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Optimal Contracts for Housing Services Purchases

Borys Grochulski

In this article, we study tradeoffs associated with homeownership and renting. We consider a model in which housing capital generates housing services, but also requires regular maintenance/upkeep effort. The eventual resale value of a home is a random variable whose distribution depends on the amount of upkeep performed on the property. In the model, we have a risk-averse agent/household who wants to purchase a flow of housing services. The household also consumes a nonhousing good and leisure. Maintenance on the piece of housing capital occupied by the household can be performed by the household itself or by an outside property manager.

We abstract in this article from taxes or other government distortions. The household contracts with a risk-neutral bank/landlord who has funds sufficient to make a lumpy housing investment. We do not assume that the bank/landlord can observe the household's effort, consumption, or savings. We show that simple renting from the bank/landlord is an optimal contract for the provision of housing services to the household, conditional on the outside property manager being hired. Conditional on the manager not being hired, it is optimal for the bank/landlord to lend the money to the household to acquire housing services by purchasing a home. In this arrangement, the bank's loan is secured by a zero-down, fixed-rate, nonrecourse mortgage that prohibits subordinated financing.

In our model, owning a home is risky because its future value is uncertain. By purchasing a home, the household exposes itself to the idiosyncratic risk in the resale value of the property. This risk is partially assumed by the bank that grants the mortgage. The mortgage contract is nonrecourse, which means that the household's mortgage liability is limited to the value of the

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collateral. Thus, the household can walk away from the house and turn its ownership over to the bank at any time without any further recourse. This is what the household chooses to do if the resale value of the property turns out to be low. If the resale value is high, the household sells the home, pays off the mortgage, and pockets the difference. The mortgage, therefore, provides partial insurance to the homeowner. If the household is to care for the house, this insurance must be partial, because the household needs an incentive to provide costly upkeep on the property. The optimal mortgage contract results from the tradeoff between the household's desire for insurance and the bank's desire to give the household adequate incentive to care for the property. Just making the purchase of a home feasible to the household, in particular, is not a goal that the mortgage contract serves in this model.

The purpose of this article is to study renting and owning as two possible ways of contracting for a household's purchase of housing services. The model we consider is useful for this purpose because the contract that the household obtains in equilibrium clearly resembles either a renting contract or an ownership contract with a mortgage, depending on the values of the parameters. The model, therefore, provides a framework in which we can discuss the costs and benefits of homeownership relative to renting.

The main lesson from the model is that owning a home must necessarily expose the owner to property upkeep costs and the idiosyncratic home resale value risk. This aspect of homeownership should not be overlooked by policymakers formulating and implementing government policy toward housing.[1] We use a simple parameterization of the model to show that for an average household, absent tax distortions or other policy interventions, renting actually dominates homeownership. Our model can be extended to study the effects of various tax-code-based and other government interventions in the housing market.

The model studied in this article is highly stylized. We assume that transaction costs associated with buying or selling real estate are zero. We assume that duration of tenancy/ownership is known in advance (no random moving shocks are allowed). In our model, the household does not face any income or employment risk. We do not distinguish between real and nominal contracts, and so we have no inflation risk. The discussion in Campbell (2006) suggests that all these factors can be important and should be examined in future work.

This article is related to two main strands of the economic literature on housing finance: the studies of optimal mortgage contracts (e.g., Dunn and Spatt 1985; Chari and Jagannathan 1989; Campbell and Cocco 2003; Piskorski and Tchistyi 2010), and the studies comparing renting and owning (Shelton 1968; Rosen and Rosen 1980; Chambers, Garriga, and Schlagenhauf

[1] Jaffee and Quigley (2010) provide an overview of government policy toward housing in the United States.

2009). This article, to our knowledge, is unique to the extent that it allows for both the choice between renting and owning and the choice of an optimal financial contract associated with either option, all in a model in which the contract structures are endogenous (derived from the fundamentals of the environment) rather than assumed exogenously. This article is also related to the large literature studying moral hazard models (see Prescott [1999] for an overview). Our model differs from the classic moral hazard problem in two respects. First, in addition to the household's upkeep effort, the principal, i.e., the bank/landlord, does not observe the household's savings, labor income, or consumption. Second, the principal has the option to circumvent the moral hazard problem altogether by hiring a property manager, whose work on the property is verifiable.

The article is organized as follows. Section 1 presents the model. Section 2 studies efficient contracts with hired upkeep. Section 3 studies efficient contracts with occupant-provided upkeep. Section 4 compares the two classes of efficient contracts in a parameterized example. Section 5 concludes.

1. THE MODEL

A household is looking to purchase housing services for a fixed period of time of length T. (The number T is intended to be a typical amount of time that a household spends living at one location, i.e., between moves from one place to another. Our parameterization in Section 4 will take T to be seven years). Time is indexed by $t \in [0, T]$.

Housing

Housing is a form of capital that works as follows. Size of a housing unit is measured in dollars of its value at $t = 0$. The gross growth of the value of a housing unit over the course of the period is a random variable denoted by R. We assume that $\Pr\{R \in \{\underline{R}, \overline{R}\}\} = 1$ with $\underline{R} < \overline{R}$ and $\Pr\{R = \overline{R}\}$ depends on the amount of effective upkeep effort, U, exerted on the property over the time interval $[0, T)$. In particular, we assume that

$$\Pr\{R = \overline{R}\} = \begin{cases} 0 & \text{if} \quad U < \hat{U}, \\ p & \text{if} \quad U \geq \hat{U} \end{cases}$$

for some $\hat{U} > 0$ and $0 < p < 1$. The number $\hat{U}$ represents adequate upkeep. Thus, if $U \geq \hat{U}$, a property of initial value H_0 will be worth at $t = T$ the amount H_T, which equals $\overline{R}H_0$ with probability p or $\underline{R}H_0$ with probability $1 - p$. If effective upkeep is inadequate, i.e., $U < \hat{U}$, the property will be worth $\underline{R}H_0$ with probability one.

We assume that upkeep must be provided at all times in $[0, T)$. In particular, if U_t is the upkeep provided at t, the effective upkeep U is given by

$$U = \min_{0 \le t < T} U_t.$$

Note that under this functional form it is a waste to provide nonconstant levels of upkeep because only the minimum level matters. Thus, we can take that U_t will be chosen to be constant: $U_t = U$ for all t.

Upkeep is generated by hours of upkeep effort/work. Upkeep effort can be delivered by either the occupant of the property or an outside manager. Let $m \in \{0, 1\}$ be the indicator of whether or not an outside manager is hired. Thus, $m = 0$ means that there is no outside manager and any upkeep on the property is up to the agent. Let h_t denote the number of man-hours of upkeep done by the occupant. Let h_t^m denote the number of man-hours of upkeep done by the manager, if one is hired. Because of the costs related to the monitoring of the manager, the manager's need to travel to the physical property site to provide services, etc., one hour of manager's upkeep effort delivers less actual upkeep service than one hour of the occupant's effort.[2] Let $\chi \le 1$ be the relative efficiency of the manager's effort. We thus have

$$U_t = \frac{(1 - m)h_t + m\chi h_t^m}{H_0}. \tag{1}$$

If the manager does the upkeep, his work is monitored, i.e., the actual amount of upkeep services delivered to the property is publicly known. The wage of a manager is w^m. If the occupant does the upkeep, only she knows how much upkeep effort she really provides (i.e., the occupant's upkeep effort is private information). In either case, because nonconstant U_t is inefficient, the upkeep effort will be constant over time, i.e., for all t, $h_t = h$ and/or $h_t^m = h^m$ for some constants h and h^m.

Households

A household has initial financial assets, A_0, and can earn a wage, w, per hour of work in outside employment. We assume that wage w is public information and constant over the interval $[0, T]$. The household has $\bar{l}$ hours to allocate between outside work, house maintenance/upkeep work, and leisure per unit of time. We will also refer to the household as the occupant, or agent.

Household preferences are over the consumption of housing services, nonhousing consumption, c_t, and leisure, l_t, over the time interval $[0, T)$, as

[2] If the occupant provides upkeep, she is not monitored. Unlike the hired manager, however, the occupant does not incur the costs of travel to the site where she provides upkeep, because she lives there. She can also develop site-specific maintenance skills over time. Hagerty (2008) describes a recent case in which hired maintenance costs turned out to be very high in a large scattered real estate investment joint venture due to the managers' moving around cost and the heterogeneity of the housing stock.

well as over its end-of-period wealth, A_T. Final wealth A_T may depend on the realization of R. Housing services are obtained by means of occupying a house. A house of initial value H_0 gives the occupant a normalized flow of H_0 units of housing services consumption at every $t \in [0, T)$. Thus, the assumption here is that housing services consumption cannot be adjusted within the period, i.e., is fixed until time T. The household maximizes an expected utility function

$$\int_0^T e^{-rt} u(c_t, l_t, H_0) dt + e^{-rT} \mathbb{E}\left[V(A_T)|U\right], \tag{2}$$

where the flow utility function, u, is assumed to be increasing in each of its three arguments; the end-of-period wealth value function, V, is assumed to be increasing and concave in end-of-period wealth A_T; and $\mathbb{E}[\cdot|U]$ denotes conditional expectation. Note that the above notation indicates that the agent consumes the housing services that the housing unit of size H_0 produces during the period, but not the housing unit itself. Note also that there is no uncertainty in this model at any $t < T$.

The riskless rate of interest is taken to be equal to the household's subjective discount rate, and, therefore, will also be denoted by r.

Banks/Lenders/Landlords

Lenders/landlords have access to funds sufficient to purchase housing capital in sufficiently large amounts, are risk neutral, and discount at the rate r. We will refer to these agents as banks.

Contracts

At $t = 0$, the household and a bank enter a contract that specifies the housing investment, H_0, and the payments, X, to be made. The flow of housing services H_0 generated by the investment H_0 will go to the household, i.e., the contract says that the agent has the right to occupy the property during the period. The contract also specifies payments X that the agent/occupant is to make to the bank. Finally, the contract specifies who is to provide upkeep on the property for the duration of the contract. In detail, the contract between them specifies the following:

1. The investment H_0 made by the bank for the use of the agent.
2. The payments from the agent to the bank that the agent will make as a payment for the flow of housing services

   $$X = \{X_0, X_T(R), (x_t)_{0<t<T}\},$$

   where X_0 and $X_T(R)$ are lumpy payments at the beginning and the end of the period, and x_t is the flow rate of payment within the period. The final payment X_T can depend on the realized return R on the housing investment. For reasons of household limited liability with no recourse, we assume that $X_T(R) \leq 0$ for any R.

3. The assignment of the upkeep duty $m \in \{0, 1\}$. If $m = 1$, then the contract says that an outside property manager is to be hired by the bank. If $m = 0$, then no outside manager is to be hired. (Note that m cannot be changed during the period.)

Note that the final payment $X_T(R)$ determines how the end-of-period value of the property, $H_T(R) = RH_0$, is to be split between the bank and the agent.

Let a contract $\{H_0, X, m\}$ be denoted by C. Given a contract C, the agent's initial assets A_0, and wage w, the agent chooses h and $(c_t, l_t)_{0 \leq t \leq T}$ so as to maximize his expected utility (2) subject to the flow constraints

$$dA_t = (rA_t + y_t - c_t - x_t)dt \quad \text{for } t \in (0, T),$$

where $y_t = w(\bar{l} - l_t - h)$, and subject to the borrowing constraints $A_t \geq 0$ for all t.

From the flow constraints, we have that

$$A_t = e^{rt}(A_0 - X_0) + \int_0^t e^{r(t-s)}(y_s - c_s - x_s)ds \tag{3}$$

for all $t < T$. Let A_{T-} denote the assets held by the agent just prior to T, i.e., before the final payment $X_T(R)$ is made. That is: $A_{T-} = \lim_{t \to T} A_t$. The agent's final wealth $A_T(R)$ is given by

$$A_T(R) = A_{T-} - X_T(R). \tag{4}$$

The borrowing constraints for $t < T$ and limited liability, $X_T \leq 0$, imply that $A_T(R) \geq 0$ for any R.

An allocation $\mathcal{A}$ is a complete description of the outcome in this contracting problem. Thus, $\mathcal{A} = \{C, h^m, h, U, (c_t, l_t, A_t)_{0 \leq t \leq T}\}$.

Let I be the indicator of adequate effort, i.e., $I = 1$ if $U \geq \hat{U}$, and $I = 0$ if $U < \hat{U}$. The expected utility an allocation gives to the agent is

$$\int_0^T e^{-rt} u(c_t, l_t, H_0)dt + e^{-rT}\mathbb{E}\left[V(A_T(R))|U\right],$$

where

$$\mathbb{E}\left[V(A_T(R))|U\right] = (1 - Ip)\, V(A_T(\underline{R})) + IpV(A_T(\overline{R})).$$

The expected profit an allocation gives to the bank is

$$-H_0 + X_0 + \int_0^T e^{-rt}(x_t - mw^m h_t^m)dt + e^{-rT}\mathbb{E}\left[H_T(R) + X_T(R)|U\right],$$

where

$$\mathbb{E}\left[H_T(R) + X_T(R)|U\right] = (1 - Ip)\left(H_0\underline{R} + X_T(\underline{R})\right) + Ip\left(H_0\overline{R} + X_T(\overline{R})\right).$$

We assume that there are potentially many banks competing against each other and, thus, all gains from trade are captured by the agent.

The timing in the model is as follows: (1) The bank offers a contract C. (2) The agent accepts and makes payments X_0 and (x_t) to the bank. If $m = 0$, the agent chooses h. If $m = 1$, the bank chooses h^m. The agent chooses $(c_t, l_t, A_t)_{0\leq t\leq T}$. (3) Return R is realized at T and the bank makes payment $-X_T(R_T)$ to the household.

Transfer and use of physical property, as well as any payments between the agent and the bank, are observable, can be enforced, and therefore are contractible. Other variables in the model are not. Thus, the bank cannot control (e.g., because it cannot observe or verify in court) the household's choices of h or $(c_t, l_t, A_t)_{0\leq t\leq T}$.

The market contract (which we will also call the competitive contract) is that contract C that gives rise to the final allocation that maximizes the expected utility for the agent subject to zero expected profit for the bank. In this article, we will characterize the competitive contract. In particular, we will show that the optimal contract with $m = 1$ is akin to renting while the optimal contract with $m = 0$ is similar to owning the house with a mortgage. With this, we can use this model to examine which contract would prevail in equilibrium, absent taxes or other distortions.

Our strategy for finding the undistorted market contract is as follows. First, we study the case of hired upkeep. Second, we study the case of occupant-provided upkeep. Finally, we compare the two conditionally optimal contracts and allocations to find the overall optimum.

2. CONTRACTS WITH HIRED UPKEEP

In this section, we consider contracts with $m = 1$. This means that upkeep is done by an outside, monitored manager. In this case, $h = 0$ and (1) reduces to

$$U = \chi h^m / H_0.$$

With property upkeep hired out (i.e., observable), the bank's problem of finding the best contract C for the agent is separable from the agent's utility maximization problem. In other words, the bank's cost of a contract with $m = 1$ is independent of any unobservable choices of the agent (in particular, it does not depend on the agent's choice of h).

The contract $C = \{H_0, X, 1\}$ is feasible if the expected profit to the bank is nonnegative. We will write C^1 to denote a contract $C = \{H_0, X, 1\}$. In the next section, we will write C^0 to denote a contract $C = \{H_0, X, 0\}$. Let Π denote the expected profit of the bank. Under a contract C^1, i.e., with $m = 1$, we have

$$\begin{aligned}\Pi(C^1) &= -H_0 + X_0 + \int_0^T e^{-rt}(x_t - w^m h^m)dt \\ &\quad + e^{-rT}\left[(1 - Ip)\left(H_0\underline{R} + X_T(\underline{R})\right) + Ip\left(H_0\overline{R} + X_T(\overline{R})\right)\right].\end{aligned}$$

The value of the expected profit under a contract with $m = 1$ can be evaluated using X, which is contractible, and h^m, which is up to the bank. Thus, this profit value does not depend on anything that is not contractible or controlled by the bank.

Because of competition, banks will offer to the agent a contract that maximizes the indirect utility of the agent. Let's call any such contract an efficient contract. In this section, we want to characterize efficient contracts among the contracts C^1, i.e., those with $m = 1$.

No Risk Exposure

It is immediately clear that an efficient contract with $m = 1$ will not expose the agent to the risk of the return on the property investment. With property upkeep provided by the bank, the agent has no control over the realization of the return, R. The agent is risk averse and the bank is risk neutral. Thus, any efficient contract in the class C^1 will specify the final payment $X_T(\overline{R}) = X_T(\underline{R})$.

Given that the agent is not exposed to the risk of the return R, his indirect utility function does not depend on the level of upkeep U that the bank chooses to provide on the property by hiring a manager. The indirect utility of the agent under the contract C^1, with X_T independent of R, is thus given by

$$\begin{aligned}&\mathcal{U}^0(H_0, X_0, X_T, (x_t)_{0\le t\le T}) \\ &= \max_{(c_t, l_t, A_t)_{0\le t\le T}} \int_0^T e^{-rt}u(c_t, l_t, H_0)dt + e^{-rT}V(A_{T-} - X_T), \qquad (5)\end{aligned}$$

where

$$A_t = e^{rt}(A_0 - X_0) + \int_0^t e^{r(t-s)}(w(\bar{l} - l_s) - c_s - x_s)ds$$

and maximization is subject to $A_t \ge 0$ at all t. Note in the above that $h = 0$.

Under a contract C^1 with X_T independent of R, the formula for the bank profit can be simplified as follows. Using $X_T(\overline{R}) = X_T(\underline{R})$, we have

$$(1 - Ip)\left(H_0\underline{R} + X_T(\underline{R})\right) + Ip\left(H_0\overline{R} + X_T(\overline{R})\right) = X_T + H_0\left(\underline{R} + Ip(\overline{R} - \underline{R})\right).$$

Also, simple algebra shows that

$$\begin{aligned}\int_0^T e^{-rt}w^m h^m dt &= w^m h^m \int_0^T e^{-rt}dt \\ &= w^m h^m D_0,\end{aligned}$$

where $D_0 = r^{-1}(1 - e^{-rT}) = \int_0^T e^{-rt}dt$ is the date-0 value of a constant payment flow of one dollar over the time interval $[0, T)$. Thus, the profit expression simplifies to

$$\begin{aligned}\Pi(C^1) &= -H_0 + X_0 - w^m h^m D_0 + \int_0^T e^{-rt}x_t dt \\ &\quad + e^{-rT}\left[X_T + H_0\left(\underline{R} + Ip(\overline{R} - \underline{R})\right)\right].\end{aligned}$$

Constant Payment Rate

Let us now consider the question of the timing of payments X. The bank is obviously interested in the present value P of payments X, which is given by

$$P = X_0 + \int_0^T e^{-rt}x_t dt + e^{-rT}X_T,$$

but the timing of payments is not important to the bank. The agent, in contrast, is concerned with the timing of payments because of the borrowing constraints $A_t \geq 0$ he faces. For a given P, the more delayed are the payments from the agent to the bank, the better for the agent. The following lemma, however, shows that spreading the payment evenly over the interval $[0, T)$ is a sufficient delay.

Lemma 1 *For any given H_0 and present value P of payments $X = (X_0, X_T, (x_t)_{0\leq t\leq T})$, the indirect utility of the agent, $\mathcal{U}^0(H_0, X_0, X_T, (x_t)_{0\leq t\leq T})$, is maximized by the payments schedule X with $X_0 = X_T = 0$ and $x_t = x$ for all $0 \leq t \leq T$.*

Proof. Ignore for a moment the nonrecourse constraint $X_T \leq 0$, i.e., allow $X_T > 0$. Suppose that all payments are loaded at the terminal date, i.e., $X_0 = x = 0$ and $X_T = e^{rT}P$. This payment schedule is the best for the agent because the payments are maximally deferred, which means that the borrowing constraints cannot bind. How would the agent behave in this problem?

The agent would choose c_t and l_t so as to attain $\mathcal{U}^0(H_0, X_0, X_T, (x_t)_{0\leq t\leq T})$ given in (5). With $X_0 = x = 0$ and $X_T = e^{rT}P$, this maximization problem reduces to

$$\max_{(c_t,l_t)_{0\leq t\leq T}} \left\{\int_0^T e^{-rt}u(c_t, l_t, H_0)dt \right.$$
$$\left. + e^{-rT}V\left(e^{rT}A_0 + \int_0^T e^{r(T-t)}(w(\bar{l} - l_t) - c_t)dt - e^{rT}P\right)\right\}.$$

The first-order (FO) necessary and sufficient conditions with respect to c_t and l_t give us

$$\begin{aligned}e^{-rt}u_c(c_t, l_t, H_0) &= e^{-rT}V'(A_T)e^{r(T-t)}, \\ e^{-rt}u_l(c_t, l_t, H_0) &= e^{-rT}V'(A_T)we^{r(T-t)},\end{aligned}$$

which simplifies to

$$\begin{aligned} u_c(c_t, l_t, H_0) &= V'(A_T), \\ u_l(c_t, l_t, H_0) &= V'(A_T)w, \end{aligned}$$

and shows that constant consumption, $c_t = c$, and leisure, $l_t = l$, are optimal for the agent.

Note now that under constant consumption and leisure, the agent saves a constant amount equal to $w(\bar{l} - l) - c$ at any point in $[0, T)$. This amount is such that the agent can make the final payment $X_T = e^{rT}P$ because his continuation value is very low for negative final assets A_T. But this also means that the agent can afford a constant payment x such that

$$\int_0^T e^{-rt} x dt = P \tag{6}$$

instead of the final payment X_T. So, for any H_0 and P, the payment schedule with $X_0 = X_T = 0$ and $x = P/D_0$ is optimal for the agent. In addition, unlike the payment schedule with $X_T = e^{rT}P$, this payment schedule does not violate the nonrecourse (agent limited liability) assumption. ■

Interpretation as a Lease/Rent Contract

At this point, we note that the contracts C^1, to which we have reduced the problem, resemble renting. In fact, under such a contract, the agent makes no initial payment to the bank and the bank makes no final payment to the agent. During the time the agent occupies the property (and therefore receives the housing services), the agent makes a constant payment to the bank. The bank provides property upkeep by hiring a manager. This contract resembles a simple lease/rent contract, where the payment x represents rent. From now on, we will call a contract of this form a rent contract.

We note here that the rent contract with a constant payment rate is not pinned down uniquely as an efficient contract with hired upkeep. Any nondecreasing payment schedule with the same present value P gives the agent the same utility because the borrowing constraints are not binding with increasing payments and so the agent can stabilize consumption and leisure while saving a constant amount sufficient to cover the payments with the present value P.

Efficient Rent Contracts

We can now discuss the efficient level of rent and the bank's upkeep policy.

With constant payment x, and $X_0 = X_T = 0$, the bank zero-profit condition gives us

$$D_0 x = H_0 + w^m h^m D_0 - e^{-rT} H_0 \left(\underline{R} + Ip(\overline{R} - \underline{R}) \right).$$

The left-hand side of this equality is the present value of the payments from the agent. The right-hand side is the cost of the initial investment, plus the present value of the upkeep expenses, less the expected present value of property resale. For any H_0, an efficient contract picks h^m such that the present value of payments from the agent is minimized and the bank breaks even. Let h^{m*} denote the efficient rate of manager upkeep hired by the bank.

The bank may choose to provide adequate upkeep or not. Thus, h^{m*} is either zero or $\hat{U}H_0/\chi$. The bank will provide adequate upkeep if and only if

$$H_0 + w^m \hat{U} H_0 \chi^{-1} D_0 - e^{-rT} H_0 \left(\underline{R} + p(\overline{R} - \underline{R})\right) < H_0 - e^{-rT} H_0 \underline{R}.$$

Eliminating H_0 and simplifying, the above reduces to

$$w^m \hat{U} \chi^{-1} D_T < p(\overline{R} - \underline{R}), \tag{7}$$

where $D_T = e^{rT} D_0$ is the date-T value of a constant payment of one dollar over the interval $[0, T)$. The above condition on the primitives of the model will be satisfied if the threshold for adequate upkeep is relatively low, or the wage of the manager is low, or relative productivity of the manager, χ, is not too low, or the expected gain in the return on the property investment from adequate upkeep is sufficiently high. We will assume (7) throughout.

Under (7), thus, $h^{m*} = \hat{U}H_0/\chi$, i.e., it is efficient in the hired-upkeep contract to provide adequate upkeep.

Substituting h^{m*} back to the zero-profit condition, we have

$$\begin{aligned} D_0 x &= H_0 + w^m \hat{U} H_0 \chi^{-1} D_0 - e^{-rT} H_0 \left(\underline{R} + p(\overline{R} - \underline{R})\right) \\ &= \alpha H_0, \end{aligned}$$

where

$$\alpha = 1 + w^m \hat{U} \chi^{-1} D_0 - e^{-rT} \left(\underline{R} + p(\overline{R} - \underline{R})\right). \tag{8}$$

The constant α is, in a sense, the price of housing services under the efficient rent contract.

We assume that $a > 0$. If it were not, the banks could invest in housing, provide upkeep by hiring a manager, sell the property at T, and turn in an expected profit without having any occupants in the property. In this situation, housing prices would adjust.

In Sum

We have shown in this section that with $m = 1$, i.e., conditional on the bank providing upkeep, a simple rent contract with a constant rent paid by the agent over the duration of the period is efficient. Under this contract, the rent payment for a house of size H_0 is given by

$$x = \frac{\alpha}{D_0} H_0. \tag{9}$$

Also under this contract, the agent chooses to have a constant consumption, c, and a constant rate of leisure consumption, l. The utility the agent attains is given by

$$D_0 u(c, l, H_0) + e^{-rT} V(A_T),$$

where final wealth,

$$A_T = e^{rT} A_0 + \int_0^T e^{r(T-t)} \left(w(\bar{l} - l) - c - \frac{\alpha}{D_0} H_0 \right) dt,$$

must be nonnegative. This expression for final wealth simplifies to

$$A_T = e^{rT} (A_0 - \alpha H_0) + D_T \left(w(\bar{l} - l) - c \right).$$

The levels of housing, leisure, and ordinary goods consumption that the agent chooses when offered a rent contract of this form can be obtained from the FO conditions of the agent's problem. These conditions are

$$\begin{aligned} D_0 u_c(c, l, H_0) &= e^{-rT} V'(A_T) D_T, \\ D_0 u_l(c, l, H_0) &= e^{-rT} V'(A_T) w D_T, \\ D_0 u_{H_0}(c, l, H_0) &= e^{-rT} V'(A_T) e^{rT} \alpha. \end{aligned}$$

Simplifying, we obtain

$$\begin{aligned} u_c(c, l, H_0) &= V'(A_T), & (10) \\ u_l(c, l, H_0) &= V'(A_T) w, & (11) \\ u_{H_0}(c, l, H_0) &= V'(A_T) \frac{\alpha}{D_0}. & (12) \end{aligned}$$

In Section 4, we will use these conditions to compute the efficient rent contract in a parametrized version of this model. There, also, we will compare this efficient contract with bank-provided upkeep (the rent contract) with an efficient contract with occupant-provided upkeep, which we study next.

3. CONTRACTS WITH OCCUPANT-PROVIDED UPKEEP

In this section, we consider the case of $m = 0$, which means that no outside property manager is hired and property upkeep is assigned to the occupant. Thus, the manager's hours are zero and upkeep U is determined by the hours of upkeep effort provided by the household. In particular, (1) reduces to

$$U = h / H_0.$$

We are looking for an efficient contract of the form $C^0 = \{H_0, X, 0\}$. Under any such contract, two cases are possible: the agent chooses to provide sufficient upkeep $\hat{U}$, or not. His incentives to provide sufficient upkeep depend on the contract C^0. In particular, they depend on the final payment, $X_T(R)$. If, as was the case under the lease/rent contract, the payment $X_T(R)$ does not depend on the realization of R, the agent has no incentive to care for the house. The bank, however, is interested in adequate upkeep because upkeep influences the expected final property value $\mathbb{E}[H_T(R)|U]$, which affects the bank's expected profit. Because the upkeep effort exerted by the agent cannot (or is too costly to) be observed by the bank or an outside enforcer (court), the amount of effort cannot be mandated by contract. Rather, for any contract C^0, the bank anticipates the amount of upkeep U that an agent will provide under C^0 and uses this value of U to evaluate its expected profit from the contract C^0. This is a version of the classic moral hazard problem.

Contracts that Do Not Provide Incentives for Adequate Upkeep

The bank may structure the contract C^0 so as to make it worthwhile for the agent to provide adequate upkeep $\hat{U}$, or not. Consider a contract C^0 not structured to give the agent incentives to provide adequate upkeep, so the agent chooses $h = 0$. The situation here is similar to the one in the renting contract, but the outside manager is not present. Because it was optimal to hire the manager to provide adequate upkeep under the renting contract, the best contract that does not encourage the agent to provide adequate upkeep must be worse than the best renting contract. Otherwise, hiring the manager for a nonzero number of hours would have been inefficient under renting, which was not the case.

Because contracts C^0 that do not provide incentives for the agent to exert adequate upkeep effort on the property are dominated by renting, there is no need to pay further attention to them.

Contracts that Provide Incentives for Adequate Upkeep

Let us now consider the contracts C^0 that provide sufficient payment incentives to the agent to provide adequate upkeep.

First, let us examine the conditions a contract C^0 must satisfy to ensure that the agent chooses adequate upkeep effort under C^0. Clearly, the agent chooses to provide upkeep effort if his total utility from optimally chosen leisure and goods consumption plan, conditional on providing the upkeep, exceeds the total utility he can obtain by not providing adequate upkeep jointly with a leisure and goods consumption plan that is optimal, conditional on not providing upkeep. It is important to note here that, because leisure and consumption are not observable to outsiders, the agent can use one leisure/consumption plan conditional on providing the upkeep effort, and another conditional on not providing it. For this reason, we need to consider two indirect utility functions that represent the agent's highest utility value conditional on the two possible upkeep effort choices.

Let $\mathcal{U}^h(C^0)$ denote the total utility the agent can obtain conditional on providing adequate upkeep. We have:

$$\begin{aligned} \mathcal{U}^h(C^0) &= \max_{(c_t,l_t)} \int_0^T e^{-rt}u(c_t, l_t, H_0)dt + e^{-rT}\mathbb{E}\left[V(A_T)|\hat{U}\right] \\ s.t. &: \quad A_t \geq 0 \text{ for all } t, \end{aligned}$$

where A_t for $t < T$ is given in (3) with $y_t = w(\bar{l} - l_t - \hat{U}H_0)$, and where, as before, A_T is given in (4). Note that this indirect utility function takes as given that (the minimum) adequate upkeep effort is provided, i.e., the agent works $h = \hat{U}H_0$ hours on upkeep (hence the notation $\mathcal{U}^h$).

Now consider the other indirect utility function, i.e., the one that represents the agent's value of not proving adequate upkeep. Note here that this value function is the same as the value function of a renter but with a final payment X_T equal $X_T(\underline{R})$ with probability one. In our model, inadequate upkeep leaves no uncertainty about the final realization of the property value (the house never gets the high return "by accident" when no upkeep is provided). Therefore, inadequate upkeep insulates the agent from risk, just like the lease/rent contract does. Thus, the indirect utility function of the agent who extends zero upkeep effort is given by

$$\mathcal{U}^0(H_0, X_0, X_T(\underline{R}), (x_t)_{0\leq t\leq T}),$$

where $\mathcal{U}^0$ is the value function of renting, given in (5).

Despite not being able to directly observe the agent's private upkeep effort choice, the bank knows that the agent will choose to provide adequate upkeep under a contract C^0 if

$$\mathcal{U}^h(H_0, C^0) \geq \mathcal{U}^0(H_0, X_0, X_T(\underline{R}), (x_t)_{0\leq t\leq T}). \tag{13}$$

This condition is often referred to as the incentive compatibility constraint, IC for short. If it holds, the contract C^0 gives the agent sufficient incentive to provide adequate upkeep because doing so is in the agent's own best interest.

Because of competition among banks, in equilibrium in which banks do not hire managers, the equilibrium contract provided to the agent, to be denoted by C^{0*}, and the house of size H_0 the agent will occupy maximize the agent's expected utility $\mathcal{U}^h(C^0)$ subject to the bank's zero-profit condition

$$-H_0 + X_0 + \int_0^T e^{-rt}x_t dt + e^{-rT}\mathbb{E}\left[H_T(R) + X_T(R)|\hat{U}\right] = 0,$$

and subject to the incentive compatibility condition (13).

Our task now is to characterize C^{0*}. The next lemma is standard in moral hazard models.

Lemma 2 *In any feasible contract $C^0 = (H_0, X)$, where $X = (X_0, (x_t), X_T(R))$, the final payment $X_T(R)$ satisfies*

$$X_T(\overline{R}) < X_T(\underline{R}).$$

Proof. Suppose $X_T(\overline{R}) \geq X_T(\underline{R})$, then the IC must be violated because by providing adequate effort the agent not only exerts himself but also runs the risk of facing a larger payment $X_T(\overline{R})$ due from him to the principal rather than facing the smaller payment $X_T(\underline{R})$ that he can guarantee himself by not providing adequate upkeep effort. ■

The intuition for this lemma is clear. For any house size H_0, the payments X must be encouraging upkeep effort, i.e., demanding a lower payment in circumstances that are indicative of effort. The high realization is indicative of adequate upkeep being done. In fact, it can only happen when upkeep is adequate. The low realization is not indicative of adequate effort as under adequate effort it only occurs with some probability less than one. Thus, the payment to the agent at T, which is given by $-X_T(R)$, must satisfy $-X_T(\overline{R}) > -X_T(\underline{R})$.

This means that the IC constraint must bind at the optimum. Why? If IC is not binding at an allocation with $X_T(\overline{R}) \neq X_T(\underline{R})$, one can make $X_T(\overline{R})$ and $X_T(\underline{R})$ closer, which provides more insurance and increases welfare of the agent, so this allocation cannot be optimal.

Reducing the Contract Space without Loss of Generality

In this subsection, we restrict attention to a particular subset of all possible contracts C^0. As we do this, however, we make sure that none of the contracts we discard dominate all of the contracts that we do not discard. The next two lemmas provide results similar to these of Lemma 1.

Lemma 3 *It is without loss of generality to only consider contracts C^0 in which the payment rate x_t is constant and $X_0 = 0$.*

Proof. Consider a feasible contract $C^0 = (H_0, X)$ with $X = (X_0, (x_t), X_T(R))$ that satisfies the IC constraint, i.e., provides the agent with incentives for adequate upkeep effort $\hat{h} = H_0\hat{U}$. We claim that the contract $C^{0c} = (H_0, X^c)$ with $X^c = (0, x^c, X_T(R))$, where

$$x^c = (X_0 + \int_0^T e^{-rt} x_t dt)/D_0,$$

is also feasible and at least as good for the agent as the original contract C^0. It clearly generates zero expected profit for the bank, provided that, under the modified payment plan X^c, the agent does not find it beneficial to shirk, i.e., deviate from providing upkeep. We claim that the fact that he does not find shirking attractive under C^0 implies that he will not find it attractive under X^c.

First, we note that under the payment plan X^c, no borrowing constraints will bind in the agent's conditional utility maximization problems defining the values of shirking and not shirking. This is simply because the agent, in either case, wants to smooth his consumption (of both goods and leisure). For a given present value of payments to the bank, the agent may be unable to perfectly stabilize his consumption if the payments are heavily front-loaded. With $X_0 = 0$ and x_t constant, it is clear that this is not the case: Because the payments to the bank are constant, the agent can perfectly smooth out his labor effort, and, thus, the borrowing constraints never bind.

Consider now the following two possible cases for the plan X in the contract C^0, where the payments x_t are not necessarily constant. One: no borrowing constraints bind in the agent's problem conditional on no-shirking. Two: some bind.

In the first case, it is clear that the utility attained under both shirking and not shirking is unaffected by the switch from the payment schedule X to X^c. The consumption path, c_t, and final assets A_{T-} that the agent chooses conditional on not shirking are unaffected by the switch from X to X^c. The same is true about consumption and assets chosen conditional on shirking. The contract C^{0c}, thus, is IC simply because C^0 is IC. The utility attained is the same under X^c and X, i.e., is not worse under X^c, as claimed.

In the second case, switching from the payment schedule X to X^c will relax the borrowing constraints in the agent's problem conditional on not shirking. The utility the agent can attain conditional on not shirking, therefore, will increase. Our conclusion is true if the utility of the agent conditional on shirking does not increase by more than it does conditional on not shirking. It is easy to see that this in fact must be the case. We sketch this argument here by giving the following two reasons: (1) Under the shirking strategy, the agent has more time to divide between leisure and work; (2) under the shirking strategy, the agent knows that the payment $-X_T$ he receives at T is state-by-state less than what he gets when he provides upkeep, so he wants to save more. Thus, when the agent shirks on the upkeep effort, he will work in outside employment more and save more. The relaxation of the borrowing constraints caused by the switch from X to X^c thus helps him less when he shirks and more when he does not shirk. ■

Lemma 4 *It is without loss of generality to only consider contracts in which* $X_T(\underline{R}) = 0$.

Proof. Suppose $X_T(\underline{R}) \neq 0$. By nonrecourse, it must be $X_T(\underline{R}) < 0$. By the Lemma 2, we have that $0 > X_T(\underline{R}) > X_T(\overline{R})$, i.e., the bank makes at least the payment $-X_T(\underline{R})$ to the agent at T in every state. Consider now the effect on the value functions of increasing uniformly $X_T(\underline{R})$ and $X_T(\overline{R})$ (toward zero, i.e., decreasing the payout that the bank will make to the agent at $t = T$), combined with a decrease in the payment rate x such that the present value of the payments to the bank is unchanged (so the bank's zero profit condition continues to hold). Under both the not shirking strategy and the shirking strategy, the agent can undo the effect of this change in the payment structure simply by saving. Since the borrowing constraints were not binding under the original payment plan and under either strategy (see the proof of the preceding lemma), they are not binding now. Therefore, such a change in the payment plan has no effect on either value function. Thus, there is no loss of generality in considering only contracts with a payment plan X such that $X_T(\underline{R}) = 0$. ■

Using these two lemmas, we can simplify the problem. We can take the payments x_t to be constant. We can take $X_0 = X_T(\underline{R}) = 0$. Under these conditions, the borrowing constraints will not bind. The agent, both under the non-shirking strategy and the shirking strategy, will choose constant rates for consumption of goods and leisure at all $t \in [0, T)$.

The value function conditional on providing upkeep is thus given by:

$$\begin{aligned}
\mathcal{U}^h(H_0, C^{0c}) &= \max_{c,l} D_0 u(c, l, H_0) + e^{-rT}\mathbb{E}\left[V(A_T(R))|\hat{U}\right] \\
s.t. \quad &: \quad A_T(\underline{R}) = e^{rT}A_0 + \int_0^T e^{r(T-t)}\left(w(\bar{l} - \hat{h} - l) - c - x\right)dt, \\
s.t. \quad &: \quad A_T(\overline{R}) = e^{rT}A_0 \\
&\qquad + \int_0^T e^{r(T-t)}\left(w(\bar{l} - \hat{h} - l) - c - x\right)dt - X_T(\overline{R}).
\end{aligned}$$

The value function conditional on shirking (zero upkeep effort) is:

$$\begin{aligned}
\mathcal{U}^0(H_0, C^{0c}) &= \max_{c,l} D_0 u(c, l, H_0) + e^{-rT}V(\tilde{A}_T) \\
s.t. \quad &: \quad \tilde{A}_T = e^{rT}A_0 + \int_0^T e^{r(T-t)}\left(w(\bar{l} - l) - c - x\right)dt.
\end{aligned}$$

The bank's zero profit condition is

$$-H_0 + xD_0 + e^{-rT}\mathbb{E}\left[H_0 R|\hat{U}\right] + e^{-rT}pX_T(\overline{R}) = 0.$$

From now on, in the class of all contracts with $m = 0$, we only consider contracts of the form $C^{0c0} = (x, X_T(\overline{R}))$ with $X_0 = X_T(\underline{R}) = 0$. Thus, for a given house size H_0, the contract specifies only the constant rate x of payment from the agent to the bank over the time interval $[0, t)$ and the final payment $-X_T(\overline{R})$ the bank makes to the agent in the state $\overline{R}$.

Solving for an Optimal Contract with $m = 0$

Now we know that we can find an optimal contract with $m = 0$ by adjusting $H_0, x, X_T(\overline{R})$ while keeping $X_0 = X_T(\underline{R}) = 0$. Because $X_T(\overline{R})$ is nonpositive, it will be useful to introduce a separate piece of notation for the payment $-X_T(\overline{R}) \geq 0$ that the agent receives at T in the state $R = \overline{R}$. Let M denote this payment to the agent.

For any fixed H_0, the problem of finding optimal x and M has a simple solution. We know that absent the IC constraint, it would be optimal to set

$M = 0$ so as to avoid exposing the agent to the risk of the realization of the return R. With the IC constraint, in order to find an optimal contract, we look for a minimal deviation from this full-insurance contract. Because we can only manipulate two numbers, x and M, subject to one linear constraint, it is clear how to proceed. We start out with $M = 0$ and gradually increase it while also increasing x sufficiently to preserve the bank's zero profit condition. At $M = 0$, the contract is not incentive-compatible. As M and x increase, however, the value of the shirking strategy decreases faster than the value of the upkeep strategy because under both strategies the agent pays x to the bank but only under the upkeep strategy does the agent have a positive probability of receiving the final payment M. We thus increase M and x to the point at which the IC constraint is satisfied for the first time. Then we stop. The contract we obtain this way makes for a minimal incentive-compatible deviation from the full-insurance contract, i.e., is efficient. If this point cannot be attained, then a house of the size H_0 is not feasible to finance with $m = 0$.

The overall problem of finding an efficient contract with $m = 0$ involves also searching over all the values H_0 that can be supported by a contract described above. In the next section, we perform this search numerically.

Interpretation as an Ownership Contract with a Mortgage

Suppose we have found an efficient contract with $m = 0$ and some H_0, x, and M. If $H_T(\overline{R}) - M \geq H_T(\underline{R})$, then we can interpret this contract as a nonrecourse mortgage contract with the agent being a homeowner and the bank being a creditor whose loan is secured by a claim (lien) on the property owned by the agent.

Recall that because $X_0 = 0$, the bank pays the whole price of the house, H_0, up front. The agent then makes payments to the bank at a constant rate, x, in return for (a) the right to use the house, and (b) the final payment, M, he receives from the bank if $H_T = H_T(\overline{R})$. Motivated by the prospect of M, the agent provides adequate upkeep on the property. Because the bank pays the initial price of the house, it is natural to think of the bank as the owner of the house, and x as of rent. But the positive final payment M the household receives is not consistent with this interpretation.

Alternatively, we can interpret x as a constant payment on a loan of size H_0 that the bank gives the agent to purchase the home. If at $t = T$ the remaining balance on the mortgage is $H_T(\overline{R}) - M$, then the final payment M in state $\overline{R}$ can be interpreted as money the agent walks away with after selling the house for $H_T(\overline{R})$ and paying off the mortgage balance $H_T(\overline{R}) - M$. In the state $\underline{R}$, the value of the house is less than the mortgage balance, and the agent receives the final payment of zero. This outcome is consistent with the owning interpretation if the mortgage is a nonrecourse mortgage, i.e., the agent can simply walk away from the house. As the agent walks, the bank's secured interest in the property lets it take ownership of the house and sell it. Because the proceeds, $H_T(\underline{R})$, are less than the bank's claim, $H_T(\overline{R}) - M$, the bank incurs a loss relative to the face value of the mortgage.

Under this interpretation of the efficient contract with occupant-provided upkeep, we can calculate the rate of interest that the bank formally charges on the mortgage, to be denoted by ρ. Let B_t be the mortgage balance at time t. Given that the loan amount at $t = 0$ is H_0, in order for the bank's claim on the property to be $H_T(\overline{R}) - M$ at $t = T$, the mortgage balance must satisfy

$$B_0 = H_0, \tag{14}$$
$$B_T = H_T(\overline{R}) - M. \tag{15}$$

For the contract to be a fixed-rate mortgage with the rate ρ, we must have

$$dB_t = (\rho B_t - x)dt,$$

i.e., the change in balance at any point in time equals the interest that the current balance accrues less the constant payment flow x made by the agent. Solving this differential equation for B_t and using the initial condition (14) we have

$$B_t = (H_0 - x\rho^{-1})e^{\rho t} + x\rho^{-1}.$$

Using the terminal balance condition (15), we get that the rate of interest charged on this mortgage is the number ρ that solves

$$H_T(\overline{R}) - M = \left(H_0 - x\rho^{-1}\right) e^{\rho T} + x\rho^{-1}.$$

The right-hand side of this expression can also be written as $e^{\rho T} H_0 - x D_T(\rho)$, where $D_T(\rho)$ is defined in the same way as D_T but using the rate ρ instead of r. Namely, $D_T(\rho) = \rho^{-1}(e^{\rho T} - 1)$.

We see that $\rho \geq r$, and $\rho > r$ whenever $H_T(\overline{R}) - M > H_T(\underline{R})$. This is intuitive: The bank breaks even at the rate r with its claim (and payoff) being $H_T(\overline{R}) - M$ in state $\overline{R}$ and $H_T(\underline{R})$ in state $\underline{R}$. If the face value of the bank's claim is increased to $H_T(\overline{R}) - M$ in both states, keeping H_0 and x fixed, the break-even condition will hold only at a higher discount rate $\rho > r$. The difference $\rho - r$ can be interpreted as a default premium compensating for the fact that in the state $\underline{R}$ the bank takes a loss, relative to the face value of its claim.

We will use this interpretation of the efficient contract with $m = 0$ in the remainder.[3]

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[3] One could consider this interpretation of the contract as a more formal implementation exercise, where the objective is to equivalently express the efficient contract with $m = 0$ in terms of instruments and contracts that are familiar and widely used in reality. In our case, these familiar instruments are (a) an ownership right, and (b) a fixed-rate, zero-down mortgage contract that does not allow subordinated financing.

It is worth noting that the mortgage loan considered here must necessarily forbid subordinated financing. The bank issuing the mortgage needs to make sure that the agent retains the final payment M, as it is this payoff in the good state that motivates the agent to provide upkeep. If, instead, the agent could take out a second mortgage such that the amount he owes on this mortgage at time T is M, the effective final payoff to the household would be zero in both states, which means that the household would not have an incentive to provide upkeep. In this case, the bank's expected profit on the first mortgage would become strictly negative.

We also note that the household has an incentive to adequately upkeep if, as we assume here, it stays in the home for T years and has a sufficient equity stake in the property at time $t = T$. It is not important in our model if the household has a stake in the home at $t < T$ ($t = 0$ in particular). Therefore, no mortgage down payment is necessary in our model.[4]

In Sum

We have shown in this section that an efficient contract with $m = 0$ can be found among contracts of the form $C^{0c0} = (x, X_T(\overline{R}))$, i.e., taking x_t constant and $X_0 = X_T(\underline{R}) = 0$. Under the assumption that $H_T(\overline{R}) + X_T(\overline{R}) \geq H_T(\underline{R})$, any such contract can be interpreted as an ownership contract with a zero-down, fixed-rate, nonrecourse mortgage that prohibits subordinated financing. Also, we have described a procedure for computing such a contract and we will use it in the next section.

4. PARAMETERIZATION

In this section, under specific parameter values, we solve for efficient renting and owning contracts, and check which one gives the agent a higher utility value, i.e., which one will prevail in equilibrium. The parameter values we take to reflect the proportions between the spending on consumption, leisure, and housing that an average, infinitely lived household would choose in a simplified, idealized situation in which the flow of housing services of any size H_t can be purchased in a continuous spot market at the cost rH_t.

Endowment and Technology

In order to model an average household, we take it to consist of two working-age members that can at most work a total of three full-time-job equivalents,

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[4] A down payment would probably be a necessary requirement in a model with a richer (stochastic) structure for the mortgage duration T.

i.e., we take $\bar{l} = 3$. We assume that one full-time job pays $w = \$40{,}000$ per year. Initial assets held by the average household, A_0, are taken to be equal to $4w$, which is \$160,000.

We take T to be seven years. The annual interest rate is taken to be 4 percent. The housing technology parameters are taken as follows:

$$\begin{aligned} \overline{R} &= \exp(rT), \\ \underline{R} &= 0.5\exp(rT), \\ p &= .92, \end{aligned}$$

which means that with adequate upkeep a house grows in value at the riskless rate with 92 percent probability, in which case the seven-year rate of return is 33.1 percent. With 8 percent probability, despite proper upkeep, the realized seven-year rate of property value growth is -33.6 percent. The threshold for adequate maintenance is taken to be $\hat{U} = \frac{.2}{4.5}$. This number means that it takes a fifth of a full-time job to properly maintain a house worth $4.5w = \$180{,}000$. The wage of a property manager will be set at $0.6w = \$24{,}000$. The parameter χ is set at 0.9 in this parameterization.

Preferences

Let's suppose the household has log preferences of the form

$$u(c, l, H) = \gamma_c \log(c) + \gamma_l \log(wl) + \gamma_H \log(rH),$$

where $\gamma_c, \gamma_l, \gamma_H$ are positive constants. Here, wl represents the real cost of leisure l consumed, and rH represents the real cost of the housing services consumed. We take the constants γ_c, γ_l, and γ_H so as to obtain reasonable expenditure shares for goods, leisure, and housing consumption under the idealized conditions in which the household purchases spot housing services H at the cost rH.

To do so, we use the following targets for parameterization. We take that the average household works $l = 1.5$, i.e., one-and-a-half full-time jobs. This means that we target the household's labor income level at \$60,000. The annual capital income of the average household is $0.04A_0 = \$7{,}200$. We target the total income, therefore, at \$67,200. The value of the home the average household occupies is taken to be $H = \$180{,}000$. Thus, for the purpose of finding a reasonable parameterization, we take that the household spends $0.04H = \$7{,}200$ on consumption of housing services, and the remaining \$60,000 on consumption of goods and services. Indirectly, the household also spends $wl = \$60{,}000$ on consumption of leisure. The total potential annual income of the household is \$127,200. We now find values $\gamma_c, \gamma_l, \gamma_H$ that are consistent with these target expenditure shares under log preferences.

In the idealized conditions we described, the household maximizes

$$\int_0^\infty e^{-rt} u(c_t, l_t, H_t) dt$$

subject to the present value budget constraint

$$\int_0^\infty e^{-rt}(c_t + wl_t + rH_t)dt = \int_0^\infty e^{-rt} w\bar{l} dt + A_0.$$

Because u is concave, the household chooses constant consumption levels c, l, and H. These levels can be found by maximizing

$$\int_0^\infty e^{-rt} u(c, l, H) dt = \frac{1}{r} u(c, l, H)$$

subject to a flow constraint (that we obtain by dividing the present value constraint by r)

$$c + wl + rH = w\bar{l} + rA_0.$$

Taking the FO conditions of this problem and solving for c, l, and H, we get

$$c\gamma_c^{-1} = wl\gamma_l^{-1} = rH\gamma_H^{-1} = 1/r\mu, \tag{16}$$

where μ is the Lagrange multiplier on the flow constraint. We want to choose γ_c, γ_l, and γ_H so that the solution hits the target values, in dollars,

$$\begin{aligned} c &= 60,000, \\ wl &= 60,000, \\ H &= 180,000, \end{aligned}$$

with $r = -\log(.96) = 0.040822$. Using the FO conditions (16), we get that the agents will indeed choose these target values if the preference parameters are

$$\gamma_c = \gamma_l = 1, \; \gamma_H = 0.12.$$

In the remainder, we therefore take $\gamma_c = \gamma_l = 1$ and $\gamma_H = 0.12$. For simplicity, we will write γ instead of γ_H.

To summarize, we take the household's preferences to be

$$u(c, l, H) = \log(c) + \log(wl) + \gamma \log(rH), \tag{17}$$

with $\gamma = 0.12$.

To obtain a terminal value function $V(A_T)$ consistent with these preferences, we now compute the value of maximizing these preferences (without any frictions or distortions) when the starting wealth is some number A_T. Substituting the FO conditions (16) to the flow constraint we have

$$(2 + \gamma)/r\mu = w\bar{l} + rA_T,$$

which gives us

$$c = wl = rH\gamma^{-1} = \frac{w\bar{l} + rA_T}{2 + \gamma},$$

and, thus,

$$
\begin{aligned}
V(A_T) &= \frac{1}{r} u(c, wl, rH) \\
&= \frac{1}{r} 2 \log\left(\frac{w\bar{l} + rA_T}{2+\gamma}\right) + \frac{1}{r}\gamma \log\left(\gamma \frac{w\bar{l} + rA_T}{2+\gamma}\right) \qquad (18)
\end{aligned}
$$

with $\gamma = 0.12$.

To summarize once more, preferences we use in this parameterization are given by the utility function u in (17) with the terminal value function V given in (18).

Results

Renting

Under these parameter values, we first calculate the value of the rent factor $x/H_0 = \alpha/D_0$, where α is defined in (8). We obtain

$$\frac{\alpha}{D_0} = 0.0362,$$

i.e., the rent payment flow level x is equal to 3.62 percent of the initial value H_0 of the rented house. This means that the present value of rent paid over the course of one year on a house of initial value of \$180,000 would be \$6,390, which corresponds to a payment of, roughly, \$533 per month. The bank's cost of hiring adequate upkeep for a house of this size is roughly \$346 per month.

With this rent level, the renting household chooses as follows

$$
\begin{aligned}
c &= 59,600, \\
lw &= 59,600, \\
y &= 63,600, \\
H_0 &= 197,600.
\end{aligned}
$$

The constant payment flow x under this contract is

$$x = 7,200,$$

which corresponds to the monthly rent on the house of roughly \$600. Under renting, the household chooses final financial assets A_T exactly equal to the initial assets A_0.

Owning

The optimal owning contract has the same constant payment flow

$$x = 7,200,$$

and the same dollar expenditures on the consumption of goods and leisure:

$$\begin{aligned} c &= 59,600, \\ lw &= 59,600. \end{aligned}$$

The optimal house size, however, is smaller:

$$H_0 = 140,400.$$

A home of this size takes the fraction

$$\hat{U} H_0 = 0.16$$

of a full-time job to adequately upkeep. The income flow of the agent under owning is

$$y = 54,000.$$

Note that the homeowning household is dissaving over the interval $[0, T)$ in terms of the financial asset holdings. Its equity position in the house, however, is growing during this time. The optimal owning contract provides the final payment in the good state

$$M = -X(\overline{R}) = 54,800,$$

which corresponds roughly to a year's worth of earnings. The final wealth of the agent is

$$\begin{aligned} A_T(\underline{R}) &= 109,600, \\ A_T(\overline{R}) &= 164,400. \end{aligned}$$

As the household dissaves, its wealth at T in the bad state $\underline{R}$ is equal to only $\frac{109.6}{160}$ =68.5 percent of its initial wealth. In the good state $\overline{R}$, its wealth is $\frac{164.4}{160}$ =102.75 percent of the initial wealth. Thus, the homeowning household is exposed to a substantial amount of risk.

The final dollar value of the home bought by the agent under the owning contract is

$$\begin{aligned} H_T(\overline{R}) &= 186,400, \\ H_T(\underline{R}) &= 93,200. \end{aligned}$$

The mortgage face value at $t = T$ is $H_T(\overline{R}) - M =$ \$131,600. Thus, over the seven years of owning, the agent pays the initial balance of \$140,400 down by only \$8,800. In the good state $\overline{R}$, the household's equity stake in its home is $\frac{54,800}{186,400}$ =29.3 percent at $t = T$. The loan's loan-to-value ratio drops from 100 percent at $t = 0$ to 70.7 percent at $t = 7$. With probability 8 percent, the bank takes at time T a loss of \$38,400, which equals 29.1 percent of its claim's face value at $t = T$ (thus, the loss-given-default ratio is close to 30 percent). The rate of interest sufficient to compensate for this loss is

$$\rho = 4.36 \text{ percent}.$$

This rate exceeds the riskless rate, r, by 28 basis points, which corresponds to about 7.3 percent of the riskless (continuously compounded) rate of $r = 4.08$ percent.

Comparing Renting and Owning

The total expected utility value provided to the agent under the renting contract turns out to be higher than the value under the owning contract. Thus, the renting contract is the contract that the bank offers to the household in equilibrium.

In the parameterization used here, the household's opportunity cost of time, $w = \$60{,}000$, is much higher than the cost of hiring property management services, which is represented by $w^m/\chi = \$26{,}666.67$. In the frictionless environment, therefore, it would clearly be efficient to hire a manager to perform home maintenance. In our model, private information makes it necessary to expose the homeowner to the home resale-value risk. The negative impact of this exposure, however, is not large enough in this parameterization to overweigh the fundamental advantage of using the agent with the lower opportunity cost of time to perform maintenance.

To get a sense of how much more attractive than owning renting is in this parameterization, we can calculate how much of its initial financial assets A_0 the household would be willing to give up in order to avoid having to live in a world in which it must own and cannot rent. Lowering the initial assets A_0 in the renter's problem decreases the utility level attained. This utility does not drop to the level attained under the owning contract before A_0 is decreased from \$160,000 to \$140,480, which is a 12.2 percent drop.

5. CONCLUSION

This article studies a simple model in which renting and owning arise endogenously as two alternative forms of contract that a household can use to purchase housing services. In this model, the household's effort, leisure, savings, and goods consumption are private information, i.e., cannot be used as conditions in the household's contract with a bank or landlord. Even if the household has savings sufficient to purchase a home outright, doing so is not optimal because of the property resale-value risk the outright homeowner faces.

Our model shows that the nonrecourse clause in mortgage lending can have a useful role in risk sharing: By taking out a nonrecourse mortgage, a household can obtain partial insurance against the idiosyncratic risk to its home resale value. Renting, however, is a contracting alternative that allows the household to hire out property upkeep and obtain full, not partial, insurance against the home resale-value risk. In this context, it is worth noting that any government policies promoting homeownership also promote undiversified risk taking by risk-averse households.

The analysis of this article can be extended in two main directions. First, one can use the current stylized model to examine the effects of various government policies on housing market outcomes. Even under the parameter values we use in this article, it is possible to characterize a set of government-provided indirect subsidies to homeowning large enough to cause the average household to switch from renting to owning in equilibrium. Second, the model itself can be extended to allow for transaction costs, shocks to the duration of occupancy, household income risk, and aggregate shocks like unexpected inflation, among others.

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