrium. Furthermore, our analysis has highlighted that labor income is subsidized in the first few periods. This feature of optimal fiscal policy gave rise to increased labor supply and allowed household consumption to be everywhere above its long-run equilibrium along its transition path. As in Chamley (1986), however, even without a debt instrument, our analysis continued to prescribe zero-capital income tax rates in the long run.

We interpret our findings to suggest that a better understanding of institutional or agency constraints that prevent the government from confiscating capital income for an extended period of time, as well as commitment problems associated with household borrowing, may be central in explaining the character of observed fiscal policies.

APPENDIX

Marcet and Marimon (1999) point out that equalities associated with feasibility constraints can generally (and in this case) be replaced with weak inequalities, with an appeal to nonsatiated preferences thereby guaranteeing that the feasibility constraints rewritten as such will also be satisfied with equality. However, an analogous argument for the equality constraints (8) and (9) is less obvious. Because of the equality signs in (8) and (9), the set of allocations satisfying these equations is not convex.

Consider the Euler equation (9). Marcet and Marimon (1999) show that it is possible to rewrite this constraint as a weak inequality in such a way that, in the optimum of the new problem, this weak inequality is satisfied as a strict equality. One can be sure, therefore, that the optimum subject to the weak inequality constraint is the same as that subject to the strict equality (9), and that one is actually solving the problem of interest.

We now provide a brief description of the arguments presented in Marcet and Marimon (1999) but refer the reader to the paper for the formal proofs. The question is whether to write the inequality associated with (9) as $\leq$ or $\geq$. Consider the case $\geq$ first, in which $c_t^{-\sigma} \geq \beta c_{t+1}^{-\sigma}\left[(1-\tau_{t+1}^k)r_{t+1}+1-\delta\right]$. The authors show that writing the inequality constraint in this way actually makes the first-best allocation feasible, so that the solution would be the unconstrained optimum, which is not the same as the Ramsey equilibrium. Hence, this option does not yield a solution equivalent to the solution under equation (9).

Next, consider the case where the inequality constraint is written as $\leq$ so that $c_t^{-\sigma} \leq \beta c_{t+1}^{-\sigma}\left[(1-\tau_{t+1}^k)r_{t+1}+1-\delta\right]$. Writing the inequality in this way reproduces the household's first-order condition if the household faced the constraint $k_t \leq \overline{k}_t$, where $\overline{k}_t$ is an upper bound imposed on households' capital position. In other words, the modified Euler equation corresponds to a setting where the policy instruments available to the planner now include the ability to set an upper limit on capital accumulation, $\overline{k}_t$. In that case, Marcet and Marimon (1999) then show that the planner will actually choose allocations where the constraint $k_t \leq \overline{k}_t$ does not bind, since the equilibrium with distortional taxes is associated with too little capital relative to the full optimum. This implies that the government will act so that $c_t^{-\sigma} \leq \beta c_{t+1}^{-\sigma}\left[(1-\tau_{t+1}^k)r_{t+1}+1-\delta\right]$ is satisfied with equality, and the optimum is then the same as the Ramsey equilibrium. Similar arguments can be made regarding constraint (8).

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