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Evolving Inflation Dynamics and the New Keynesian Phillips Curve

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In most industrialized economies, periods of above average inflation tend to be associated with above average economic activity, for example, as measured by a relatively low unemployment rate. This statistical relationship, known as the Phillips curve, is sometimes invoked when economic commentators suggest that monetary policy should not try to suppress signs of inflation. But this interpretation of the Phillips curve implicitly assumes that the statistical relationship is structural, that is, the relationship will not break down during periods of persistently high inflation. Starting in the mid-1960s, Friedman and Phelps argued that the Phillips curve is indeed not structural and the experience of the United States and other countries with high inflation and low GDP growth in the late 1960s and 1970s has subsequently borne out their predictions.

Various theories have been proposed to explain the Phillips curve and most of these theories agree that there is no significant long-term tradeoff between inflation and the level of economic activity. One theory that provides a structural interpretation of the short-term inflation-unemployment relationship, and that has become quite popular over the last ten years among central bank economists is based on explicit models of nominal price rigidity. The most well-known example of this theory is the New Keynesian Phillips Curve (NKPC).

In this article, I evaluate how well a structural NKPC can account for the changing nature of inflation in the United States from the 1950s to today. First, I document that changes in average inflation have been associated with

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changes in the dynamics of inflation as measured by inflation persistence and the co-movement of inflation with measures of real activity that the NKPC predicts are relevant for inflation. Then I argue that the NKPC with fixed structural parameters cannot account for these changes in the inflation process. I conclude that the NKPC does not provide a complete structural interpretation of the Phillips curve. This is troublesome since the changed inflation dynamics are related to changes in average inflation, which are presumably driven by systematic monetary policy. But if the NKPC is not invariant to systematic changes of monetary policy, then its use for monetary policy is rather limited.

In models with nominal rigidities, sticky-price models for short, monopolistically competitive firms set their prices as markups over their marginal cost. Since these firms are limited in their ability to adjust their nominal prices, future inflation tends to induce undesired changes in their relative prices. When firms have the opportunity to adjust their prices they will, therefore, set their prices contingent on averages of expected future marginal cost and inflation. The implied relationship between inflation and economic activity is potentially quite complicated, but for a class of models one can show that to a first-order approximation current inflation is a function of current marginal cost and expected future inflation, the so-called NKPC. The coefficients in this NKPC are interpreted as structural in the sense that they are likely to be independent of monetary policy.

In the U.S. economy, inflation tends to be very persistent, in particular, it tends to be at least as persistent as is marginal cost. At the same time, inflation is not that strongly correlated with marginal cost. This observation appears to be inconsistent with the standard NKPC since here inflation is essentially driven by marginal cost, and inflation is, at most, as persistent as marginal cost. But if inflation is as persistent as is marginal cost then the model also predicts a strong positive correlation between inflation and marginal cost. One can potentially account for this observation through the use of a hybrid NKPC which makes current inflation not only a function of expected future inflation, but also of past inflation as in standard statistical Phillips curves. With a strong enough backward-looking element, inflation persistence then need not depend on the contributions from marginal cost alone.

Another feature of U.S. inflation is that average inflation has always been positive, and it has varied widely: periods of low inflation, such as the 1950s and 1960s, were followed by a period of very high inflation in the 1970s, and then low inflation again since the mid-1980s. Cogley and Sbordone (2005, 2006) point out that the NKPC relates inflation and marginal cost defined in terms of their deviations from their respective trends. In particular, the standard NKPC defines trend inflation to be zero. Given the variations in average U.S. inflation, Cogley and Sbordone (2005, 2006) then argue that accounting for variations in trend inflation will make deviations of inflation from trend less persistent. Furthermore, as Ascari (2004) shows, the first-order approximation of the NKPC needs to be modified when the approximation is taken at a positive inflation rate.

I build on the insight of Cogley and Sbordone (2005, 2006) and study the implications of a time-varying trend inflation rate for the autocorrelation and cross-correlation structure of inflation and marginal cost. In this I extend the work of Fuhrer (2006) who argues that the hybrid NKPC can account for inflations's autocorrelation structure only through a substantial backward-looking element. In this article, I argue that a hybrid NKPC, modified for changes in trend inflation, cannot account for changes in the autocorrelation and cross-correlation structure of inflation and marginal cost in the United States.

The article is organized as follows. Section 1 describes the dynamic properties of inflation and marginal cost in the baseline NKPC and the U.S. economy. Section 2 describes and calibrates the hybrid NKPC, and it compares the autocorrelation and cross-correlation structure of inflation and marginal cost in the model with that of the 1955–2005 U.S. economy. Section 3 characterizes the inflation dynamics in the NKPC modified to account for nonzero trend inflation. I then study if the changes of inflation dynamics, associated with changes in trend inflation comparable to the transition into and out of the high inflation period of the 1970s, are consistent with the changing nature of inflation dynamics in the U.S. economy for that period.

1. INFLATION AND MARGINAL COST IN THE NKPC

Inflation in the baseline NKPC is determined by expectations about future inflation and a measure of current economic activity. There are two fundamental differences between the NKPC and more traditional specifications of the Phillips curve. First, traditional Phillips curves are backward looking and relate current inflation to lagged inflation rates. Second, the measure of real activity in the NKPC is based on a measure of how costly it is to produce goods, whereas traditional Phillips curves use the unemployment rate as a measure of real activity. More formally, the baseline NKPC is

$$\hat{\pi}_t = \kappa_0 \hat{s}_t + \beta E_t \left[\hat{\pi}_{t+1}\right] + u_t, \tag{1}$$

where $\hat{\pi}_t$ denotes the inflation rate, $\hat{s}_t$ denotes real marginal cost, $E_t \hat{\pi}_{t+1}$ denotes the expected value of next period's inflation rate conditional on current information, u_t is a shock to the NKPC, β is a discount factor, $0 < \beta < 1$, and κ_0 is a function of structural parameters described below. The baseline NKPC is derived as the local approximation of equilibrium relationships for a particular model of the economy, the Calvo (1983) model of price adjustment.

For the Calvo model one assumes that all firms are essentially identical, that is, they face the same demand curves and cost functions. The firms are monopolistically competitive price setters, but can adjust their nominal prices only infrequently. In particular, whether a firm can adjust its price is random, and the probability of price adjustment is constant. Random price adjustment introduces ex post heterogeneity among firms, since with nonzero inflation a firm's relative price will depend on how long ago the firm last adjusted its price. Since firms are monopolistically competitive they set their nominal (and relative) price as a markup over their real marginal cost, and since firms can adjust their price only infrequently they set their price conditional on expected future inflation and marginal cost.

The NKPC is a linear approximation to the optimal price-setting behavior of the firms in the Calvo model. Furthermore, the approximation is local to a state that exhibits a zero-average inflation rate. The inflation rate $\hat{\pi}_t$ should be interpreted as the log-deviation of the gross inflation rate from one, that is, the net-inflation rate, and real marginal cost $\hat{s}_t$ should be interpreted as the log-deviation from its long-run mean. For a derivation of the NKPC, see Woodford (2003).[1] The optimal pricing decisions of firms with Calvo-type nominal price adjustment are reflected in the parameter κ_0 of the NKPC,

$$\kappa_0 = \frac{1-\alpha}{\alpha}\left(1-\alpha\beta\right), \tag{2}$$

where α is the probability that a firm cannot adjust its nominal price, $0 \leq \alpha < 1$.

The shock to the NKPC is usually not derived as part of the linear approximation to the optimal price-setting behavior of firms. Most of the time the shock is simply "tacked on" to the NKPC, although it can be interpreted as a random disturbance to the firms' static markup. Given the absence of serious microfoundations of the cost shock one would not want the shock to play an independent role in contributing to the persistence of inflation. We, therefore, assume that the shock to the NKPC is i.i.d. with mean zero.[2]

Persistence of Inflation in the NKPC

The NKPC represents a partial equilibrium relationship within a more comprehensive model of the economy. Thus, inflation and marginal cost will be simultaneously determined as part of a more complete description of the economy. Conditional on the equilibrium process for marginal cost we can, however, solve equation (1) forward by repeatedly substituting for future inflation and obtain the current inflation rate as the discounted expected value

[1] The NKPC approximated at the zero inflation rate is also a special case of the NKPC approximated at a positive inflation rate. For a derivation of the latter, see Ascari (2004), Cogley and Sbordone (2005, 2006), or Hornstein (2007).

[2] The shock to the NKPC is often called a "cost-push" shock, but this terminology can be confusing since the shock is introduced independently of marginal cost.

of future marginal cost

$$\hat{\pi}_t = \kappa_0 \sum_{j=0}^{\infty} \beta^j E_t \left[\hat{s}_{t+j}\right] + u_t. \tag{3}$$

The behavior of the inflation rate, in particular its persistence, is therefore closely related to the behavior of marginal cost. To get an idea of what this means for the joint behavior of inflation and marginal cost, assume that equilibrium marginal cost follows a first-order autoregressive process [AR(1)],

$$\hat{s}_t = \delta \hat{s}_{t-1} + \varepsilon_t, \tag{4}$$

with positive serial correlation, $0 < \delta < 1$, and ε_t is an i.i.d. mean zero shock with variance σ_ε^2. This AR(1) specification is a useful first approximation of the behavior of marginal cost since, as we will see below, marginal cost is a highly persistent process. For such an AR(1) process the conditional expectation of marginal cost j-periods-ahead is simply

$$E_t \left[\hat{s}_{t+j}\right] = E_t \left[\delta \hat{s}_{t+j-1} + \varepsilon_{t+j}\right] = \delta E_t \left[\hat{s}_{t+j-1}\right] = \ldots = \delta^j \hat{s}_t. \tag{5}$$

Substituting for the expected future marginal cost in (3), we get

$$\hat{\pi}_t = \kappa_0 \sum_{j=0}^{\infty} \beta^j \delta^j \hat{s}_t + u_t = \frac{\kappa_0}{1 - \beta\delta} \hat{s}_t + u_t = a_0 \hat{s}_t + u_t. \tag{6}$$

This is a reduced form relationship between current inflation and marginal cost. The relationship is in reduced form since it incorporates the presumed equilibrium law of motion for marginal cost, which is reflected in the fact that the coefficient on marginal cost, a_0, depends on the law of motion for marginal cost. If the law of motion for marginal cost changes, then the relation between inflation and marginal cost will change.

Given the assumed law of motion for marginal cost, inflation is positively correlated with marginal cost and is, at most, as persistent as is marginal cost. The second moments of the marginal cost process are

$$E \left[\hat{s}_t \hat{s}_{t-k}\right] = \delta^k \frac{\sigma_\varepsilon^2}{1 - \delta^2} = \delta^k \sigma_s^2, \tag{7}$$

where σ_s^2 is the variance of marginal cost. The implied second moments of the inflation rate and the cross-products of inflation and marginal cost are

$$\begin{aligned} E \left[\hat{\pi}_t \hat{\pi}_{t-k}\right] &= a_0^2 E \left[\hat{s}_t \hat{s}_{t-k}\right] + I_{[k=0]} \sigma_u^2 = \delta^k \left(a_0 \sigma_s\right)^2 + I_{[k=0]} \sigma_u^2, && (8) \\ E \left[\hat{\pi}_t \hat{s}_{t+k}\right] &= a_0 E \left[\hat{s}_t \hat{s}_{t+k}\right] = \delta^k a_0 \sigma_s^2, && (9) \end{aligned}$$

where $I_{[.]}$ denotes the indicator function. The autocorrelation coefficients for inflation and the cross-correlations of inflation with marginal cost are

$$Corr\left(\hat{\pi}_t, \hat{\pi}_{t-k}\right) = \delta^k \frac{a_0^2}{a_0^2 + \sigma_u^2/\sigma_s^2}, \text{ and} \tag{10}$$

$$Corr\left(\hat{\pi}_t, \hat{s}_{t+k}\right) = \delta^k \frac{a_0}{\left[a_0^2 + \sigma_u^2/\sigma_s^2\right]^{1/2}}. \tag{11}$$

As we can see, the autocorrelation coefficients for inflation are simply scaled versions of the autocorrelation coefficients for marginal cost, and the scale parameter depends on the relative volatility of the shocks to the NKPC and marginal cost. If there are no shocks to the NKPC, $\sigma_u = 0$, then inflation is an AR(1) process with persistence parameter δ, and it is perfectly correlated with marginal cost. If, however, there are shocks to the NKPC, $\sigma_u > 0$, then inflation and marginal cost are imperfectly correlated and inflation is less persistent than is marginal cost.

Inflation and Marginal Cost in the U.S. Economy

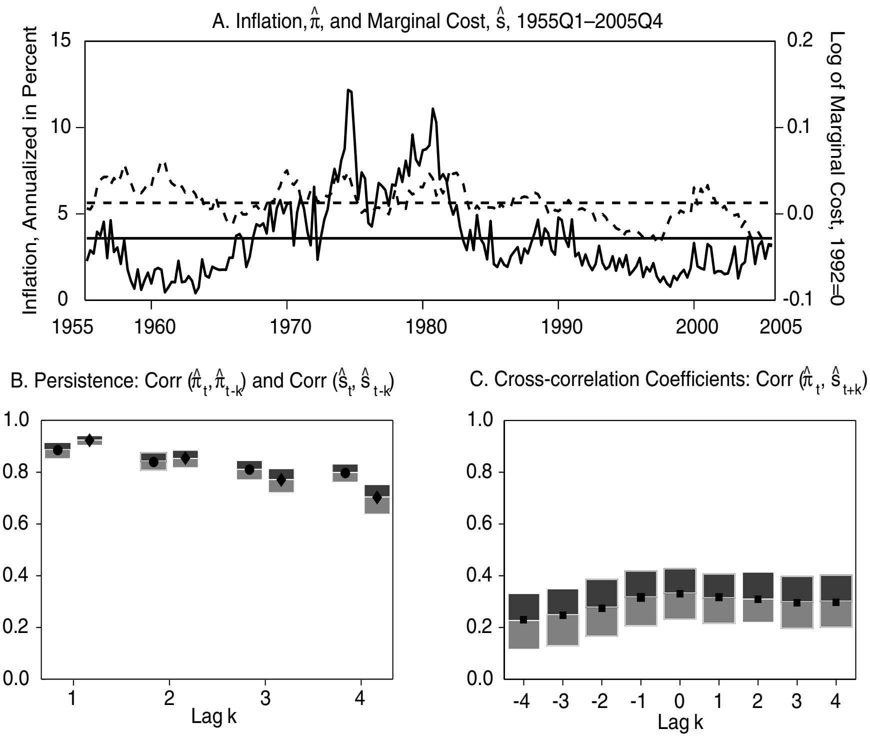
In order to make the NKPC operational, we need measures of the inflation rate and marginal cost. For the inflation rate we will use the rate of change of the GDP deflator.[3] We measure aggregate marginal cost through the wage income share in the private nonfarm business sector. This choice can be motivated as follows. Suppose that all firms use the same production technology with labor as the only input. In particular, assume that the production function is Cobb-Douglas, $y = zn^{\omega}$, with constant input elasticity ω. Then the nominal marginal cost is the nominal wage divided by the marginal product of labor

$$S_t = \frac{W_t}{MPL_t} = \frac{W_t}{\omega y_t / n_t}, \tag{12}$$

and nominal marginal cost is proportional to nominal average cost. We use the unit labor cost index for the private nonfarm business sector as our measure of average labor cost. Deflating nominal average cost with the price index of the private nonfarm business sector yields real average labor cost, that is, the labor income share. The log deviation of real marginal cost from its mean is

[3] This is the most commonly used price index in the implementation of the NKPC. Other price indices used include the price index of the private nonfarm business sector or the price index for Personal Consumption Expenditures (PCE), the consumption component of the GDP deflator. Although the choice of price deflator affects the results described below, the differences are not dramatic, e.g., Galí and Gertler (1999). We should also note that only consumption based indices, such as the PCE index, are commonly mentioned by central banks in their communications on monetary policy.

Figure 1 Inflation and Marginal Cost in the United States, 1955–2005



Notes: Inflation and marginal cost are defined in the Appendix. The solid line in Panel A represents the inflation rate and its sample mean, and the dashed line represents marginal cost and its sample mean. In Panel B, the circles (diamonds) denote the sample autocorrelations for inflation (marginal cost). In Panel C, the squares denote the cross-correlations of inflation and marginal cost. In Panels B and C, the boxes denote the 5-percentile to 95-percentile range of the statistic calculated from 1,000 bootstraps of the data.

then equal to the log-deviation of the labor income share from its mean

$$\hat{s}_t = \frac{\widehat{W_t n_t}}{P_t y_t}. \tag{13}$$

The detailed source information for our data is listed in the Appendix.

In Figure 1.A, we graph the quarterly inflation rate and marginal cost for the time period 1955Q1 to 2005Q4. Inflation varies widely over this time period, from about 1 percent at the low end in the early 1960s, to more than 10 percent in the 1970s, with a 3 1/2 percent average inflation rate, Table 1, column 1. Inflation and marginal cost are both highly persistent, the first-order autocorrelation coefficient is about 0.9 for both variables, Figure 1.B. To the

Table 1 Inflation and Marginal Cost

Sample	$\bar{\pi}$	$\sigma_{\hat{\pi}}$	$\bar{s}$	$\sigma_{\hat{s}}$	$\bar{\delta}_{\hat{\pi}}$	$\bar{\delta}_{\hat{s}}$	Corr $(\hat{\pi}, \hat{s})$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1955Q1–2005Q4	3.6	2.4	0.013	0.021	0.94	0.93	0.33
					[0.88,0.99]	[0.89,0.98]	[0.23,0.43]
1955Q1–1969Q4	2.5	1.4	0.023	0.018	0.97	0.89	-0.12
					[0.83,0.98]	[0.79,1.00]	[-0.30,0.05]
1970Q1–1983Q4	6.5	2.2	0.024	0.016	0.80	0.72	0.29
					[0.62,0.98]	[0.56,0.88]	[0.10,0.46]
1984Q1–1991Q4	3.2	0.9	0.011	0.007	0.60	0.73	0.10
					[0.20,1.03]	[0.51,0.95]	[0.09,0.34]
1992Q1–2005Q4	2.1	0.7	-0.009	0.018	0.76	0.92	-0.06
					[0.50,1.02]	[0.81,1.02]	[-0.32,0.22]

Notes: Columns (1) and (2) contain the average annualized inflation rate, $\bar{\pi}$, and its standard deviation, $\sigma_{\hat{\pi}}$. Columns (3) and (4) contain the average values and standard deviation of marginal cost, $\bar{s}$ and $\sigma_{\hat{s}}$. Marginal cost is in log deviations from its normalized 1992 value. Columns (5) and (6) contain the sum of the autocorrelation coefficients of a univariate OLS regression with four lags for inflation respectively marginal cost, $\bar{\delta}_{\hat{\pi}}$ and $\bar{\delta}_{\hat{s}}$. Column (7) contains the contemporaneous correlation coefficient between inflation and marginal cost. For the sum of autocorrelation coefficients and the correlation coefficient, columns (5), (6), and (7), we list the 5th and 95th percentile of the respective bootstrapped statistic with 1,000 replications in brackets.

extent that the autocorrelation coefficients of inflation do not decline as fast as the ones for marginal cost, inflation appears to be somewhat more persistent than marginal cost. Levin and Piger (2002) use an alternative measure of persistence in their analysis of inflation in the United States, namely the sum of lagged coefficients in a univariate regression of a variable on its own lags. This measure also yields estimates of significant and similar persistence for inflation and marginal cost, Table 1, columns 5 and 6. Inflation and marginal cost tend to move together. The cross-correlations between inflation and marginal cost are positive, 0.33 contemporaneously and above 0.2 at all four lags and leads, Table 1, column 7, and Figure 1.C. Although the co-movement between inflation and marginal cost is significant, it is not particularly strong.[4]

As we have shown previously, in the basic NKPC model, persistence of inflation and marginal cost, and co-movement of inflation with marginal cost go together. The observation that inflation is about as persistent as marginal cost, but only weakly correlated with marginal cost then seems to be inconsistent with the basic NKPC. We now study if two modifications of the basic

[4] The positive cross-correlation coefficients are significant for all four lags and leads. Based on 1,000 bootstraps the 5-percentile to 95-percentile ranges of the coefficients do not include zero, Figure 1.C.

NKPC can resolve this apparent inconsistency. The first approach is to make the NKPC more like a standard Phillips curve by directly introducing lagged inflation. The second approach argues that some of the observed inflation persistence is spurious. Extended apparent deviations of the inflation rate from the sample average inflation rate, for example in the 1970s, are interpreted as sub-sample changes in the mean inflation rate. This approach then suggests that the NKPC has to be modified to take into account changes in trend inflation. We will discuss these two approaches in the following sections.

2. A HYBRID NKPC

The importance of marginal cost for inflation persistence will be reduced if there is a source of persistence that is inherent to the inflation process itself. Two popular approaches that introduce such a backward-looking element of price determination into the NKPC are "rule-of-thumb" behavior and indexation. For the first approach, one assumes that a fraction ρ of the price-setting firms do not choose their prices optimally, rather they index their prices to past inflation. For the second approach one assumes that firms who do not have the option to adjust their price optimally simply index their price to a fraction ρ of past inflation.[5] The two approaches are essentially equivalent and for the second case the NKPC becomes

$$(1 - \rho L)\,\hat{\pi}_t = \beta E_t\left[(1 - \rho L)\,\hat{\pi}_{t+1}\right] + \kappa_0 \hat{s}_t + u_t, \tag{14}$$

where L is the lag operator, $L^j x_t = x_{t-j}$ for any integer j.

This modification of the NKPC is also called a hybrid NKPC since current inflation not only depends on expected inflation as in the baseline NKPC, but it also depends on past inflation as in a traditional Phillips curve. The dependence on lagged inflation introduced through backward-looking price determination is called "intrinsic" persistence since it is an exogenous part of the model structure. Complementary to intrinsic persistence is "extrinsic" inflation persistence which comes through the marginal cost process that drives inflation. To the extent that monetary policy affects marginal cost, it influences extrinsic inflation persistence.

Note that the hybrid NKPC, equation (14), is of the same form as the basic NKPC, equation (1), except for the linear transformation of inflation, $\tilde{\pi}_t = \hat{\pi}_t - \rho\hat{\pi}_{t-1}$, replacing the actual inflation rate. Forward-solving equation (14), assuming again that marginal cost follows an AR(1) process, as in equation (4), then yields the following expression for $\tilde{\pi}_t$:

$$\hat{\pi}_t - \rho\hat{\pi}_{t-1} = \frac{\kappa_0}{1 - \beta\delta}\hat{s}_t + u_t = a_0 \hat{s}_t + u_t. \tag{15}$$

[5] "Rule-of-thumb" behavior was introduced by Galí and Gertler (1999); inflation indexation has been used by Christiano, Eichenbaum, and Evans (2005).

For this specification, inflation can be more persistent than marginal cost because current inflation is indexed to past inflation.

The autocorrelation coefficients for the linear transformation of inflation, $\tilde{\pi}_t$, are the same as defined in equation (10), but the autocorrelation coefficients for the inflation rate itself are now more complicated functions of the persistence of marginal cost and the intrinsic inflation persistence. In Hornstein (2007), I derive the autocorrelation and cross-correlation coefficients for inflation and marginal cost,

$$\begin{aligned} Corr\left(\hat{\pi}_t, \hat{\pi}_{t-k}\right) &= \frac{\left(\sigma_u/\sigma_s\right)^2 A\left(k;\rho\right) + a_0^2 B\left(k;\rho,\delta\right)}{\left(\sigma_u/\sigma_s\right)^2 A\left(0;\rho\right) + a_0^2 B\left(0;\rho,\delta\right)} \text{ and} && (16)\\ Corr\left(\hat{\pi}_t, \hat{s}_{t+k}\right) &= \frac{a_0 C\left(k;\rho,\delta\right)}{\left[\left(\sigma_u/\sigma_s\right)^2 A\left(0;\rho\right) + a_0^2 B\left(0;\rho,\delta\right)\right]^{1/2}}, && (17)\end{aligned}$$

where

$$\begin{aligned} A\left(k;\rho\right) &= \rho^k \frac{1}{1-\rho^2},\\ B\left(k;\rho,\delta\right) &= \left[\delta^k - \frac{\rho}{\delta}\frac{1-\delta^2}{1-\rho^2}\rho^k\right]\frac{1}{\left(1-\rho/\delta\right)\left(1-\rho\delta\right)},\\ C\left(k;\rho,\delta\right) &= \delta^k \frac{1}{1-\rho\delta} \text{ if } k \geq 0, \text{ and}\\ C\left(k;\rho,\delta\right) &= \left[\delta^{-k} - \rho^{-k}\frac{\rho}{\delta}\frac{1-\delta^2}{1-\rho\delta}\right]\frac{1}{1-\rho/\delta} \text{ if } k < 0.\end{aligned}$$

Inflation Persistence in the Hybrid NKPC

Inflation persistence for the hybrid NKPC depends not only on the persistence of marginal cost and intrinsic inflation persistence, δ and ρ, but also on the relative volatility of the shocks to the NKPC and marginal cost, σ_u/σ_s, and the reduced form coefficient on marginal cost, a_0. In order to evaluate the implications of the hybrid NKPC for inflation dynamics we, therefore, need estimates of the structural parameters of the NKPC and the relative standard deviation of the NKPC shock. In the following, I study the implications of two alternative calibrations. The first calibration is based on generalized method of moments (GMM) estimates of the structural parameters, α, β, and ρ, and an estimate of the relative volatility of the NKPC shocks that is implicit in the GMM estimates. This calibration has only limited success in matching the autocorrelation and cross-correlation properties of inflation and marginal cost. For the second calibration, I then set intrinsic persistence and the relative volatility of the NKPC shock to directly match the autocorrelation and cross-correlation properties of inflation and marginal cost.

Table 2 New Keynesian Phillips Curve Estimates, 1960 Q1–2005 Q4

	α	ρ	β	$\hat{\pi}_{t-1}$	$\hat{\pi}_{t+1}$	$\hat{s}_t$
(1)	0.901	0.164	0.990	0.141	0.851	0.010
	(0.028)	(0.124)	(0.028)	(0.091)	(0.087)	(0.007)
(2)	0.897	0.469	0.944	0.325	0.654	0.012
	(0.021)	(0.095)	(0.043)	(0.046)	(0.048)	(0.005)

Notes: This table reports estimates of the NKPC approximated at a zero inflation rate, equation (14). The first three columns contain estimates of the structural parameters: price non-adjustment probability, α, degree of inflation indexation, ρ, and time discount factor β. The next three columns contain the implied reduced form coefficients on marginal cost, and lagged and future inflation when the coefficient on current inflation is one. The first row represents estimates of the moment conditions from equation (14). The second row represents estimates of the moment conditions from equation (14) when the coefficient of contemporaneous inflation is normalized to one. The covariance matrix of errors is estimated with a 12 lag Newey-West procedure. Standard errors of the estimates are shown in parentheses.

Galí, Gertler, and López-Salido (2005) (hereafter referred to as GGLS) estimate the hybrid NKPC for U.S. data using GMM techniques.[6] I replicate their analysis for the hybrid NKPC (14) using the data on inflation and marginal cost for the time period 1960–2005. The instrument set includes four lags of the inflation rate, and two lags each of marginal cost, nominal wage inflation, and the output gap.[7] The results reported in Table 2 are not exactly the same as in GGLS, but they are broadly consistent with GGLS. The time discount factor, β, is estimated close to one, and the coefficient on marginal cost, $\kappa_0 = 0.01$, is smaller than for GGLS. The small coefficient on marginal cost translates to a relatively low price adjustment probability: only about 10 percent, $1-\alpha$, of all prices are optimally adjusted in a quarter. Similar to GGLS the estimated degree of inflation indexation depends on the normalization of the GMM moment conditions. For the first specification, when equation (14) is estimated directly, we find a relatively low degree of indexation to past inflation, $\rho = 0.16$. For the second specification, when the coefficient on current inflation in equation (14) is normalized to one, we find significantly more indexation, $\rho = 0.47$.

We construct an estimate of the volatility of shocks to the NKPC in two steps. First, we regress current inflation $\hat{\pi}_t$ on the set of instrumental variables. The instrumental variables contain only lagged variables, that is, information

[6] Other work that estimates the NKPC using the same or similar techniques includes Galí and Gertler (1999) and Sbordone (2002). See also the 2005 special issue of the *Journal of Monetary Economics* vol. 52 (6).

[7] The data are described in detail in the Appendix.

Table 3 Calibration

	Parameter	Calibration	
		(1)	(2)
β	Time Discount Factor	0.99	0.99
α	Probability of No Price Adjustment	0.90	0.80
ρ	Price Indexation	0.45	0.86
σ_u/σ_s	Relative NKPC Shock Volatility	0.10	2.97
δ	Marginal Cost Persistence	0.90	0.90

available in the previous period. We then use this regression to obtain an estimate of the expected inflation rate conditional on available information, $E_t\hat{\pi}_{t+1}$, and substitute it together with the information on current inflation and marginal cost, and the estimated parameter values in equation (14), and solve for the shock to the NKPC, u_t. The calculated standard deviation of the shock is about $1/10$ of the standard deviation of marginal cost.[8]

Based on the GMM estimates for the second specification of the moment conditions, I now choose a parameterization of the hybrid NKPC with some intrinsic inflation persistence, Table 3, column 1.[9] For the persistence of marginal cost, I choose $\delta = 0.9$, which provides a reasonable approximation of the autocorrelation structure of marginal cost for the period 1955 to 2005.

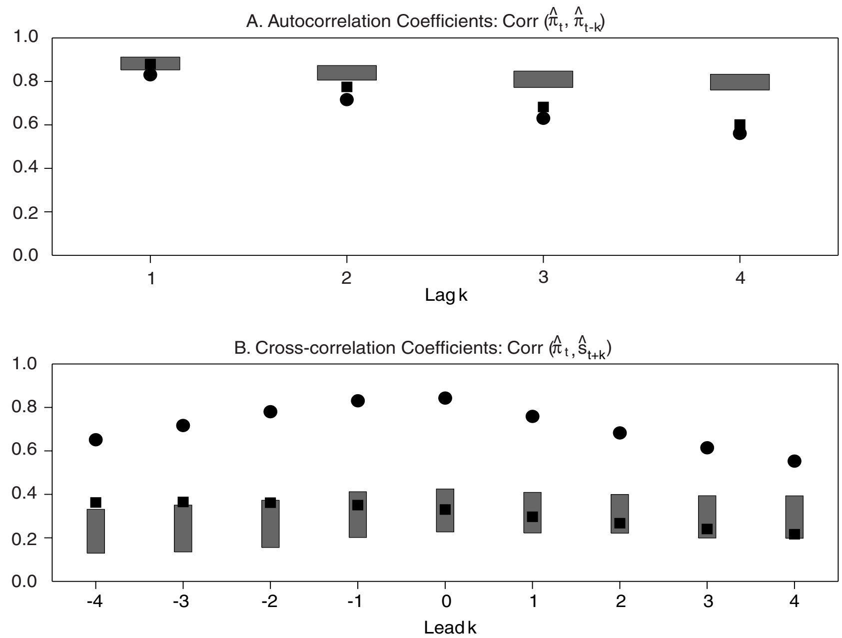
We can now characterize the inflation dynamics implied by the hybrid NKPC. The bullet points in Figure 2 display the first four autocorrelation coefficients of inflation and the cross-correlation coefficients of inflation with marginal cost implied by the calibrated model. Figure 2 also displays the bootstrapped 5th to 95th percentile ranges for the autocorrelation and cross-correlation coefficients of inflation and marginal cost for the U.S. economy from Figure 1.B and 1.C. As we can see, the model does not do too badly for the autocorrelation structure of inflation: the first-order autocorrelation coefficient of inflation is just outside the 5th to 95th percentile range, but then the autocorrelation coefficients are declining too fast relative to the data.[10] The model does generate too much co-movement for inflation and marginal cost

[8] Depending on the parameter estimates, $\sigma_u = 0.0019$ for specification one and $\sigma_u = 0.0025$ for specification two. For either specification the serial correlation of the shocks is quite low, the highest value is 0.2. Fuhrer (2006) argues for a higher relative volatility of the NKPC shock, about 3/10 of the volatility of marginal cost.

[9] Choosing a lower value for indexation based on specification, one would generate less inflation persistence.

[10] Fuhrer (2006) assumes a three times larger relative volatility of the NKPC shocks and, therefore, requires substantially more intrinsic persistence, that is, a higher ρ, in order to match inflation persistence.

Figure 2 Inflation Dynamics for the Hybrid NKPC



Notes: The circles (squares) denote autocorrelations and cross-correlations from calibration 1 (2) of the hybrid NKPC. The boxes denote the 5-percentile to 95-percentile range of the statistic calculated from 1,000 bootstraps of data.

relative to the data: the predicted contemporaneous correlation coefficient is about 0.8, well above the observed value of 0.3.

Given the failure of the GMM-based calibration to account for the autocorrelation and cross-correlation structure of inflation and marginal cost, I now consider an alternative calibration that exactly matches the first-order autocorrelation of inflation and the contemporaneous cross-correlation of inflation and marginal cost. As I pointed out above, the estimated price adjustment probability of 10 percent per quarter is quite low. Other work suggests higher price adjustment probabilities, about 20 percent per quarter, e.g., Galí and Gertler (1999), Eichenbaum and Fisher (2007), or Cogley and Sbordone (2006).[11] For the alternative calibration I, therefore, assume that $\alpha = 0.8$. Conditional

[11] The NKPC specification in equation (14) is based on constant firm-specific marginal cost. Eichenbaum and Fisher (2007) and Cogley and Sbordone (2006) consider the possibility of increasing firm-specific marginal cost. Adjusting their estimates for constant firm-specific marginal cost yields $\alpha = 0.8$.

on an unchanged time discount factor, β, this implies a coefficient on marginal cost, $\kappa_0 = 0.05$, which represents an upper bound of what has been estimated for hybrid NKPCs.

I now choose intrinsic persistence, ρ, and the relative volatility of the NKPC shock, σ_u/σ_s, to match the sample first-order autocorrelation coefficient of inflation, $Corr\left(\hat{\pi}_t, \hat{\pi}_{t-1}\right) = 0.88$, and the contemporaneous correlation of inflation and marginal cost, $Corr\left(\hat{\pi}_t, \hat{s}_t\right) = 0.33$. This procedure yields a very large value for inflation indexation, $\rho = 0.86$, which makes inflation persistence essentially independent of marginal cost. A very high relative volatility of the NKPC shock, $\sigma_u/\sigma_s = 2.97$, can then reduce the co-movement between inflation and marginal cost without affecting inflation persistence significantly. The implied parameter values of this calibration are summarized in the second column of Table 3.

The autocorrelation and cross-correlation structure of the alternative calibration is represented by the squares in Figure 2. With few exceptions the cross-correlations predicted by the alternative calibration stay in the 5th to 95th percentile ranges of the observed cross-correlations. The autocorrelation coefficients continue to decline at a rate that is faster than observed in the data.

3. THE CHANGING NATURE OF INFLATION

The behavior of inflation has changed markedly over time, Table 1, column (1). Inflation tended to be below the sample mean in the 1950s and 1960s, average inflation was about 2.5 percent, but inflation increased in the second half of the 1960s. In the 1970s, inflation increased even more, averaging 6.5 percent and reaching peaks of up to 12 percent. In the early 1980s, inflation came down fast, averaging 3.2 percent from 1984 to 1991. Finally, in the period since the early 1990s, inflation continued to decline, but otherwise remained relatively stable, averaging about 2 percent.[12]

Most observers attribute the changes in average inflation since the 1960s to changes in monetary policy, as represented by different chairmen of the monetary policy committee of the Federal Reserve System. We have the "Burns inflation" of the 1970s, the "Volker disinflation" of the early 1980s, and the "Greenspan period" with a further reduction and stabilization of inflation from the late 1980s to 2005. Interestingly enough, these substantial changes in the mean inflation rate were not associated with comparable changes in mean marginal cost: average marginal cost differs by at most 3 percent across the sub-samples, Table 1, column 3.

[12] I choose 1970 as the starting point of the high inflation era since mean inflation before 1970 is relatively close to the sample mean. The year 1984 is usually chosen as representing a definite break with the high inflation regime of the 1970s, e.g., Galí and Gertler (1999) or Roberts (2006). Levin and Piger (2003) argue for a break in the mean inflation rate in 1991.

In the following, we will first show that allowing for changes in mean inflation rates affects the inflation dynamics as measured by the autocorrelation and cross-correlation structure. Since it appears that accounting for changes in the mean inflation rate affects the dynamics of inflation, we investigate whether the average inflation rate around which we approximate the optimal price-setting behavior of the firms in the Calvo model affects the dynamics of the NKPC.

Inflation Dynamics and Average Inflation[13]

The persistence and co-movement of inflation and marginal cost have varied across decades. In Figure 3, we display the autocorrelations and cross-correlations of inflation and marginal cost for the four periods we have just mentioned: the 1960s, 1970s, 1980s, and the period beginning in 1992.

In the 1960s, both inflation and marginal cost are highly persistent, with inflation being somewhat more persistent than marginal cost: the autocorrelation coefficients for inflation do not decline as fast as the ones for marginal cost. But in the following periods, it appears as if the persistence of inflation declines, at least relative to marginal cost. This decline of inflation persistence is especially noticeable for the first- and second-order autocorrelation coefficients from 1984 on, Figure 3, A.3 and A.4.[14]

The positive correlation between inflation and marginal cost in the full sample hides substantial variation of co-movement across sub-samples. The 1970s is the only period with a strong positive correlation between inflation and marginal cost, Figure 3, B.2. At the other extreme are the 1960s when the correlation between inflation and marginal cost is negative for almost all leads and lags, Figure 3, B.1. In between are the remaining two sub-samples from 1984 on, in which the correlation between inflation and marginal cost tends to be positive, but only weakly so.

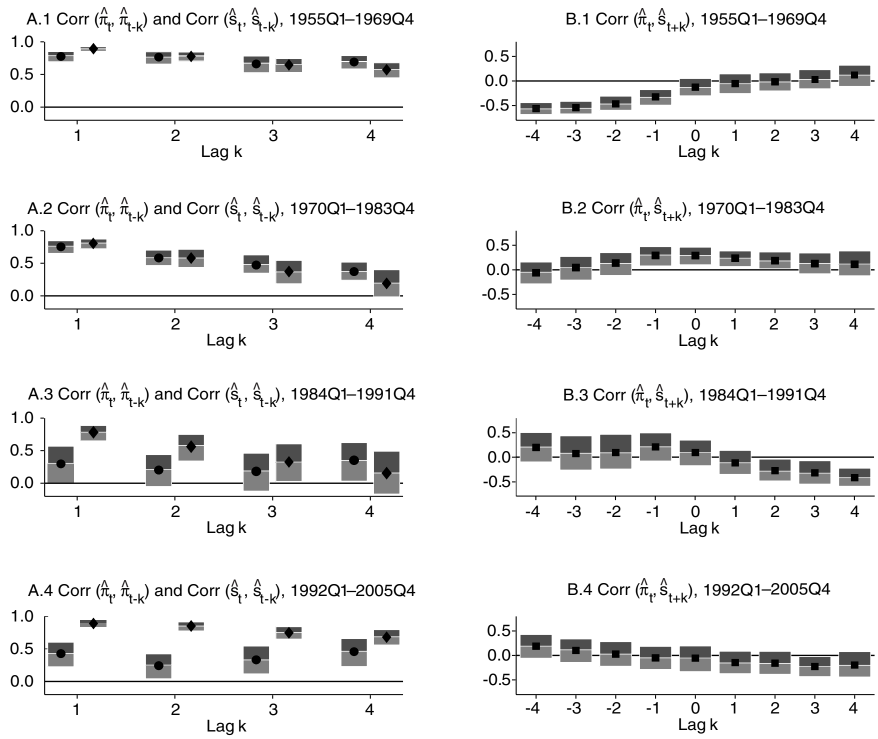
The NKPC at Positive Average Inflation

How should we interpret these changes in the time series properties of inflation and marginal cost? In particular, what do these changes tell us about the NKPC as a model of inflation? The decline in persistence is especially intriguing since it coincides with the decline of the average inflation rate. Most observers

[13] Articles that discuss changes in the inflation process include Cogley and Sargent (2001), Levin and Piger (2003), Nason (2006), and Stock and Watson (2007). Roberts (2006) and Williams (2006) relate the changes in the inflation process to changes in the Phillips curve.

[14] We should note, however, that the sum of autocorrelation coefficients from univariate regressions in the inflation rate and marginal cost do not indicate statistically significant changes in the persistence of inflation or marginal cost across subperiods, Table 1, columns 5 and 6.

Figure 3 Inflation and Marginal Cost Dynamics Over Time



Notes: In Panel A, the circles (squares) denote the sub-sample autocorrelations for inflation (marginal cost). In Panel B, the diamonds denote the cross-correlations of inflation and marginal cost. In Panels A and B, the boxes denote the 5-percentile to 95-percentile range of the statistic calculated from 1,000 bootstraps of the sub-sample data.

attribute the reduction of the average inflation rate to monetary policy, but should one also attribute the reduced inflation persistence to monetary policy?

From the perspective of the reduced form NKPC with no feedback from inflation to marginal cost, equation (15), monetary policy is unlikely to have affected the persistence of inflation. In this framework, monetary policy works through its impact on marginal cost, but if anything, marginal cost has become more persistent rather than less persistent since the 1990s. We now ask if this conclusion may be premature since it relies on an approximation of the inflation dynamics in the Calvo model around a zero-average inflation rate. If one approximates the inflation dynamics around a positive-average inflation rate, then inflation persistence depends on the average inflation rate, even when the other structural parameters of the environment remain fixed.

The modified hybrid NKPC for an approximation at the gross inflation rate $\bar{\pi} \geq 1$ is

$$E_t \left[\left(1 - \lambda_1 L^{-1}\right) \left(1 - \lambda_2 L^{-1}\right) (1 - \rho L)\, \hat{\pi}_t \right] = \kappa_1 E_t \left[\left(1 + \phi L^{-1}\right) \hat{s}_t \right] + u_t. \tag{18}$$

The derivation of (18) is described in Hornstein (2007).[15] The NKPC is now a third-order difference equation in inflation and involves current and future marginal cost. The coefficients $\lambda_1, \lambda_2, \phi$, and κ_1 are functions of the underlying structural parameters, α, β, ρ, and a new parameter θ, representing the firms' demand elasticity. Furthermore, the coefficients also depend on the average inflation rate, $\bar{\pi}$, around which we approximate the optimal pricing decisions of the firms.

The modified hybrid NKPC (18) simplifies to the hybrid NKPC (14) for zero net-inflation, $\bar{\pi} = 1$. As we increase the average inflation rate, inflation becomes less responsive to marginal cost in the modified NKPC. In Figure 4.A, we plot the coefficient on marginal cost κ_1 in the modified NKPC as a function of the average inflation rate for our two calibrations of the hybrid NKPC. In addition to the parameter values listed in Table 3, we also have to parameterize the demand elasticity of the monopolistically competitive firms, θ. Consistent with the literature on nominal rigidities, we assume that $\theta = 11$, which implies a 10 percent steady-state markup. For both calibrations, the coefficient on marginal cost declines with the average inflation rate, Figure 4.A. This suggests that everything else being equal, inflation will be less persistent and less correlated with marginal cost at higher inflation rates, since marginal cost has a smaller impact on inflation. The first calibration with a low price adjustment probability represents an extreme case, in that respect, since the coefficient on marginal cost converges to zero. On the other hand, for the second calibration with a higher price adjustment probability, the coefficient on marginal cost is relatively inelastic with respect to changes in the inflation rate.

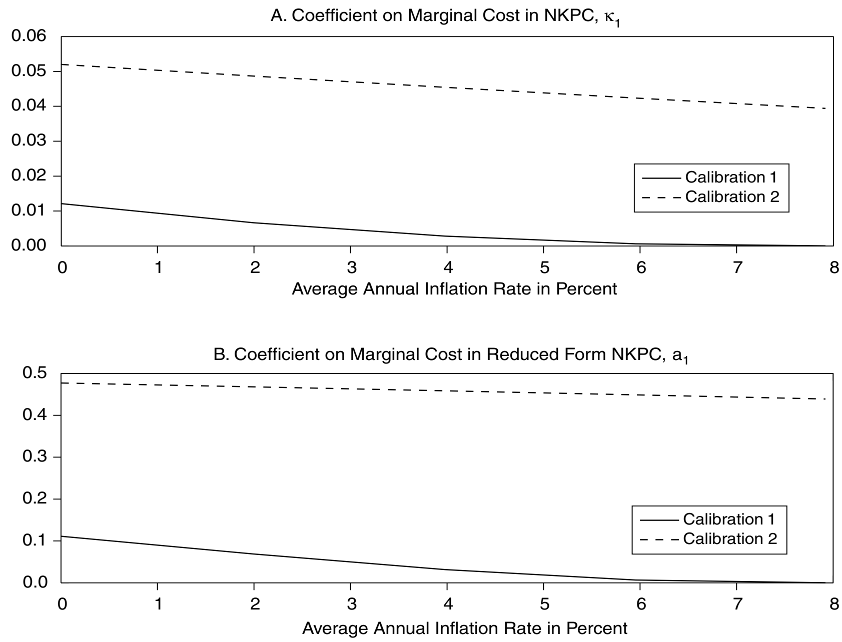
Assuming that marginal cost follows an AR(1) with persistence δ such that the product of δ and the roots of the lead polynomials in equation (18) are less than one, $|\delta\lambda_i| < 1$, we can derive the reduced form of the modified NKPC as

$$(1 - \rho L)\, \hat{\pi}_t = \kappa_1 \frac{1 + \delta\phi}{(1 - \lambda_1\delta)(1 - \lambda_2\delta)} \hat{s}_t + u_t = a_1 \hat{s}_t + u_t. \tag{19}$$

This expression is formally equivalent to the reduced form of the hybrid NKPC, equation (15), but now the coefficient a_1 is a function of the average inflation rate. Since inflation becomes less responsive to marginal cost in the NKPC

[15] Ascari (2004) and Cogley and Sbordone (2005, 2006) also derive the modified NKPC, but choose a different representation. Their representation is based on the hybrid NKPC, equation (14), and adds a term that involves the expected present value of future inflation.

Figure 4 The NKPC and Changes in Average Inflation

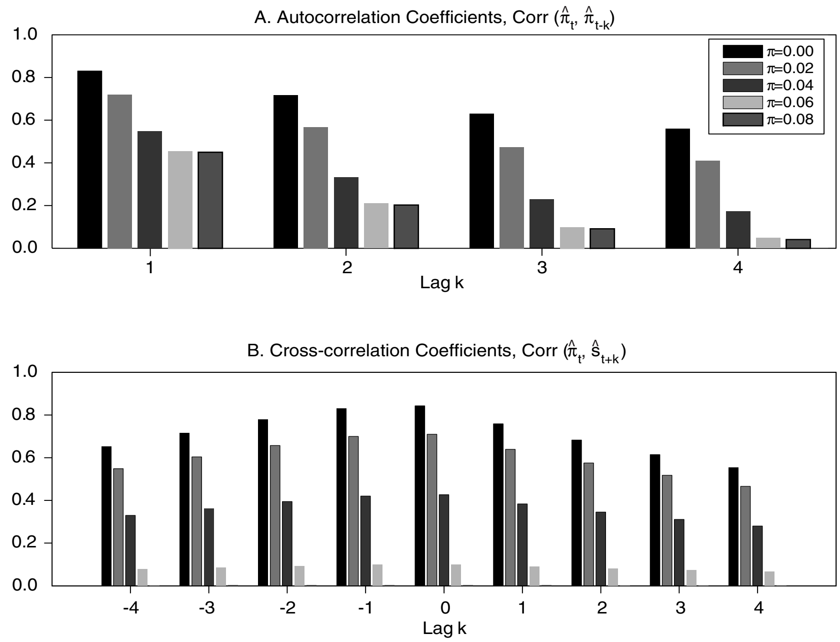


when the average inflation rate increases, inflation in the reduced form NKPC also becomes less responsive to marginal cost: a_1 declines with the average inflation rate, Figure 4.B. As with the coefficient on marginal cost in the NKPC, κ_1, the coefficient on marginal cost in the reduced form NKPC, a_1, declines much more for the first calibration with the relatively low price adjustment probability. This feature is important since the autocorrelations and cross-correlations of inflation depend on the average inflation rate only through the responsiveness of inflation to marginal cost, a_1.

We now replicate the analysis of Section 2 and calculate the first four autocorrelation coefficients of inflation and the cross-correlation coefficients of inflation with marginal cost when the average annual inflation rate varies from 0 to 8 percent.[16] In Figures 5 and 6, we display the autocorrelation and cross-correlation coefficients for the two calibrations. With a low price adjustment probability, the first calibration, an increase of the average inflation rate substantially reduces the persistence of inflation and its co-movement with marginal cost, Figure 5. Even moderately high annual inflation rates, about 4

[16] For the parameter values used in the calibration, the "weighted" roots of the lead polynominal are less than one for all of the average annual inflation rates considered.

Figure 5 The Effects of Average Inflation, Calibration 1

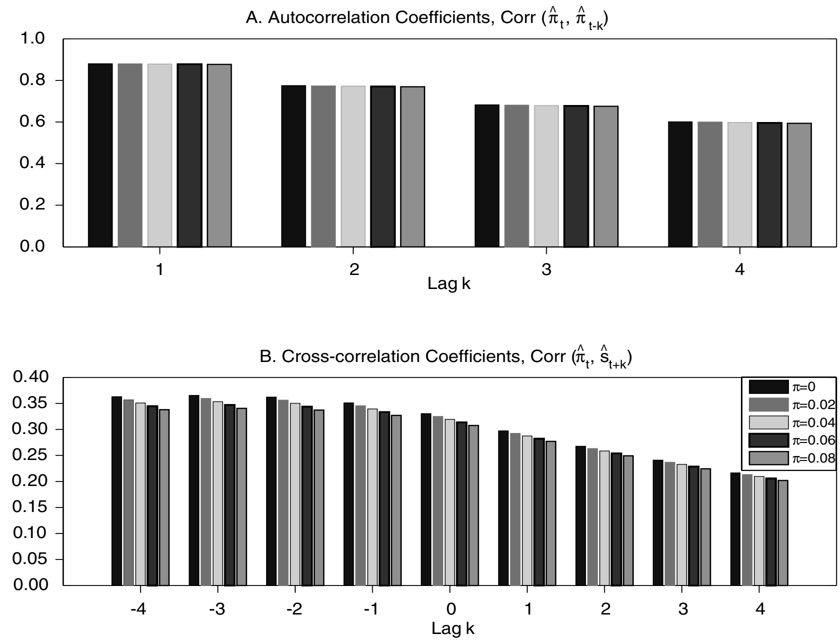


percent, reduce the first-order autocorrelation and the contemporaneous cross-correlation by half. This pattern follows directly from equations (16) and (17) and the fact that the coefficient a_1 converges to zero for the first calibration. With a higher price adjustment probability, the second calibration, a higher average inflation rate also tends to reduce persistence and co-movement of inflation, but the quantitative impact is negligible, Figure 6. Again, this pattern conforms with the limited impact of changes in average inflation on the reduced form coefficient of marginal cost.

Changing U.S. Inflation Dynamics and the Modified NKPC

Based on the modified NKPC, can changes in average inflation account for the changing U.S. inflation dynamics? Not really. There are two big changes in the average inflation rate between sub-samples of the U.S. economy. First, average inflation increased from 2.5 percent in the 1960s to 6.5 percent in the 1970s, and second, average inflation subsequently declined to 3.2 percent in the 1980s. These changes in average inflation were associated with significant changes in the persistence of inflation and the co-movement of inflation with marginal

Figure 6 The Effects of Average Inflation, Calibration 2



cost. Yet, the predictions of the modified NKPC for inflation persistence and co-movement based on the observed changes in average inflation are inconsistent with the observed changes in persistence and co-movement.

On the one hand, a calibration with relatively low price adjustment probabilities, the first calibration, predicts big changes for persistence and co-movement in response to the changes in average inflation, but the changes either do not take place or are opposite to what the model predicts. In response to the increase of the average inflation rate from the 1960s to the 1970s, inflation persistence and co-movement should have declined substantially, but persistence did not change and co-movement increased. Indeed the correlation between inflation and marginal cost switches from negative, which is inconsistent with the NKPC to begin with, to positive. In response to the reduction of average inflation in the 1980s, the model predicts more inflation persistence and more co-movement of inflation and marginal cost. Yet again, the opposite happens. Inflation persistence declines, at least the first- and second-order autocorrelation coefficients decline, and the correlation coefficients between inflation and marginal cost decline.

On the other hand, a calibration of the modified NKPC with relatively high price adjustment probabilities, the second calibration, cannot account for any quantitatively important effects on the persistence or co-movement of inflation based on changes in average inflation.

4. CONCLUSION

We have just argued that a hybrid NKPC, modified to account for changes in trend inflation, has problems accounting for the changes of U.S. inflation dynamics over the decades. One way to account for these changes of inflation dynamics within the framework of the NKPC is to allow for changes in the model's structural parameters. For example, inflation indexation, that is, intrinsic persistence, could have increased and decreased to offset the effects of a higher trend inflation in the 1970s. This pattern of inflation indexation in response to the changes in trend inflation looks reasonable. However, attributing changes in the dynamics of inflation to systematic changes in the structural parameters of the NKPC makes this framework less useful for monetary policy analysis. This is troublesome since several central banks have recently begun to develop full-blown Dynamic Stochastic General Equilibrium (DSGE) models with versions of the NKPC as an integral part. Ultimately, these DSGE models are intended for policy analysis, and for this analysis it is presumed that the model elements, such as the NKPC, are invariant to the policy changes considered. Based on the analysis in this article, it then seems appropriate to investigate further the "stability" of the NKPC before one starts using these models for policy analysis.

APPENDIX

We use seasonally adjusted quarterly data for the time period 1955Q1 to 2005Q4. All data are from HAVER with mnemonics in parentheses. From the national income accounts we take real GDP (GDPH@USECON) and for the GDP deflator we take the chained price index (JGDP@USECON). From the nonfarm business sector we take the unit labor cost index (LXNFU@USECON), the implicit price deflator (LXNFI@USECON), and the hourly compensation index (LXNFC@USECON). All of the three nonfarm business sector series are indices that are normalized to 100 in 1992.

We define inflation as the quarterly growth rate of the GDP deflator and marginal cost as the log of the ratio of unit labor cost and the nonfarm business price deflator. We construct the instruments for the GMM estimation other than lagged inflation and marginal cost following Galí, Gertler, and López-Salido (2005). The output gap is the deviation of log real GDP from a quadratic trend, and wage inflation is the growth rate of the hourly compensation index.

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