Economic Quarterly—Volume 93, Number 2—Spring 2007—Pages 111–142

Interest on Reserves and Daylight Credit

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Banks hold reserves in the form of account balances at the central bank and vault cash. The average aggregate reserves of depository institutions in the United States during 2005 was $46 billion. Banks use these reserves to settle payments to other banks (and other participants in financial markets) during the day. In 2005, the average daily value of Fedwire fund transfers—the primary means by which banks transfer funds to one another—was approximately $2 trillion; that is, nearly 50 times the quantity of reserves. When reserves do not pay interest overnight, banks face an opportunity cost from holding reserves overnight. However, if overnight overdrafts resulting from ending the day with insufficient reserves imply a penalty (in terms of higher interest rates or other types of penalties), then holding reserves may also be associated with the benefit of avoiding potential overdrafts. On average, during 2005 banks held a total of $1.7 billion in excess reserves; that is, reserves in excess of required reserves (see Table 1).

In September 2006, Congress passed legislation that authorized the Federal Reserve to pay interest on banks' reserve balances, beginning in 2011. The legislation also granted the Board of Governors additional flexibility in setting reserve requirements for depository institutions after October 1, 2011. According to this new legislation, the Federal Reserve can pay interest on all types of balances, including required reserves, supplemental reserves, and contractual clearing balances, held by or for depository institutions at a reserve bank. Such interest, if authorized by the Board, may be paid at least once each

We would like to thank Mike Dotsey, Marvin Goodfriend, Bob Hetzel, Todd Keister, Jeff Lacker, Antoine Martin, Jamie McAndrews, David Mills, Brian Minton, Ned Prescott, John Walter, and Alex Wolman for useful comments on a previous draft. We are also grateful to many of our colleagues at the Richmond Fed for helpful discussions and feedback. All errors are our own. The views expressed in this article do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System. E-mails: Huberto.Ennis@rich.frb.org and John.Weinberg@rich.frb.org.

Table 1 Total Reserves at Depository Institutions (Million $)

Date	Total Reserves	Non-Borrowed Reserves	Required Reserves	Excess Reserves
June 2003	42,025	41,864	39,980	2,046
December 2003	42,949	42,903	41,906	1,043
June 2004	45,720	45,540	43,787	1,933
December 2004	46,848	46,785	44,938	1,909
June 2005	45,950	45,701	44,176	1,774
December 2005	45,406	45,237	43,497	1,909
June 2006	45,067	44,814	43,282	1,785
October 2006	41,756	41,528	40,058	1,698

Notes: Federal Reserve Statistical Release H.3, Table 2, Aggregate Reserves of Depository Institutions and the Monetary Base (not adjusted for changes in reserve requirements and not seasonally adjusted) (http://www.federalreserve.gov/releases/h3/).

calendar quarter at a rate or rates not to exceed the general level of short-term interest rates.

This new legislation represents a significant change in policy that could affect the choices that banks make about reserve holdings. And, since most central banks conduct monetary policy by intervening in the daily market for banks' reserves, this change could affect the implementation of monetary policy as well by altering the behavior of the demand for reserves. Since paying interest reduces the opportunity cost for a bank of being "stuck" with unused reserves overnight, banks may become willing to hold greater reserves. But the demand for reserves depends not only on this opportunity cost, but also on the benefit of avoiding the need to borrow to make up for a reserves shortfall. It is also likely that the demand for reserves depends on the nature of the payments for which the reserves will be used.

In the settlement of payments during a business day, banks' reserves are supplemented by access to intraday credit from the Fed. If one bank seeks to send funds in excess of its reserve balance through the Fedwire system to another bank, the sender incurs a daylight overdraft. So reserves and daylight credit act as a substitute means of funding transfers during the day. The treatment of reserves overnight, though, can influence the degree to which banks rely on daylight credit to cover their daylight payment activity.

The opportunity cost of holding reserves is most directly affected by the central bank's interest rate policy. A bank's willingness to substitute away from reserves for payment purposes is directly affected by the terms on which the central bank provides daylight credit. In this article, we are interested in the link between these terms and the terms on overnight reserves. We provide a simple model of the demand for reserves by banks (in line with the classic contribution by Poole 1968) and study the implications of paying interest on reserves on the conduct of monetary policy and the use of daylight credit by banks.

One important public policy dimension with regard to daylight credit is absent from our model. Specifically, the model abstracts from credit risk incurred by the central bank. When banks settle payments by drawing on central bank credit, the result is to shift credit risk exposure from private counterparties to the central bank. Central banks have a number of tools available for managing this exposure, from the pricing of daylight credit to the imposition of credit caps or collateral requirements. To address these and other public policy questions adequately would require a more complete, general equilibrium model. The model we examine is meant to isolate some key forces that we think would be at work in the joint determination of the demand for reserves and for daylight credit in a reasonable, more general, model. Understanding the forces driving daylight credit is important because of the potential for overuse of underpriced central bank credit and the associated misallocation of risk.

Before presenting the model, we discuss in Section 1 some basic observations about reserves, payments, and credit in the Fed's large-value payment system. Section 2 introduces the basic model of banks' demand for reserves and the determination of the equilibrium interest rate in the market for reserves. Banks' demand for reserves in our model is purely voluntary. No reserve requirements are assumed. The reason banks hold reserves in our model is because reserves are useful for making payments. The alternative assets, in our case bonds, have a positive overnight rate of return premium but cannot be used to make payments. If the bank does not have enough reserves to settle its payments, it has to resort to central bank credit. Overnight overdrafts, in particular, are subject to a penalty rate that banks want to avoid paying. In other words, banks hold reserves to limit their exposure to overdraft penalties.

In Section 3, we introduce the central bank's ability to pay interest on unused reserves. We show how interest on reserves allows the central bank to fix the market interest rate at a target level by "flooding" the market with reserves and fixing the interest on reserves at the chosen target. This policy was first proposed by Goodfriend (2002) and the model provides a formalization of his argument. The model also allows a precise description of an alternative approach to paying interest on reserves. In this approach, the central bank pays a rate at a fixed spread *below* the target market rate, which, together with an overnight lending rate at a fixed spread *above* the target, creates a "corridor" around the market rate.

Sections 2 and 3 consider the demand for reserves in the absence of a potential payments-related need for daylight credit. However, as noted by Lacker (2006) and as suggested by the interdependence discussed above, the ability of the central bank to pay interest on reserves may have relevant implications for the daylight credit policy that the central bank may find optimal.

Section 4, then, extends the model in Sections 2 and 3 to take into consideration the determinants of the daylight credit decisions of banks. We show how interest on reserves can motivate banks to economize the use of daylight credit without reducing their access to liquidity during the day.

Our simple model allows us to demonstrate a number of interesting features of the mechanics of the markets for reserves. For instance, in a corridor system, there are circumstances in which a central bank can implement a change in the target market rate without changing the supply of reserves, simply by moving its lending rate and its rate on reserves together. But this result requires that aggregate demand for reserves—which is driven in the model by aggregate payment requirements—be relatively stable. With greater variability in demand, the task of implementing the target rate is simplified by the approach proposed by Goodfriend of paying interest at the target rate. When intraday variation in the timing of payments is added, which creates a potential demand for daylight credit, eliminating the opportunity cost of holding reserves by paying interest at the target rate has the added effect of greatly reducing the demand for daylight credit.

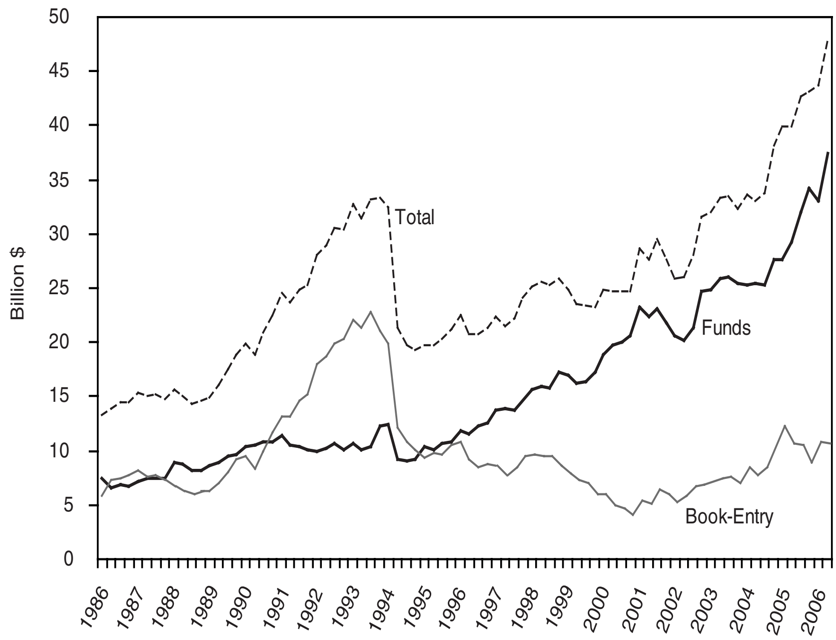
1. U. S. PAYMENTS AND RESERVES

Systems for clearing and settling large-value payments among banks are often categorized according to their approach to settlement. Systems in which payments are settled one-by-one through the transfer of central bank money throughout the day are typically referred to as real-time gross settlement systems (RTGS). The alternative is net settlement, in which payments are held until the end of a settlement period, typically a day, and only net obligations are actually transferred. Zhou (2000) provides a good introduction to these differences, and Kahn and Roberds (1999) discuss in detail the comparative advantages and disadvantages of the alternative systems.

A notable difference between these two alternative ways of organizing (large-value) payment systems is that a daily net settlement arrangement involves the creation of intraday credit exposures among its members. By contrast, in an RTGS system, bilateral obligations are extinguished throughout the day. Because of possible mismatches in the timing of receipts and payments during the day, participants in an RTGS system may demand credit to cover early payments when they are expecting later receipts. In some of these systems, intraday credit is provided by the central bank.

For the most part, large-value payments in the United States are executed using one of the two main systems: Fedwire and CHIPS (Clearing House Interbank Payments System). Fedwire has two subsystems: Fedwire Funds Transfer and Fedwire Book-Entry Securities. The Fedwire Funds Transfer system is a real-time gross settlement system of funds transfers across Federal Reserve accounts of participants. The Fedwire Book-Entry Securities system

Figure 1 Average Daylight Overdraft (Quarterly Data)



Notes: Federal Reserve Board Payment System Risk data. Average daylight overdrafts are calculated based on a 21.5-hour Fedwire operating day (http://www.federalreserve.gov/paymentsystems/psr/).

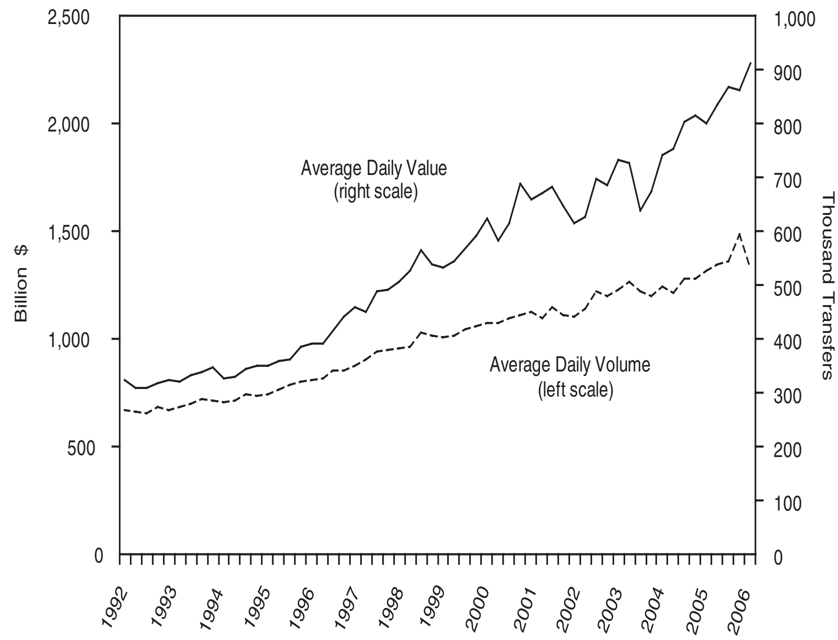
is a real-time, delivery-versus-payment, gross settlement system that allows for the immediate, simultaneous (electronic) transfer of government securities against payment.[1]

CHIPS is a bank-owned payment system operated by the New York Clearing House to clear and settle business-to-business transactions. On January 22, 2001, CHIPS converted from an end-of-day, multilateral net settlement system to one that provides final settlement for all payment orders as they are released. Payment instructions submitted to the queue that remain unsettled at the end of the day, known as the residual, are tallied on a multilateral net basis. Banks pre-fund their CHIPS payments with a Fedwire transfer from their reserve accounts at the Fed at the beginning of the day.

To facilitate the normal flow of payments in the system, the Federal Reserve provides daylight credit to depository institutions. In this context, the Federal Reserve has adopted an explicit program to control the use of intraday

[1] A delivery-versus-payment system is a mechanism that ensures that the final transfer of one asset occurs if and only if the final transfer of another asset occurs.

Figure 2 Fedwire Funds Transfers (Quarterly Data)



Notes: Federal Reserve Board Fedwire Funds Service quarterly data. The average daily volume and average daily value of transfers are based on the number of business days in each period (http://www.federalreserve.gov/paymentsystems/fedwire/).

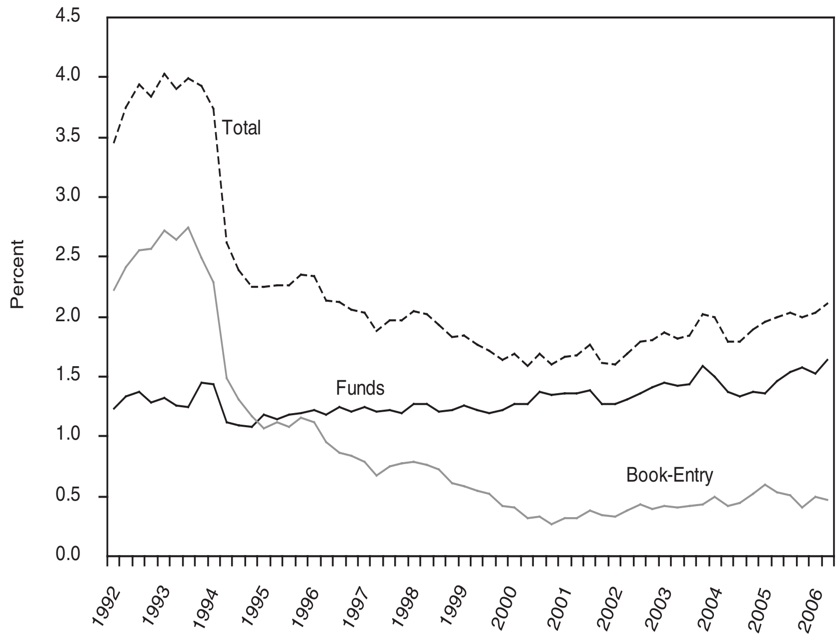
credit, the Payments System Risk (PSR) policy (Coleman [2002] provides a good introduction to the evolution of the PSR policy of the Fed). The two main instruments of the PSR policy are the imposition of net debit caps and interest rate fees on daylight overdrafts. The objective is to limit excessive use of daylight credit and, therefore, reduce the Fed's exposure to credit risk.

In 1985, the Fed introduced net debit caps for the first time. Net debit caps limit the maximum daylight overdraft position that a depository institution can incur in its Federal Reserve account. These debit caps did not have a great influence on the expansion of daylight credit that was taking place at the time, and, in 1994, the Federal Reserve started imposing a minute-by-minute interest charge on the average daylight overdraft that each institution incurred during the business day.

Figure 1 shows the large drop on average daylight overdraft after that change in policy.[2] (For a careful statistical analysis of the effect of caps and

[2] Most of the reduction in daylight overdrafts after the change in policy in 1994 was due to a reduction in securities-related overdrafts. Charging fees provided a strong incentive for securities

Figure 3 Average Daylight Overdraft as a Percentage of Average Daily Volume of Transfers (Quarterly Data)



Notes: Federal Reserve Board data (see Figures 1 and 2). Average daylight overdrafts are calculated based on a 21.5-hour Fedwire operating day (http://www.federalreserve.gov/paymentsystems/psr/).

fees on the level of daylight overdrafts in the United States, see Hancock and Wilcox 1996. See also Mills and Nesmith 2006.) It is important to note, however, that the number and value of Fedwire transactions has been trending upwards during this period (see Figure 2) and that, in fact, the value of the average daylight overdraft, as a percentage of the value of total transactions over Fedwire, has remained relatively stable at around 1.8 to 2.0 percent for the last ten years, displaying perhaps a slight upward trend (see Figure 3).

The need for credit in the payment system is determined by banks' holdings of reserves. The reserve positions of banks are, in turn, determined by an array of factors, including legal reserve requirements and the price of borrowing reserves, either on the federal funds market or from the Fed's discount window.

dealers to adopt practices that reduced the use of intraday credit; in particular, they substantially revised their repo settlement practices. Net debit caps were reduced by 25 percent in early 1988 and daylight credit initially fell (approximately 5.5 percent) but then continued growing at an accelerated pace until 1994, as seen in Figure 1. The data on peak daylight credit present a similar pattern to that for average daylight credit presented here.

Desired reserves could also depend on the cost of borrowing from the Fed within the day. For a given level of payment activity, daylight overdrafts will typically decrease as reserve holdings of banks increase. The model in the next section provides a first step in formalizing some of the relationships among payments, demand for reserves, and interest rates, which are essential for understanding how modern payment systems function.

2. A SIMPLE MODEL

We start our analysis with a very simple model that allows us to capture some of the tradeoff faced by banks in their management of reserves. In fact, in the next two sections we abstract from issues related to daylight credit and keep the model and analysis as simple as possible. These sections provide a good introduction to the main forces driving the determination of the interest rate in the market for reserves. Later, in Section 4, we extend the model in a natural way and discuss the connection among daylight credit, reserve management by banks, and the market interest rate.

We start our study of the simple model by first describing the decision problem faced by a typical bank. The solution to this problem delivers the demand for reserves for each individual bank (Poole 1968). After that, we consider the situation in which there are many banks and aggregate their demands to obtain the demand for reserves in the market. Finally, we study the determination of interest rate as a result of a standard equalization of (aggregate) demand and supply.

The Bank's Problem

Let us start with a simplified setup. Suppose the bank has a given amount of funds, F, that will be used to execute some payments for the same amount. While the the total amount of payments, F, is known with certainty, payments can happen at the end of the day or next morning. The bank can decide to hold these funds in either of two possible assets, reserves (R) or bonds (B). That is, we have that

$$F = R + B.$$

Let P be the payment that the bank has to make at the end of the day. The next-morning payment will then be equal to $F - P$. Suppose that bonds cannot be used to settle payments, and the bank must decide the allocation of funds between reserves and bonds before knowing the exact amount P. Also, for simplicity, assume that payments today are only credited in the recipient's account the next day; that is, the bank does not expect to receive new funds that would increase its end-of-day balances. Then, if the amount of reserves

R held by the bank is lower than the required end-of-day payment P, the bank incurs an overnight overdraft for the value $P - R$.[3]

For concreteness, assume that the size of the end-of-day payment P is uniformly distributed in the interval $\left[0, \overline{P}\right]$. The assumption about the distribution of P is just for the sake of simplicity; it implies that the size of the payment can take any value in the interval $\left[0, \overline{P}\right]$, with the probability of observing any particular one of these values being the same. More importantly, the probability that P is smaller than an arbitrary value $x \in \left[0, \overline{P}\right]$ is given by $p(x) = x/\overline{P}$, and the average value of P conditional on being greater than x is given by

$$E_{x+}P = \frac{\overline{P} + x}{2}.$$

Let r be the (overnight) rate of return on bonds and r_o the interest rate on overnight overdrafts. We consider the case where the overnight overdraft rate implies a penalty; that is, $r_o > r$. Reserves give no return but can be used to cover part (or all) of the payment P. Throughout this article we assume that $F > \overline{P}$.

The overnight expected return for the bank, denoted by Π, is then given by

$$\Pi = [1 - p(R)] \left[rB - r_o \left(E_{R+}P - R\right)\right] + p(R)rB.$$

The first term tells us that, with probability $1 - p(R)$, the bank needs to make an end-of-day payment P greater than R and, hence, the bank has to incur an overdraft. The expected overdraft is given by the amount $E_{R+}P - R$. With probability $p(R)$, the payment P is smaller than the total reserves held by the bank and the bank just gets the normal return on its bond holdings r, B.

Rearranging the expression for the bank's return we have that

$$\Pi = rB - [1 - p(R)] \, r_o \left(E_{R+}P - R\right).$$

Using the equation $F = B + R$ and substituting the expression for $E_{R+}P$ and $p(R)$, we can rewrite the expression for Π as

$$\Pi = r(F - R) - \left(1 - \frac{R}{\overline{P}}\right) r_o \left(\frac{\overline{P} - R}{2}\right),$$

which again can be rewritten as

$$\Pi = rF - \left[rR + \frac{r_o}{2\overline{P}} \left(\overline{P} - R\right)^2\right].$$

[3] Alternatively, we could interpret P as the value of the required payments net of any new balances arriving late in the day. Our simplistic assumption about turnover of reserve flows facilitates the analysis of equilibrium but is not essential for the results. However, within-the-day turnover brings about a number of other interesting issues that we do not discuss in this article (see Beyeler et al. 2006 for a careful study regarding this issue).

The bank will choose its level of reserves, R, to maximize its overnight expected return Π. Then, when $r_o > r > 0$, the demand for reserves by the typical bank is given by

$$R^* = \frac{(r_o - r)}{r_o}\overline{P}. \tag{1}$$

This expression tells us that, when the interest rate on bonds, r, increases, the bank will lower the amount of reserves held (the opportunity cost of holding reserves is higher). Also, as the size of the possible payments increases (that is, as $\overline{P}$ increases), *ceteris paribus*, the bank will choose to hold higher levels of reserves (reserves are more likely to be useful in avoiding overdrafts). It is a little less obvious to see, yet still true, that if the value of the overdraft interest rate, r_o, increases, the optimal level of reserves, $R,^*$ also increases. Finally, notice that for $r \geq r_o$ the bank will demand zero reserves and for $r = 0$ the bank will hold any amount of reserves between $\overline{P}$ and F.

We have assumed that the total amount of funds held by the bank is fixed, and equal to F.[4] Under this assumption, equation (1) has an alternative interpretation. The equation tells us that the penalty premium on overnight overdrafts, given by $r_o - r$, determines the *composition* of the bank's portfolio between bonds and reserves. Reserves are held to avoid paying the penalty premium. However, reserves do not gain interest overnight. Hence, holding reserves also has an opportunity cost. The bank balances these costs and benefits to determine the optimal composition of its portfolio. In this simple model, the only reason for banks to hold reserves is to avoid paying the overnight penalty rate. If $r_o = r$, then there are no benefits of holding reserves (while there is still an opportunity cost), and the proportion of funds held as reserves is zero.

The Market for Reserves

Normally, there are many banks interacting in the market and deciding their optimal level of reserves.[5] In principle, we can aggregate all their demands to obtain the market demand. Recall that the demand for reserves of bank i is given by

$$R_i^* = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{r_o}\overline{P}_i & \text{if} \quad r_o > r > 0 \\ \left[\overline{P}_i, F\right] & \text{if} \quad r = 0. \end{cases}$$

[4] Then, in fact, we could normalize F to unity, in which case R and B could be interpreted as the proportion of funds held in reserves and bonds, respectively.

[5] See Bartolini et al. (2005) for a more detailed discussion of the overnight federal funds market in the United States where over 7,500 institutions with accounts at the Federal Reserve borrow and lend reserve balances on an uncollateralized basis.

Note that r_o and r are market prices common to all banks, but the distribution of likely end-of-day payments may differ across banks, and hence $\overline{P}_i$ may differ across banks.

For simplicity we will assume that there is a large number (a continuum) of banks (with mass equal to one). Then, when $r_o > r > 0$ the total demand for reserves in the market is given by

$$R^d = \int_0^1 R_i^* di = \frac{(r_o - r)}{r_o} E\overline{P},$$

where $E\overline{P}$ is the average value (across banks) of the maximum possible required payment. Note that in our simple model, the expected payment requirement for bank i is given by $\overline{P}_i/2$ and the average across banks is then equal to $E\overline{P}/2$. Hence, the variable $E\overline{P}$ characterizes the level of payment requirements in the economy.

Let us also assume that, each day, the aggregate volume of end-of-day payments can be in either of two possible states, high or low.[6] In other words, some days the required payments (on average) tend to be high, and some days they tend to be low. We capture this idea by allowing $E\overline{P}$ to take two possible values, $E\overline{P}_H$ and $E\overline{P}_L$, with $E\overline{P}_H > E\overline{P}_L$ and the probability of $E\overline{P}_H$ equal θ (hence, the probability of $E\overline{P}_L$ equals $1 - \theta$).

Here, for simplicity, we assume that banks know the level of aggregate required payments before choosing their demand for reserves. Note, then, that for a given value of r_o, the aggregate demand for reserves is a function of the interest rate r and the level of required payments indexed by $E\overline{P}_j$. Then, we can write $R_j^d(r) \equiv R^d(r; E\overline{P}_j)$ with j being equal to H and L.

By buying and selling bonds in exchange for reserves, the central bank controls the relative supply of reserves available in the system. Given a value of the supply of reserves, R^s, there exists an interest rate r^* that clears the market; that is, there exists an interest rate r_j^* such that

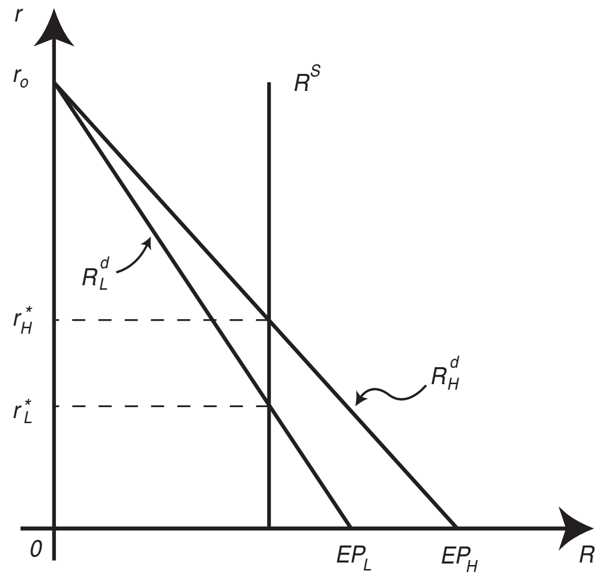
$$R_j^d(r_j^*) = R^s, \tag{2}$$

with $j = H, L$. Figure 4 provides a graphic representation of this market-clearing condition.

Equation (2) defines an implicit function $r_j^*(R^s)$ for the market-clearing interest rate given a supply of reserves, R^s. In particular, if $R^s \in (0, E\overline{P}_L)$

[6] One possible interpretation of the fluctuations in the aggregate (net) volume of payments by banks is that the fluctuations originate in flows between the government and the banking system. Dotsey (1991) and Guthrie and Wright (2000) take this interpretation in their study of New Zealand's system. See also Bartolini, Bertola, and Prati (2002) for a study of the U.S. system that uses a similar interpretation.

Figure 4 The Market for Reserves



we have that

$$r_j^*(R^s) = r_o \left(1 - \frac{R^s}{E\overline{P}_j}\right). \tag{3}$$

Then, a higher supply of reserves, R^s, implies a lower market interest rate. Also, it is easy to see that (for a given R^s) the market interest rates satisfy $r_H^* > r_L^*$. In other words, if the central bank were to fix the supply of reserves, R^s, the interest rate would be higher in periods of high payment requirements and lower in periods of low payment requirements.

We can also think of this relationship between the market interest rate and the supply of reserves in a slightly different way. Suppose that the central bank wants the interest rate to be equal to some target level r_T. Then, there is a level of the supply of reserves R_j^s such that

$$r_j^*(R_j^s) = r_T.$$

In particular, we have that if $r_T \in (0, r_0)$ then

$$R_H^s = \frac{r_o - r_T}{r_o} E\overline{P}_H > \frac{r_o - r_T}{r_o} E\overline{P}_L = R_L^s;$$

that is, to maintain a given target interest rate, the central bank has to provide a higher supply of reserves in periods of high payment requirements.

Now, suppose that the central bank has a target interest rate but, for some reason, has to decide the supply of reserves before knowing whether the level of payment requirements in the banking system will be high or low. In principle, the central bank will try to predict the level of demand for reserves. However, the predictions may not be perfect. In this case, a possible strategy the central bank could follow is to fix the supply of reserves such that the *average* interest rate equals the target rate. The market rate, then, will fluctuate around the target rate, being higher than the target rate in periods of high demand (that is, when $E\overline{P} = E\overline{P}_H$) and lower than the target rate during period of low demand (that is, when $E\overline{P} = E\overline{P}_L$).

To see this in the model, notice that the central bank would choose the supply of reserves R^{sT} such that

$$\theta r_H^*(R_T^s) + (1-\theta)\, r_L^*(R_T^s) = r_T. \tag{TC}$$

To simplify the exposition, let us define the variable $\eta_j \equiv 1/E\overline{P}_j$ and concentrate our attention on the case where r_j^* is lower than r_o and positive for both $j = H$ and $j = L$.[7] Then, using expression (3) we have that the average interest rate can be rewritten as follows:

$$\theta r_H^*(R_T^s) + (1-\theta)\, r_L^*(R_T^s) = \theta r_o \left(1 - \eta_H R_T^s\right) + (1-\theta)\, r_o \left(1 - \eta_L R_T^s\right),$$

Reorganizing terms, the target condition (TC) becomes

$$r_o \left(1 - \overline{\eta} R_T^s\right) = r_T,$$

where $\overline{\eta} = \theta\eta_H + (1-\theta)\,\eta_L$. Equivalently, we can rewrite the above condition as

$$R_T^s = \frac{r_o - r_T}{r_o} \frac{1}{\overline{\eta}},$$

which tells us that to implement a higher average (target) rate the central bank will need to provide a lower supply of reserves. Note that $\eta_H \leq \overline{\eta} \leq \eta_L$ and that $\overline{\eta}$ is a decreasing function of θ. This property of $\overline{\eta}$, in turn, implies that when the probability of a high demand for reserves increases, the central bank, to target the same average rate of interest, will need to supply a higher amount of reserves. Finally, note that the market interest rate will be given by

$$r_j^*(R_T^s) = r_o \left(1 - R_T^s \eta_j\right).$$

Using the expression for R_T^s we have that

$$r_j^* = r_T + (r_o - r_T)\left(\frac{\overline{\eta} - \eta_j}{\overline{\eta}}\right),$$

which implies that $r_H^* \geq r_T$ (since $\overline{\eta} \geq \eta_H$), $r_L^* \leq r_T$ (since $\overline{\eta} \leq \eta_L$), and $r_L^* < r_H^*$. These inequalities confirm our previous claim stating that the market

[7] While the other possible cases are similar, they are less interesting.

rate will be higher than the target rate in periods of high demand and lower than the target rate during periods of low demand.

3. INTEREST ON RESERVES

In the previous section, we considered the case in which reserves held by banks and not used in payments yielded no interest overnight. In general, banks hold reserves as balances in an account at the central bank, and in principle the central bank could pay interest on those unused reserves. We consider this possibility in this section.

There are different ways the central bank can pay interest on reserves. Here we concentrate on one possible scheme that has been discussed in policy circles (see Lacker 2006). Under this scheme, the central bank automatically pays interest overnight on all unused reserves held by banks at the end of the day after all payments have been executed. We call this scheme a sweep facility.

Overnight Sweep of Reserves

Suppose unused reserves are "swept" overnight into bonds that pay an interest rate r_s. Then, banks obtain a return of r_s on the amount $R - P$, whenever this difference is positive. In this case, the overnight expected return for the bank is given by

$$\Pi(R) = [1 - p(R)]\left[rB - r_o\left(E_{R_+}P - R\right)\right] + p(R)\left[rB + r_s\left(R - E_{R_-}P\right)\right],$$

where $E_{R_+}P$ is the expected value of P conditional on being greater than R, and $E_{R_-}P$ is the expected value of P conditional on being smaller than R. Here, it is important to note that if $R \geq \overline{P}$ then $p(R) = 1$, $E_{R_+}P = 0$, and $E_{R_-}P = \overline{P}/2$. After some manipulations, the expression for $\Pi(R)$ can be rewritten as

$$\begin{aligned}\Pi(R) &= rF - r\frac{\overline{P}}{2} \\ &\quad - [1 - p(R)]\,(r_o - r)\left(E_{R_+}P - R\right) \\ &\quad + p(R)(r_s - r)\left(R - E_{R_-}P\right).\end{aligned}$$

The second term in this expression ($-r\overline{P}/2$) is the average forgone interest from making the required end-of-day payment. The third term is the cost of covering the high end of the distribution of payments with overnight overdrafts, and the fourth term is the (potential) net benefit of getting the sweep interest rate on unused reserves (on the low end of the distribution of required payments).

Recall that $F \geq \overline{P}$. Then, if $r_s > r$, it is clear that the bank would choose the level of R to equal F; that is, the bank would maintain all its funds in the form of reserves. To see this, note that, for all $R \geq \overline{P}$ the overnight expected return is given by

$$\Pi(R \mid R \geq \overline{P}) = rF - r\frac{\overline{P}}{2} + (r_s - r)\left(R - \frac{\overline{P}}{2}\right),$$

and when $r_s > r$ we find that $\Pi(F) > \Pi(R)$ for all $R \leq F$ and, hence, $\Pi(R)$ is maximized at $R = F$.[8] It is not hard to see that even if $r_s = r$ we still find that $\Pi(F) \geq \Pi(R)$ for all $R \leq F$. However, as long as R is greater than $\overline{P}$, the bank is indifferent over the composition of its portfolio; that is, the bank makes the same return independent of how much of its funds are held in reserves (as long as they are enough to cover all possible end-of-day payments).

When $r_s < r \leq r_o$, the bank's demand for reserves is given by

$$R^* = \frac{(r_o - r)}{(r_o - r_s)}\overline{P},$$

where R^* is the (interior) value of R that maximized Π.[9] Note that, since in this case $r_s < r$, we find that $R^* < \overline{P}$ and, for some high possible realizations of the size of the payment P, the bank will not have enough reserves and will take an overnight loan at the penalty rate r_o.

The demand for reserves of an individual bank is then given by

$$R^* = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{(r_o - r_s)}\overline{P} & \text{if} \quad r_o > r > r_s \\ \left[\overline{P}, F\right] & \text{if} \quad r = r_s \\ F & \text{if} \quad r < r_s. \end{cases}$$

The Market for Reserves Under Sweeps

Using the demand function for individual banks, we can aggregate across banks and obtain the market demand for reserves under a sweep system. Following the aggregation procedures used before, we find that

$$R^d = \begin{cases} 0 & \text{if} \quad r \geq r_o \\ \frac{(r_o - r)}{(r_o - r_s)}E\overline{P} & \text{if} \quad r_o > r > r_s \\ \left[E\overline{P}, F\right] & \text{if} \quad r = r_s \\ F & \text{if} \quad r < r_s. \end{cases}$$

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[8] Here we are not allowing the bank to hold a negative position on bonds. If banks could short-sell bonds then r_s would be the effective "floor" of the market interest rate. None of our results depend on this assumption.

[9] See Whitesell (2006) for a similar analysis that would correspond to the case when P has a normal distribution. Also, Dotsey (1991) and Guthrie and Wright (2000) use versions of this theory to explain monetary policy implementation by the Reserve Bank of New Zealand.

As long as the supply of bonds is positive, the equilibrium interest rate r^* cannot be lower than r_s. The reason for this result is that if $r^* < r_s$ then all banks will want to hold all their funds as reserves. In this case, the demand for bonds in the market is equal to zero and the market for bonds does not clear (since the supply was positive). Figure 5 illustrates the determination of the market-clearing interest rate under a sweeps system. Note that if the supply of reserves by the central bank R^s is greater than $E\overline{P}$ and less than F, then the market-clearing interest rate r^* equals r_s.

Two important insights, useful in understanding monetary policy implementation, result from studying the determination of interest rates in a market in which banks have available a sweep facility that allows them to earn interest on reserves. The first insight is related to the role of the net supply of reserves in a so-called "corridor" system (see Guthrie and Wright 2000 and Whitesell 2006 for recent, more thorough discussions of corridor systems).[10] The second insight, discussed extensively by Goodfriend (2002) (see also Woodford 2000), is related to the advantages of "flooding" the market with reserves as a means of targeting a specific market interest rate.

In fact, the corridor and Goodfriend systems have been regarded as two alternative schemes for the implementation of interest rate policy. Next, we provide a brief introduction to these systems in the context of our model. While abstracting from many important issues, we believe that the discussion that follows can be helpful in understanding the relative advantages of each of the systems.

The Corridor System

For simplicity, let us concentrate on the case where $E\overline{P}$ is constant. Suppose that the monetary authority wishes to target a given rate r_T. One alternative is to use a corridor system, in which the overnight overdraft and sweep rates are set as follows:

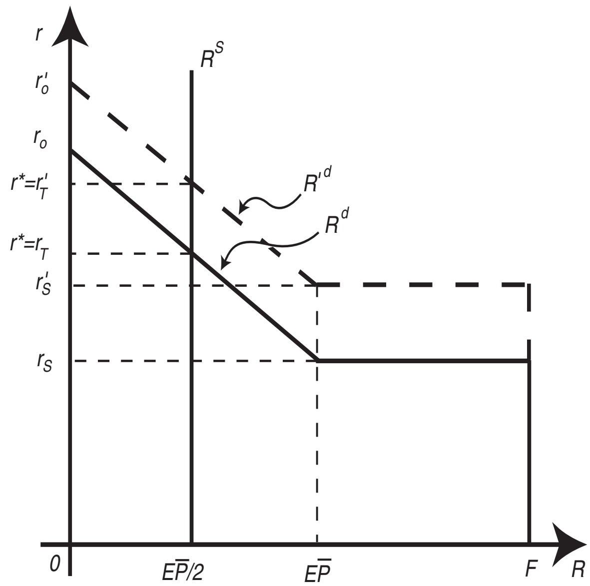
$$r_o = r_T + \frac{\delta}{2} \text{ and } r_s = r_T - \frac{\delta}{2}.$$

We will call δ the size of the corridor. Then, by setting the supply of reserves equal to $E\overline{P}/2$, the monetary authority can drive the market interest rate r^* to equal the target rate r_T.[11] What is most interesting about this system is that, to the extent that the value of $E\overline{P}$ is fairly stable, the monetary authority can drive the market rate to *any* target it wishes, just by changing proportionally the rates r_o and r_s (or, in other words, given the size of the corridor, by

[10] See also Berentsen and Monnet (2006) for a general equilibrium analysis of a corridor system.

[11] To see this, substitute the formula for the demand of reserves (the case when $r_o > r > r_s$) in the equation $R^d = R^s = E\overline{P}/2$.

Figure 5 The Corridor System



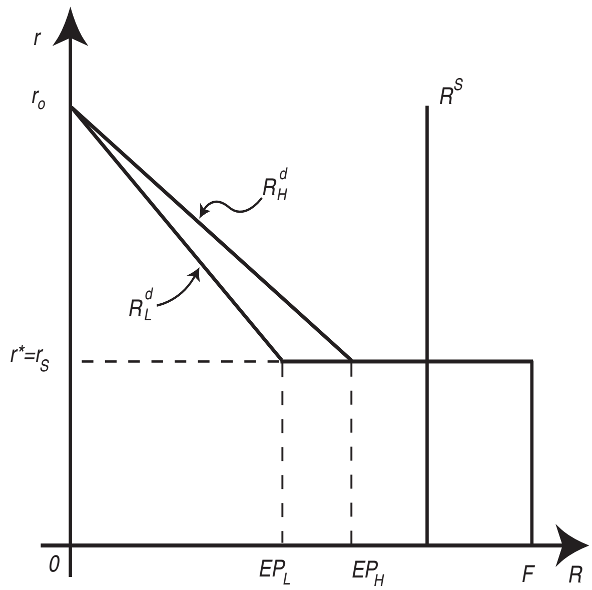
changing the target rate r_T), without changing (in any significant way) the supply of reserves. In fact, the market rate will jump to the new target just as a consequence of its announcement.

Figure 5 illustrates this case. If the monetary authority intends to increase the target rate from r_T to r'_T, then it only needs to increase proportionally the rates r_o and r_s (to r'_o and r'_s, respectively). The demand for reserves, as a result, will shift upward (in a parallel manner) and even if the supply of reserves remains unchanged (at the $E\overline{P}/2$ level) the market will clear at the higher, desired rate r'_T.

Alternatively, the corridor may be centered at the market interest rate (and not the target rate).[12] In such a case, Guthrie and Wright (2000) explain how the central bank can still use announcements to influence the overnight market interest rate without the need for explicit open market operations. Their explanation uses an arbitrage argument based on the expectations hypothesis of the term structure of interest rates. The key element in Guthrie and Wright's theory is the ability of the central bank to use open market operations, if necessary, to influence the overnight rates in the future. They call their

[12] Recall that, for example, when demand is stochastic, the market rate can be different from the target rate.

Figure 6 The Goodfriend System



strategy "threat-based monetary policy" (i.e., a threat to influence future rates, if necessary).

The Goodfriend System

Consider now the case where $E\overline{P}$ can take two values, $E\overline{P}_H > E\overline{P}_L$, as in the second part of Section 2. Then, the central bank can make the market interest rate always equal to a given target rate r_T by fixing the sweep rate $r_s = r_T$ and supplying $R^s > E\overline{P}_j$ for $j = H, L$ (see Figure 6). Clearly, the forecasting process required to assure that $R^s > E\overline{P}_j$ for $j = H, L$ is much simpler than the one that requires forecasting the exact values of $E\overline{P}_j$ for $j = H, L$.

The Goodfriend system requires that banks hold large amounts of reserves, which may result in large interest payments associated with the sweep facility.[13] The corridor system is less subject to this qualification, but when the payment requirements by banks fluctuate (as represented by fluctuations in $E\overline{P}$ in the model), the interest rate will be harder to target precisely. An

[13] See Goodfriend (2002, Section IV) for a careful discussion of these issues. Paying interest on reserves tends to encourage banks to substitute reserves for bond holdings in their balance sheet. Goodfriend discusses how the central bank could exploit the yield curve to finance interest on reserves by increasing its holdings of longer-term bonds (which, in principle, will result from the initial exchange of bonds for reserves when banks increase their demand for the latter).

exhaustive discussion of the pros and cons of corridor systems is beyond the scope of this article. Whitesell (2006) provides some interesting perspective on these issues. Here, it is sufficient to note that even when $E\overline{P}$ fluctuates, if the fluctuations are not very significant, a corridor system still allows the central bank to change the (average) target rate (by announcement) without major revisions to the supply of reserves.

4. DAYLIGHT CREDIT

The previous section dealt with the decision of banks, which, as the end of the day approaches, do not want to find themselves holding unused reserves that will earn zero interest overnight. To discuss the issue of daylight credit and how it relates to the end-of-day decisions, we need to extend the model to include some daytime decisions. However, the analysis in the previous section will constitute an integral part of the analysis in this more complicated case.

The Bank's Problem

We start the analysis, again, by studying the decisions of an individual bank. Relative to the bank's problem in the previous section, we add an extra decision that will allow us to capture some of the tradeoffs faced by the bank during the day. In particular, we will consider the situation in which the bank expects to make *two* payments before the night. We will denote by P^E the early payment and P^L the late payment. Both payments, as before, are uniformly distributed in the interval $\left[0, \overline{P}\right]$.

The bank starts the day with a given amount of funds F, with $F \geq 2\overline{P}$. These funds are allocated to holdings of bonds B_1 and reserves R_1. The bank observes the value of P^E and is required to settle this payment (real-time gross settlement). If $P^E > R_1$, then the bank needs to obtain daylight credit. Let r_e be the daylight credit interest rate. After that, with probability q, the bank finds a counterparty to trade bonds for reserves and adjust the composition of its portfolio. Later, with potentially an adjusted portfolio, the bank faces the arrival of a second payment P^L and is required to settle that payment. No new trading opportunities (or chances to adjust the portfolio) exist after the second payment. The left-over (positive or negative) balances are carried overnight.

First, we consider the case in which unused reserves earn no interest (and no sweep service is in place). To solve the problem of the bank, we start by studying the decision of the bank in the later part of the day when it finds a counterparty (that is, with probability q). Let us define the value Π_2 as follows:

$$\Pi_2^*\left(P^E\right) = r\left(F - P^E\right) + \max_{R_2}\left\{-rR_2 - [1 - p(R_2)]\, r_o\left(E_{R_2+}P^L - R_2\right)\right\},$$

where R_2 is the amount of reserves chosen by the bank in the rebalancing stage (after finding the counterparty). The maximization problem in the expression for Π_2^* is the same as the one studied in the previous section and describes the quantity of reserves the bank would like to carry to fulfill the late payment P^L (before knowing the exact value of that payment).

We are now ready to describe the decision problem of the bank choosing daylight reserves R_1. The final return of the bank will depend on the value of the early payment P^E and the late payment P^L, relative to the chosen value of reserves R_1. The bank does not know the value of the required payments at the time of choosing R_1 so the value of P^E may turn out to be lower or greater than R_1. In fact, under some values of the interest rates r and r_o, the optimal value of R_1 may be larger than $\overline{P}$, in which case the bank always has enough reserves to cover the early payments P^E (and no daylight credit is used).

For given values of P^E and R_1, we describe in the Appendix the expected payoff to a bank that does not get to rebalance its portfolio after its early payment. We denote this payoff by $\pi_1\left(P^E, R_1\right)$. The total expected payoff to the bank also involves the payoffs when it can rebalance its portfolio and the charges from using daylight credit, if there is any. Explicitly, if P^E is greater than R_1, then the bank's total return is given by

$$\Pi_1\left(P^E, R_1\right) = q\Pi_2^*\left(P^E\right) - r_e(P^E - R_1) + (1-q)\,\pi_1\left(P^E, R_1\right),$$

where the second term represents the interest paid on daylight credit.

If P^E is lower than R_1, then the bank's total return after making the early payment is given by

$$\Pi_1\left(P^E, R_1\right) = q\Pi_2^*\left(P^E\right) + (1-q)\,\pi_1\left(P^E, R_1\right).$$

Here, since $P^E < R_1$, the bank does not need daylight credit and the interest rate r_e does not appear in the expression.

The bank will choose daylight reserves R_1 to maximize the expected value (over possible realizations of P^E) of the total return $\Pi_1\left(P^E, R_1\right)$.[14] The solution to this problem is given by

$$R_1^* = \begin{cases} \frac{\left[r_e^2+2(1-q)r_oc\right]^{\frac{1}{2}}-r_e}{(1-q)r_o}\overline{P} & \text{if} \quad r_o > r \geq 0.5r_o, \\ \left[2-\left(\frac{2r}{r_o}\right)^{\frac{1}{2}}\right]\overline{P} & \text{if} \quad 0.5r_o > r > 0, \end{cases}$$

[14] It is worth mentioning that to perform these computations one needs to take into account that, for some interest rates, the optimal amount of reserves R_1 would be greater than $\overline{P}$. In such case, the bank will not require daylight credit regardless of its realization of P^E.

where $c = (1 - q)(r_o - r) + r_e$. Note that R_1^* is equal to $\overline{P}$ and continuous in r at $r = 0.5r_o$. If $r = 0$ then the bank is indifferent between holding any amount of reserves in the interval $\left[2\overline{P}, F\right]$.[15]

Note first that given all other values of the relevant variables, increases in the value of the market interest rate r decrease the value of R_1^*. In other words, the demand for reserves of an individual bank is a decreasing function of the interest rate, as in the previous sections. Similarly, the demand for reserves is an increasing function of the size of the highest possible payment $\overline{P}$.

It is also not hard to show that if $r \in (0.5r_o, r_o)$, then R_1^* is an increasing function of r_e. That is, when the interest rate on daylight overdrafts increases, the bank holds more reserves. When $r \in (0, 0.5r_o)$, the demand for reserves does not depend on the interest rate on daylight credit because the bank chooses a level of reserves $R_1^* > \overline{P}$ and never incurs a daylight overdraft.

The last comparative statics that we consider is with respect to the probability of being able to rebalance the portfolio after the early payment, that is, the probability q. In this model the optimal amount of reserves, as long as it is smaller than $\overline{P}$, is increasing in q. The reason for this result is that the probability of holding unused reserves overnight increases as q decreases (this is because rebalancing is not possible and the value of both payments P^E and P^L may happen to be low). For a high opportunity cost of holding unused reserves (that is, for high values of r), the bank will lower reserves if these are more likely to become excess overnight reserves.

When $r \in (0.5r_o, r_o)$, the average daylight credit incurred by the bank can be computed as

$$DC = \int_{R_1}^{\overline{P}} \frac{P - R_1}{\overline{P}} dP = \frac{\left(\overline{P} - R_1\right)^2}{2\overline{P}}.$$

Clearly, this quantity decreases when R_1 increases. In this model then, an increase in the interest rate on daylight credit tends to increase the level of reserves and, hence, decrease the average daylight credit incurred by banks.

The Market for Reserves

The overnight interest rate on bonds, r, will result from the interactions late in the day between banks that get to rebalance their portfolio and the central bank. The demand for reserves early in the day results from the anticipation by banks of the value that the interest rate will take in these late-in-the-day interactions. This is the case since what matters to banks is the opportunity cost of holding

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[15] Note that when $r = r_o$, we have that R_1^* is positive. Hence, in contrast with the analysis in the previous section, in this case r could potentially be greater than r_o and the demand for reserves still be positive. Recall that reserves now also allow the bank to economize in daylight credit. We do not study the (unusual) case of $r > r_o$ here.

reserves given by the overnight interest rate on bonds, r. In our model, banks can perfectly predict this interest rate, r. Clearly, this result is a simplification. In reality, the overnight interest rate tends to fluctuate during the day (although such fluctuations are not very significant in the United States). Bartolini et al. (2005) extensively document the behavior of the overnight interest rate in the U.S. federal funds market and provide interesting discussions of the reasons for the observed interest rate fluctuations.[16]

In our model, only a proportion q of the banks are active late in the day. Then, the aggregate demand for reserves (late in the day) is given by

$$R_2^d = q \int_0^1 R_{2i}^* di.$$

Given the supply of reserves, R_2^s, provided by the central bank (also, late in the day), the market-clearing interest rate, r^*, will be such that $R_2^d(r^*) = R_2^s$. Note that we have assumed here that those reserves that have been used to make early payments during the day do not become available as new reserves for the recipient until the next day. Also, in most cases, the central bank does not intervene in the bond market late in the day. Assuming that it does intervene, as we do in this article, simplifies the exposition but is not essential for the argument. In summary, these extreme assumptions are just simplifications that keep the market-clearing conditions easy to manipulate. There are, of course, alternative ways of setting up the market-clearing condition that would result in similar conclusions.

Note that the demand for reserves obtained in this way will behave similarly to the demand obtained in Sections 2 and 3. Then, it is easy to demonstrate that, for large enough values of R_2^s, the market interest rate r^* will be lower than $0.5r_o$ and, hence, there will be no demand for daylight credit. However, as in the previous section, if the demand for reserves is not perfectly predictable, some fluctuations in the market interest rate will persist. Also, the market interest rate that implies no demand for daylight credit may be too low, relative to some specific target that the central bank may have in mind. In the rest of this section, we demonstrate that a sweep facility that amounts to paying interest on reserves can resolve these two potential shortcomings.

Overnight Sweeps and Daylight Credit

Suppose the central bank automatically sweeps overnight all unused reserves held by banks into bonds that pay a return r_s, which is also fixed by the central

[16] Many of the key elements in Bartolini et al. (2005) discussions are captured in our model in a stylized and easy-to-study manner (for example, the inability by banks to perfectly predict their payment needs late in the day and the risk of overdraft faced by banks that might not be able to find a lender before the market closes). We abstract from studying within-the-day interest rate fluctuations but believe that our model, after some minor modifications, could be used as a first step in a formal study of these issues.

bank. Then, the demand for reserves by an individual bank is given by

$$R_1^* = \begin{cases} \frac{\left[r_e^2+2(1-q)(r_o-r_s)c\right]^{\frac{1}{2}}-r_e}{(1-q)(r_o-r_s)}\overline{P} & \text{if} \quad r \geq 0.5\,(r_o+r_s)\,, \\ \left[2-\left(\frac{2(r-r_s)}{r_o-r_s}\right)^{\frac{1}{2}}\right]\overline{P} & \text{if} \quad r < 0.5\,(r_o+r_s)\,, \end{cases}$$

where again $c = (1-q)(r_o - r) + r_e$. Here, if $r = r_s$ then the bank is indifferent among holding any amount of reserves in the interval $\left[2\overline{P}, F\right]$. Note that, not surprisingly, the demand for reserves under a system with no sweeps is the same as the one in a system where the interest on sweeps, r_s, is set to equal zero.

For $r \geq r_s$ the demand is decreasing in r and continuous (and, in particular, when $r = 0.5\,(r_o + r_s)$ the demand for reserves R_1^* is equal to $\overline{P}$). Hence, when the interest rate r is smaller than $0.5\,(r_o + r_s)$, the bank holds enough reserves to never require daylight credit (since $P^E < R_1$ for all possible values of P^E).

If the central bank wishes to set the market interest rate at a given target level r_T, and simultaneously drive the use of daylight credit to zero, then an effective mechanism is to set $r_s = r_T < r_o$ and supply enough reserves to make $r = r_s$. This is basically the same Goodfriend idea that we explained for the simpler model with only one payment per period (see Figure 6). Here, the Goodfriend system has the added (potential) benefit of significantly reducing the demand for daylight credit by banks.

5. DISCUSSION AND EXTENSIONS

In this section, we discuss some important aspects of the interbank payment system that were left out in our simple model. First, we discuss how the model would work in the presence of reserve requirements. After that, we discuss the important issue of credit risk that partly motivates many of the most significant policy questions in this general subject. Finally, we provide some discussion of a few other assumptions that are associated with important issues related to the workings of the market for reserves. In all the cases, we make an explicit effort to provide adequate references to the relevant literature that extend the analysis in this article.

Reserve Requirements

Our model of the demand for reserves by banks does not rely on the imposition of reserve requirements. However, reserve requirements are a common feature of many payment systems and, in particular, of the U.S. system. A thorough discussion of the functioning of the market for reserves when there are reserve requirements is beyond the scope of this article. Here, we only present a short introduction to the issue that exploits the simplicity of our model (see Whitesell 2000 and Bartolini, Bertola, and Prati 2002 for more comprehensive, related studies).

Let us go back to the simpler setup of Section 3 and suppose that the central bank imposes a minimum reserve requirement equal to $\underline{R}$. Also assume that banks that cannot satisfy the reserve requirement have to pay the overnight overdraft rate r_o per units of reserve deficiency (i.e., borrowed reserves).[17] In such a case, the demand for reserves would take the form in Figure 7. Clearly, if the supply of reserves is lower than $\underline{R}$ then banks would only agree to hold bonds if the rate of return for holding bonds, r, is greater than or equal to r_o. In fact, if r is greater than r_o then it is better to hold only bonds and pay the penalty rate r_o to obtain borrowed reserves that cover the reserve requirement. Some of the return from the interest payments accrued on bonds can later be used to cover the interest on borrowed reserves.

Hence, if both bonds and reserves are to be held in equilibrium the market interest rate, r, must equal r_o when the supply of reserves is lower than $\underline{R}$. If the supply of reserves is greater than $\underline{R}$ then the analysis is similar to the one in Section 2, where banks choose their balance to cover the reserve requirement and the expected late payment P.

In reality, banks could face some uncertainty about the value of $\underline{R}$ since it depends on their holding of deposits subject to reserve requirement, a variable that is not fully predictable at all times. In the United States, the system is set up so as to minimize this uncertainty. Banks have to satisfy an average level of reserves over a reserve maintenance period, where the required reserves are calculated based on the holding of deposits in a previous period. Also, failure to meet the requirement implies a penalty that is different than the overnight overdraft rate. While the analysis would be more complicated in this case, the basic logic described here would still apply.[18]

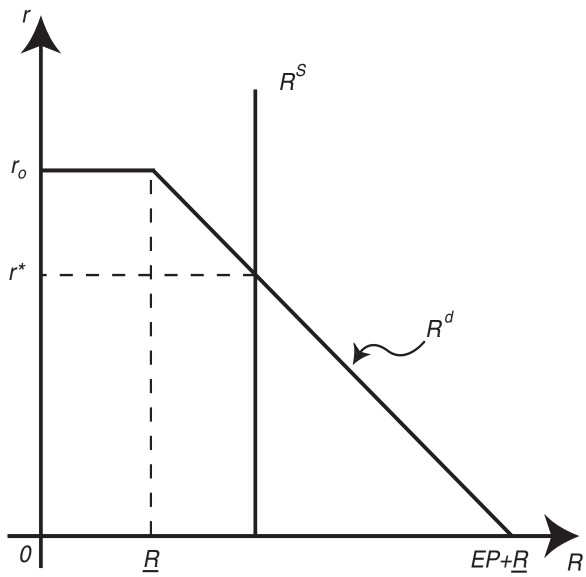
Credit Risk

The model does not deal with the role of credit risk in determining the behavior and outcomes in the payment system. Clearly, paying attention to credit risk considerations would be essential to reach any definite policy conclusion. For

[17] This is not exactly how deficiencies in reserve requirements are treated in the U.S. system, but this simpler case serves for the purpose of illustration.

[18] Whitesell (2000) analyzes a similar model with a two-day reserve requirement maintenance period and a corridor system. See also Bartolini, Bertola, and Prati (2002) for a model more closely motivated by the features of the U.S. system.

Figure 7 Reserve Requirements and the Market for Reserves



example, credit risk would play a role in explaining why the central bank may want to economize on bank usage of daylight credit (see Kahn and Roberds 1999 for a description of a model in which credit risk plays a crucial role). Zhou (2000), following the original contribution by Freeman (1996), shows that if credit risk is not relevant, then the intraday interest rate r_e should be set to equal zero in the optimal policy.[19] The model in this article artificially abstracts from credit risk and is not designed to provide direct insight into the optimal determination of the rates r_o, r_e, and r_T.[20] Rather, it shows how, given the values for the relevant interest rates, a system of sweeps, which amounts to paying interest on reserves, can facilitate the implementation of a target rate r_T.

An important feature of the U.S. intraday credit policy that was left out of the analysis is the imposition of quantitative limits (or "caps") on the amount of intraday credit. At any time during the day, each bank should hold intraday

[19] This conclusion also depends on the assumption that daylight credit will be exclusively devoted to payment purposes and cannot be diverted into short-term speculative investment.

[20] Optimal policy in payment arrangements is a difficult question (see Zhou 2000, Temzelides and Williamson 2001, Martin 2004, Mills 2006, and Berentsen and Monnet 2006 for recent contributions). Even the basic question of whether the central bank should play a role in the payment system does not have a commonly accepted answer among academic economists. Green (1999) provides a careful study of this important issue.

credit to an amount that does not exceed the bank's cap.[21] In the model, there is no role for caps. In part, this is a consequence of our explicit abstraction of any credit risk consideration. To a first approximation, caps are imposed to limit the ability of banks to take large negative positions in their accounts at the central bank when they are likely to fail. Temzelides and Williamson (2001) provide a related justification for caps in a dynamic model with explicit informational frictions (see also Koeppl, Monnet, and Temzelides 2004).

Two other instruments that can be used to limit the credit risk exposure associated with the provision of daylight credit are daylight interest rates and collateral. Interest rates on daylight credit have other implications (aside from accounting for credit risk) for the management of reserves by banks. Even though our model does not take into account credit risk, we have considered the case of positive daylight credit interest rates to study such alternative implications on banks' management of reserves. Lacker (1997) points out that interest on daylight credit could reduce the distortion created by not paying interest on reserves, and in this case finds that daylight overdraft should be charged an interest rate at least as high as the market rate.

Collateral, in the form of repos, could certainly be used in the environment of our model. Recall that total funds, F, are allocated between bonds and reserves. Since F is greater than P^E, the sum of bonds and reserves is also greater than P^E. Therefore, even if payment P^E is greater than reserves, the bank can always use bonds to collateralize the necessary daylight credit.[22] Since there is no explicit consideration of credit risk in our model, even though the use of collateral is possible, it is inconsequential (see Martin 2004 and Mills 2006 for environments in which collateral requirements play an important role).

There are other ways to influence the amount of daylight credit in the system. For example, McAndrews and Rajan (2000) propose the use of explicit policies to encourage synchronization of payments during the day and suggest that such policies would tend to limit banks' reliance on daylight credit to cover intraday payments (see also Martin and McAndrews 2006).

Other Important Assumptions

In the model, the connection between the payment of interest on reserves and daylight credit comes from the fact that banks can only adjust their portfolio

[21] There is some flexibility implicit in these caps. Basically, banks can exceed the caps in unusual situations. After repeated violations of the cap, banks can be placed under a strict system of monitoring and, if necessary, some of their requested payments may be rejected to avoid overdrafts. See Federal Reserve System (2005) for details.

[22] There is no opportunity cost of holding collateral in the model. For a model exploiting a mechanism similar to this one, but where collateral bears an opportunity cost, see Berentsen and Monnet (2006).

during the day with some probability. In other words, banks face a trade friction in the asset market that limits their ability to adjust their holdings of reserves. Interestingly, there is a growing amount of literature that aims at capturing these trading frictions in financial markets (see Duffie, Gârleanu, and Pedersen 2005, Weill 2005, and Lagos and Rocheteau 2006, for example). In principle, we can expect that some of the ideas in this article will extend to environments that more closely follow the new literature on financial markets with (search) frictions.

The model also assumes that the size of payments P^E and P^L and the probability of being able to adjust the portfolio after a payment, q, are exogenous and cannot be modified by the bank. However, in principle the bank could influence the size and timing of payments at some cost. This flexibility is not present in the model and, if introduced, would highlight the potentially distortionary effects of certain payment system policies, as for example, not paying interest on reserves (see Lacker 1997 for a model in which this type of distortion is possible). Similarly, the efforts to find counterparties to trade and adjust portfolios are also part of an explicit decision by banks facing costs and benefits that are implied by the system in place. If we change the system, for example to evaluate different policies, such decisions by banks may also change. The model abstracts from this type of so-called "Lucas critique" effect.

The sizes of P^E and P^L can be interpreted as proxies for the volume of payment requirements arriving early and late in the day. These values are random in the model. Since the size of the payments are random, the bank cannot perfectly predict them. However, there is no relevant decision in the model that influences the values of P^E and P^L that are likely to be observed. It is in this sense that we say that P^E and P^L are exogenous. In the real world, banks have some degree of discretion regarding the timing of payments during the day. McAndrews and Rajan (2000) discuss some evidence that suggests that U.S. banks actively synchronize payments to affect payment flows during the day. The discretion over the timing of payments opens the door to strategic behavior by banks. A number of articles have formally studied the possibility of delays and gridlocks in real-time gross settlement systems (see Bech and Garratt 2003, Martin and McAndrews 2006, Mills and Nesmith 2006, and Beyeler et al. 2006 for recent contributions).[23]

[23] See Rochet and Tirole (1996) and Freixas and Parigi (1998) for discussions of contagion and systemic risk in payment systems. Furfine (2003) provides an interesting empirical investigation of interbank exposures in the United States.

6. CONCLUSION

The model in this article is not suitable to analyze the welfare economics of the topic, as the underlying real economic activity that drives payment needs (and hence the demands for reserves and daylight credit) is hidden from view and treated as exogenous. Still, even the partial equilibrium analysis of this article points to some tentative conclusions. By paying interest on reserves at less than the market rate of interest, the central bank essentially imposes a tax on reserves. This tax encourages banks to hold no more reserves than is necessary to just meet their payment needs. But uncertainty in the timing of payments means that "just meeting" only happens by accident. Hence, the desire to hold down reserves leads banks to demand central bank credit. The provision of such credit presents the central bank with a new set of challenges, from finding an appropriate price to managing the credit risk exposures that could result.

In short, like any tax, a tax on reserves creates distortions, including distortions in the use of daylight credit. Against these, a complete analysis would have to consider what costs might be associated with the greater reserve holdings implied by the Goodfriend system of paying interest at the target market rate. Our model does not address this issue, but to the extent that such costs are small, our analysis suggests the optimality of eliminating the tax on reserves.

In a more general context, Goodfriend (2006) discusses the role of interest on reserves as part of a comprehensive proposal for determining and implementing an optimal rate of inflation in the economy. He argues that the provision of currency card accounts, combined with the payment of interest on banks' reserves, would allow the monetary authority to achieve the optimum quantity of money (as in the Friedman rule) at any inflation rate. This would, in turn, free the monetary authority to set the inflation rate at the optimum level based only on considerations related to the existence of price rigidities (relative price distortions and mark-ups) and the zero-lower-bound on nominal interest rates.

This article has emphasized the interdependence of banks' demand for daylight credit in the payment system and for overnight reserves. The use of reserves as the medium of settlement for interbank payments means that changes in the central bank's treatment of overnight reserves could also affect the operation of the intraday, interbank settlement system. If one of the forces driving the demand for daylight credit has been the desire by banks to avoid the opportunity cost of holding sterile reserves, then reducing that opportunity cost by paying interest on reserves should reduce the demand for such credit. We have considered a simple model that deals with these interdependencies explicitly and allows us to better understand their origin and consequences.

APPENDIX

Let us denote by π_1 the expected payoff to the bank if it does not get a chance to rebalance its portfolio after the first payment, P^E. To specify this expected payoff, there are three relevant ranges of P^E that need to be considered. First, when $P^E < R_1 - \overline{P}$, the amount of extra reserves R_1 left after making the early payment P^E is enough to cover all possible late payments. This can only happen if $R_1 > \overline{P}$, of course. In this case, no overnight overdraft will be incurred and the expected payoff to the bank is given by

$$\pi_1\left(P^E, R_1\right) = rB_1.$$

Under a sweep system this payoff becomes

$$\pi_1\left(P^E, R_1\right) = rB_1 + r_s\left(R_1 - P^E - \frac{\overline{P}}{2}\right),$$

where the second term represented the interest earned on unused reserves overnight.

When $\max\left\{0, R_1 - \overline{P}\right\} \leq P^E < \min\left\{R_1, \overline{P}\right\}$, there are always some possible values of P^L such that the bank will have to incur an overnight overdraft. In this case, the expected payoff π_1 is given by

$$\pi_1\left(P^E, R_1\right) = rB_1 - \left[1 - p\left(R_1 - P^E\right)\right] r_o\left(P^E + E_{(R_1 - P^E)+}P^L - R_1\right).$$

Under a sweep system we need to add to this payoff the expected interest earned on unused reserves, which is given by

$$p\left(R_1 - P^E\right) r_s\left(R_1 - P^E - E_{(R_1 - P^E)-}P^L\right),$$

where $E_{(R_1 - P^E)-}P^L$ represents the conditional expectation over values of P^L smaller than $R_1 - P^E$.

Finally, if $R_1 < \overline{P}$, then it can happen that P^E is greater than R_1 and, in this case, the bank will incur an overnight overdraft equal to the sum of the daylight credit balance ($P^E - R_1$) and the full amount of the second payment P^L. The expected payoff π_1 is then given by

$$\pi_1\left(P^E, R_1\right) = rB_1 - r_o(P^E + \frac{\overline{P}}{2} - R_1),$$

where $\overline{P}/2$ represents the average value of P_L. In this case, the bank does not hold any unused reserves and, hence, the payoff is the same under a sweep system.

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