THE THEORY OF

MULTIPLE EXPANSION OF DEPOSITS:

WHAT IT IS AND WHENCE IT CAME

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Beginning students of banking must grapple with a curious paradox: the banking system can multiply deposits on a given base of reserves yet none of its member banks can do so. Let the reserve-to-deposit ratio be, say, 20 percent and the system can, by making loans, create $5 of deposit money per dollar of reserves received. By contrast, the individual bank receiving that same dollar on deposit can lend out no more than 80 cents of it. How does one reconcile the banking system's ability to multiply loans and deposits with the individual bank's inability to do so? Fully answering this question required the intellectual efforts of at least six economists writing in the period 1826-1921. The story of their contributions is the story of the evolution of the theory of the multiple expansion of deposits.

At the heart of banking theory is the notion of the multiple expansion of bank deposits. This idea consists of two interrelated parts. The first explains how the banking system as a whole creates deposits by making loans equal to a multiple of its cash reserves, the multiplier being the inverse of the reserve-to-deposit ratio. The second shows how the individual bank contributes to this expansion, not by multiplying its own deposits, but rather by making loans and losing reserves through the clearinghouse to other banks so that they too can expand. Taken together, these components reconcile the banking system's ability to multiply loans and deposits with the individual bank's inability to do so. For the individual bank, far from expanding its loans by several times any new cash deposits received, lends out only the fraction of those deposits remaining after required reserves have been set aside.

The preceding ideas are fairly well known. Many economics textbooks explain why a banking system having a required reserve ratio of, say, twenty percent can create five dollars of deposit money per dollar of cash reserves while at the same time no individual bank can lend more than eighty cents per dollar of deposits received. What the texts do not explain, however, is the origin and development of the theory. The result is to convey the impression that the theory has always existed in its present form, having been fully and correctly articulated from the start. Nothing, however, could be further from the

truth. On the contrary, as Lloyd Mints notes in his authoritative *A History of Banking Theory* (1945), "The problem of the manner in which the banking system increases the total volume of the circulating medium, while at the same time the lending power of the individual banks is severely limited, has proved to be one of the most baffling for writers on banking theory" [10, p. 39]. Far from understanding how loans generate deposits, bankers throughout the nineteenth and early twentieth centuries insisted that banks lend only the funds entrusted to their care and therefore could not possibly multiply deposits. Economists, on the other hand, often went to the opposite extreme, arguing that individual banks were simply small-scale versions of the banking system at large and thus could multiply deposits per dollar of reserves just as the system does. Both views were wrong. Not until the 1820s did a more plausible view start to emerge. And not until the 1920s was it finally stated in a way that fully convinced the economics profession and thus enabled the theory to gain widespread acceptance. In an attempt to provide historical perspective and to show how earlier writers resolved the paradox of a banking system doing what none of its members could do, this article traces the evolution of the theory between those two dates. Before doing so, however, it reviews the essentials of the theory as a prerequisite to identifying what earlier writers had to say about them.

The Theory of Deposit Expansion

Suppose for simplicity that the banking system consists of a single monopoly bank constrained by a required reserve-to-deposit ratio r and desiring to be fully loaned up. Suppose further that the public never wishes to convert deposits into currency so that no cash withdrawals occur when deposits expand. Because the bank cannot lose reserves through the clearinghouse to other banks (of which there are none) or to cashholders via withdrawal, it faces no restriction on its ability to expand loans and deposits other than the requirement that it hold r percent of its deposits in reserves. Thus upon the receipt of C dollars of new reserves it can instantly expand loans and deposits D up to the full limit allowed by the reserve ratio—that is, up to the amount $D = (1/r)C$, where $(1/r)$, the inverse of the reserve ratio, is the deposit expansion multiplier. In this way the system as a whole multiplies deposits per dollar of reserves.

Next suppose that the system consists of many small banks, each of which loses through the clearinghouse reserves equal to the full amount of loans made. Because of these adverse clearing balances, no bank can safely lend out more than $(1-r)$ of each dollar of deposits received, this sum being the amount remaining after r percent has been put in required reserve. Thus the first individual bank receiving C dollars of new cash deposits lends $(1-r)C$ of that amount after putting rC dollars in reserve. When borrowers write checks on the proceeds of the loans in favor of recipients who deposit the checks in a second group of banks, the latter banks gain $(1-r)C$ dollars in new deposits. They in turn keep r percent of the new deposits in reserve and lend out the remaining $(1-r)$ percent so that their loans equal $(1-r)(1-r)C$. This amount they lose through the clearinghouse to a third group of banks whose deposits accordingly rise by $(1-r)^2C$, and who, after setting aside a fraction r for reserve, lend out the remaining $(1-r)^3C$. And so it goes from bank to bank in ever-diminishing amounts until excess reserves are zero and all the new cash reserves C are absorbed in backing deposits in the ratio of 1 to r. Summing over the successive groups of banks in the dwindling, never-ending chain gives total new deposits D for the system of $D = [1 + (1-r) + (1-r)^2 + (1-r)^3 + \ldots + (1-r)^n]C$ which, when the number of banks n gets large, converges to the limit $D = (1/r)C$, the same expression that holds for the single monopoly bank.

In short, multiple expansion occurs in the multibank case because the excess reserves that form the basis for loans, though lost to the individual bank, are not lost to the system as a whole. They are simply transferred to other banks that use them for further expansion. As the expansion proceeds from bank to bank, each institution retains the reserves required to back the new deposits that brought it the extra reserves in the first place and lends out the remainder. The result is multiple expansion, the same as that achieved in the monopoly case. The only difference is that in the multibank case each individual bank does not multiply its own deposits. Rather it creates them for other banks by making loans and allowing its reserves to shrink to a fraction of the initial deposit. In a word, the banking system collectively multiplies deposits per dollar of new reserves while the small individual bank fractionalizes reserves per dollar of new deposits.

Historical Evolution

Having outlined the theory itself, we are now prepared to trace its origin and development. Retrospectively, one can discern a certain logical progression. First came the perception that deposits are a multiple of reserves, followed by a rudimentary exposition of the lending, redeposit, and multiplier aspects of the expansion mechanism. Next appeared a specification of the limits to deposit expansion and a definition of the limit value of the multiplier. There followed an analysis of how expansion spreads from bank to bank in a multibank system. Then came the first algebraic statement of the theory followed by the first clear distinction between the expansion power of a monopoly bank and a competitive bank. Finally came the persuasive restatement of the theory that, by consolidating, refining, and elaborating its key ideas, established it in mainstream banking analysis. Each stage saw a different innovator—Pennington, Torrens, Joplin, Marshall, Davenport, and Phillips are the key names here—advance the theory.

Multiple Deposits Recognized

The initial step in the theory's evolution came in the eighteenth century when writers such as John Law (1671-1729), Bishop Berkeley (1685-1753), and Alexander Hamilton (1755-1804) observed that bank deposits were several times larger than the underlying cash base and inferred from this that banks create deposits (see O'Brien [11, p. 15]). These writers, however, did not explain the mechanism that works to multiply deposits. They simply assumed that multiple deposit expansion would somehow occur for both the individual bank and the banking system as a whole. They failed to state that deposit

multiplication occurs through the successive lending and redeposit of excess reserves. Not until 1826 was this point made clear.

James Pennington (1777-1862)

It was James Pennington, a British currency expert and confidential monetary advisor to the government, who advanced the theory into its second stage. He did so with his rudimentary exposition of the lending, redeposit, and multiplier mechanics of deposit expansion. His contribution appears in his 1826 memorandum to the English statesman and financier William Huskisson. There he shows (1) that with fractional reserve banking cash deposits produce excess reserves, (2) that such excess reserves lead to loans, and (3) that the proceeds of the loans when redeposited in the system augment the volume of deposits per dollar of cash base. To illustrate these points he argued that if banks receive a cash deposit of which half must be held in reserve the rest will go to purchase earning assets (loans and investments). The sellers of these assets will, upon receiving the cash, redeposit it in their banks thus increasing the volume of deposits. At the end of this first round of the expansion process, the cash reserves of the banks will be the same as before, but the sum total of deposits—including the initial cash deposit plus the additional deposits created by loan—will already be increased by fifty percent. In his words:

> of the money entrusted to their [bankers'] care.... if a reserve of one half were sufficient...the other half would be employed in discounting bills [i.e., making loans]....But the Persons to whom these advances...were made, would, for their own convenience, deposit the money...in the hands of their respective bankers, and the aggregate amount of the outstanding [deposit] balances...would...be encreased 50 per cent....The money due to all the depositors would be 50 per cent more than it was previously to the commencement of these operations...[12, pp. xlv-xlvi].

Pennington did not trace the expansion process beyond the first round. But he did indicate how the individual bank contributes to expansion in a multibank system. He pointed out that as one bank expands its loans it either recovers the proceeds in the form of redeposits or else it loses reserves to other banks so that they too can expand. Either way, deposits increase. As he put it in a letter published in Volume 2 of Thomas Tooke's *History of Prices* (1838), if, after a bank receives an initial cash deposit and makes a loan,

> a cheque be drawn upon the...banker for the amount of the advance....[and] be paid into his hands by some other depositor, and placed at the credit of that other

> depositor...the whole amount of the book credits [i.e., deposits] of that banker will be increased to the extent of this new advance. And even if the cheque be paid into the hands of some other banker, the [initial] amount of the book credits of the banker who has paid the cheque will not be diminished, while the book credits, as well as the reserved fund of the banker, to whom it is paid, will be increased by its amount [13, p. lvi].

In other words, reserves lost by one bank show up as new deposits in another. In this way deposits gradually multiply on the given increase in the reserve base as it shifts from bank to bank. To illustrate, he showed that if the first bank in a system of two identical banks lends and loses through the clearinghouse half its initial cash reserve to the second that subsequently does the same, deposits of both banks expand although the reserve base remains unchanged [12, pp. xlvii-xlviii].

Pennington's failure to trace the expansion process to its completion accounts for his failure to specify the limit value of the multiplier. Far from defining it as the reciprocal of the reserve ratio, he was content merely to demonstrate that its value was greater than one. He also denied that he viewed the multiplier as a rigid mechanical relationship. This view was attributed to him by Robert Torrens, who cited Pennington as the source of the notion that London banks always hold in the form of notes of the Bank of England a one-fifth cash reserve against deposits, resulting in a multiplier of five. In correcting Torren's misapprehension, Pennington said:

> It never occurred to me, as appears to have been supposed by Colonel Torrens, that every million of notes issued by the Bank of England forms the basis of five millions of deposits; and that every million withdrawn from circulation, by the Bank, occasions a five-fold diminution of those deposits. On the contrary, it is perfectly consistent with my view of the subject, to suppose that the deposit accounts of the London bankers may be materially diminished, while the circulation of the Bank of England is greatly enlarged, or vice versa [13, p. lii].

Pennington contended that bankers' desired reserve ratios (and thus the multiple relationship between deposits and reserves) vary with the state of business confidence. In so doing, he originated the notion of a flexible multiplier.

Pennington's contemporaries quickly grasped the significance of his pioneering work. Torrens referred to it as "a subject of the greatest practical importance" [19, p. 12]. The Banking School likewise shared this opinion. While not accepting his definition of deposits as money, they used his notion of a flexible multiplier to argue that the credit superstructure (of which deposits were the chief component) could expand

and contract independently of the narrow monetary base such that control of the base did not imply control of the superstructure.

Robert Torrens (1780-1864)

Pennington was the first to outline the lending, redeposit, and multiplier aspects of bank credit creation. But Robert Torrens was the first to specify the limits to deposit expansion and to define the limiting value of the multiplier. Torrens, a professional soldier, newspaper proprietor, member of Parliament, promoter of schemes for the colonization of Australia, co-discoverer of the theory of comparative advantage, and one of the ablest monetary theorists of his generation, presented his analysis in his 1837 *Letter to Lord Melbourne*. There, in a section bearing the caption "A given amount of circulating Cash becomes the basis of a much greater amount of Bank Deposits," he wrote that deposits expand until they reach that particular ratio to reserves that bankers deem "safe and legitimate" [19, p. 16]. In other words, the desired deposit/reserve ratio together with the available quantity of reserves fixes the upper limit to expansion. He also explained how deposits grow up to this limit. Stressing the successive lending and redeposit of excess reserves, he wrote that given

a reserve...in coin...more than sufficient to meet... occasional demands....a part of this coin would be again advanced upon securities, and would be again returned upon the banks, in the form of new deposits, restoring their reserve...to the original sum...[19, p. 15].

It follows that

Whatever sums they may advance upon securities in the morning, the same sums will be returned to them in the evening, in the form of new deposits; and in this way the amount of their deposits must continue to increase, until they bear that proportion to the fixed amount of the returning cash, which the experience of the bankers may suggest as safe and legitimate [19, p. 16].

That is, expansion proceeds via the successive lending and redeposit of excess cash reserves until the desired deposit/reserve ratio is attained.

As for the deposit multiplier itself, Torrens expressed it as the inverse of the reserve ratio. He saw, for example, that a reserve ratio of one-tenth would produce a multiplier of ten. Observing that

in ordinary times, one-tenth, or even one-twentieth, of the money deposited with a banker, is a sufficient rest [reserve] for meeting occasional demands; and that nine-tenths, or even nineteenth-twentieths, of the sums deposited with a bank may be lent out on securities [19, p. 18],

he concluded:

I should not be arguing on an extreme case, were I to assume that the cash originally deposited...with bankers, will be successively re-issued upon securities, by the banks, and successively returned to them, in the form of new deposits, until the proportion between the amount of the deposits, and the amount of the cash, is as ten to one [19, pp. 18-19].

Here is the first clear statement of the multiplier as the reciprocal of the reserve ratio.

In his theoretical analysis, Torrens treated the multiplier as a potentially variable magnitude, fluctuating in value from a high of twenty to a low of five depending on the state of business confidence and its impact on bankers' desired deposit/reserve ratios. As he put it, these ratios

will necessarily vary with the variations of commercial confidence. When trade is prosperous, when few failures are occurring, and when commercial bills are promptly paid as they fall due, bankers might consider it safe to continue to re-issue, upon securities, the cash returning upon them as deposits, until the proportion between their deposits and their cash, became as fifteen to one, or even as twenty to one. In periods of commercial pressure, on the other hand, bankers would be disposed to contract their liabilities, until the deposits...bore to their cash a proportion, not exceeding seven to one, or even five to one [19, pp. 17-18].

Owing to these potential multiplier fluctuations, "a fixed amount of circulating money may be the basis of a fluctuating amount of credit money" [19, p. 17]. Yet in his practical policy analysis he treated the multiplier (or deposit/reserve ratio) as a more-or-less-fixed constant, arguing that control of the reserve base constituted automatic control of the deposit superstructure.

This last idea proved especially influential. The Currency School used it to argue that bank reserves controlled an inverted credit pyramid (with deposits the chief component) resting on a gold and banknote base. Through the writings of the Currency School, Torrens's doctrines of deposit multiplication on a reserve base and deposit control via that base became sufficiently well established by the mid-nineteenth century to be bequeathed to future generations of monetarists (see O'Brien [11, p. 16]). In short, the modern monetarist notion of base control derives straight from Torrens by way of the Currency School.

Thomas Joplin (1790-1847)

The next step in the theory's evolution was taken by Thomas Joplin, a British banker and co-originator of the principle of "metallic fluctuation" around which much of nineteenth century monetary contro-

versy raged. He advanced a view markedly different from Torrens's of the way deposits expand to the limit set by bankers' desired reserve ratios. As documented above, Torrens focused on the lending-redeposit mechanism of the banking system as a whole; he did not trace the expansion process from bank to bank. He merely stated that banks as a group expand loans, then recoup the proceeds in the form of redeposits, and then expand again and again until the limit is reached. He did not identify individual banks nor did he mention the distribution of reserves among them.

By contrast, Joplin explained how expansion proceeds from one bank to the next, each lending out its excess reserve and losing it to another bank which also expands and so on until excess reserves are eliminated and all cash is absorbed in backing deposits at the ratio desired by bankers. Joplin developed his analysis in his 1841 book *The Cause and Cure of Our Commercial Embarrassments*. He starts out by establishing the limits to expansion and defining the deposit multiplier as the inverse of the reserve ratio.

> Every banker...has therefore the power of creating bank money, and...there is no other limit to the exercise of this power than his own prudence....I apprehend that bank money is always created by the bankers to the full extent that prudence will permit. If one-fifth of their deposits in cash be sufficient to meet any demand for payment by their depositors, for every thousand pounds of cash deposited with them, they discount to the extent of £5,000, and create £5,000 of bank money [7, pp. 33, as quoted in Mints 10, p. 105].

He then proceeds to trace the expansion process across a succession of banks until the limit is reached. Assuming a reserve ratio of 20 percent, he states that a bank receiving a new cash deposit of £1,000 will immediately put £200 in reserves and lend out the remaining £800. The borrowers, upon receiving this sum,

> pay the amount, we shall assume, to the credit of their account with some other banker, who...finds his cash increased £800, and his deposits £800, and he has in consequence £640 to spare, which he lends accordingly. This again being paid into another bank, the same operation, again occurs, and so it goes on from bank to bank until the thousand pounds has created for itself deposits to the extent of £5,000 [7, pp. 33-34, as quoted in Mints 10, p. 105].

Here are all the elements found in modern textbook treatments of the multiple expansion process: (1) the initial cash deposit that generates excess reserves, (2) the lending out and subsequent loss of those reserves to other banks who repeat the process, (3) the resulting diminution of excess reserves at each successive bank as they are absorbed in backing the extra deposits created by their arrival, and (4) the cumulative rise in deposits until they reach their limit ratio to cash reserves, at which point excess reserves vanish. All that was missing was a mathematical statement of the process.

Alfred Marshall (1842-1924)

The mathematical statement referred to above constituted the next stage of the theory. The key name here is that of the great English neoclassical economist Alfred Marshall, who provided the algebraic basis for the theory and who used the standard mathematical technique to derive the deposit expansion multiplier as the summation of a geometrical series. Marshall used the symbol n to denote the multiplier, defined by him as the ratio of deposits to reserves (i.e., the inverse of the reserve ratio). In a note scribbled in the margin of his personal copy of Robert Giffen's *Stock Exchange Securities* (1877), he wrote:

> Let it [bankers' desired reserve/deposit ratio] be 1/n th:
> Let A be the original amount of deposits without credit:
> then credit can be spread to $A\left[1 + \frac{n-1}{n} + \left(\frac{n-1}{n}\right)^2 + \ldots\right] = A \cdot n$. This is true if there is only one bank,
> as well as if many, except that if there are many banks n cannot be very large in any one bank, while on the other hand if the banks pool their reserves (theoretically or practically) they count as cash what they have in the pool and the pool lends much of that again [quoted in Eshag 4, pp. 9-10].

He elaborates the substance of this brief note in his evidence before the Gold and Silver Commission of 1887. He says:

> I should consider what part of its deposits a bank could lend and then I should consider what part of its loans would be redeposited with it and with other banks and, vice versa, what part of the loans made by other banks would be received by it as deposits. Thus I should get a geometrical progression; the effect being that if each bank could lend two-thirds of its deposits, the total amount of loaning power got by the banks would amount to three times what it otherwise would be. If it could lend four-fifths, it will then be five times; and so on. The question how large a part of its deposits a bank can lend depends in a great measure on the extent to which the different banks directly or indirectly pool their resources [8, p. 37, as quoted in Eshag 4, p. 10].

In these passages Marshall makes three main points. First, to find the multiplier, one simply adds to each dollar of initial cash deposit the proportion of that dollar that successive banks can lend as it goes in dwindling amounts from bank to bank. In this con-

nection it should be noted that the terms $\frac{n-1}{n}$, $\left(\frac{n-1}{n}\right)^2$, etc., of Marshall's equation are the same as the terms $(1-r)$, $(1-r)^2$, etc., which show the proportion of each dollar of initial deposit that successive banks can lend out after required reserves have been set aside. The resulting multiplier, Marshall notes, is the same whether the system is composed of a single monopoly bank or many small competing banks. Second, the proportion of its deposits a bank can lend is determined by its reserve ratio. If that ratio is, say, one-fifth, the bank can lend out the remaining four-fifths of its deposits. Third, reserve ratios and the resulting power to lend vary by type of bank. Small isolated banks, because of their potentially greater exposure to cash drains and adverse clearings, will operate with larger reserve ratios than big banks or those having ready access to a central reserve pool.

Herbert Joseph Davenport (1861-1931)

The theory progressed to its sixth stage with University of Missouri economist H. J. Davenport's distinction between the expansion power of a single monopoly bank versus that of a small competitive bank in a multibank system. "Modern developments," writes F. A. Hayek, "follow the exposition of H. J. Davenport" [6, p. 153]. On page 261 of his *Economics of Enterprise* (1913) Davenport shows that a monopoly bank in a closed community can do what a whole banking system can do but what a competitive bank cannot do, namely multiply loans and deposits per dollar of cash reserves received. The monopoly bank, he says, loses no reserves to other banks; all checks written on it return in the form of redeposits. Consequently the only restriction on its ability to expand is that it keep r percent of cash reserves against deposits. Thus upon the receipt of C dollars of new reserves it can expand deposits D up to the limit $D = (1/r)C$.

To illustrate, he shows that a new monopoly bank, being the only bank in an isolated town and facing a reserve requirement of 15 percent, will, upon opening for business, engineer a 6⅔-fold expansion of loans and deposits per dollar of initial cash reserves contributed by the stockholders. He then applies this same multiplier to a cash deposit of $100,000, showing how the bank puts $15,000 in reserve, lends out an amount equal to six and two-thirds of the remaining $85,000, and realizes a deposit expansion (primary plus loan-derived) of $666,666. The monopoly bank, he explains, expands up to the limit allowed by the reserve ratio for one reason: it loses

no reserves through the clearinghouse or through cash drain.

> For the...customers of the bank make payments through checks upon the bank, and these credits are deposited in turn to the credit of other customers....And if some customers draw out cash, other customers will probably receive it and return it to the bank [3, p. 261].

Having described the multiplicative power of a monopoly bank, he turns his attention to the competitive bank. He notes that a competitive bank cannot expand to the extent of a monopoly bank since its attempts to do so will result in reserve losses through the clearinghouse. The competitive bank, he says, cannot expect the proceeds of its loans to be redeposited with it. On the contrary,

> When the check drawn by the borrowing depositor may be deposited in other banks and collected by them against the lending bank, its granting of credits rapidly draws down its reserves to swell the reserves of its competitors [3, p. 263].

These reserves, he notes, go to other banks, which also try to expand; in this way the system as a whole ultimately expands in the same ratio as the monopoly bank. He also suggests that when all banks simultaneously expand their loans approximately in balance, their reserve losses will tend to cancel each other.

> Each bank, as it, in turn lends to its customers, is losing reserves to other banks, but is, in turn, gaining reserves at the expense of the other banks—if at the same time the banking activity of these other banks is maintained [3, p. 287].

To the extent this happens, the group of banks together can (like a monopoly bank) quickly expand to the limit allowed by the reserve ratio.

Chester Arthur Phillips (1882-1976)

The theory of deposit expansion reached its zenith with the publication of C.A. Phillips's *Bank Credit* in 1921. There in the famous Chapter III entitled "The Philosophy of Bank Credit" he stated the theory with a power, precision, and completeness unmatched by his predecessors. In particular, it was Phillips more than anyone else who brought home to the economics profession the crucial distinction between the reserve loss of a competitive bank that expands its loans versus multiple expansion by the banking system as a whole. In so doing, he advanced the theory in at least three ways.

First, he refuted the view, held by Horace White, H. D. McLeod, and other banking writers of the time, that an individual bank multiplies its deposits

on a given reserve base just as the banking system does. Not so, said Phillips. An individual bank can not multiply deposits. For its attempts to do so by making loans of several times the amount of new reserves received will simply result in reserve losses to other banks equal to the amount of the loans made (or slightly less if a small fraction of the loans returns to the bank as deposits). No bank, he said, could tolerate such losses that imperiled its legal reserve position.

> Let us suppose that the Hanover National Bank of New York acquires a deposit of $1,000,000 in gold imported and lends $10,000,000 to its customers, an amount suggested by the approximate ratio of 1 to 10 between reserves and deposits....Perhaps not more than $100,000 out of all the checks drawn against the $10,000,000 borrowed would be deposited at the Hanover National Bank. The remainder of the manifold loans supposedly extended on the basis of the imported gold...would represent cash that the bank would lose through unfavorable clearing house balances, an amount that would be scattered widely among the banks of the system. It is clear that an individual bank attempting to lend greatly in excess of the amount of an addition to its reserves would do so at its peril [14, pp. 37-38].

Second, he explained with greater rigor and exactness than his predecessors how the individual bank contributes to systemwide multiple expansion even though it cannot itself multiply deposits. "How," he asked, "can a given amount of cash become the basis of manifold loans and deposits in a banking system if the acquisition of that amount by an individual bank has little or no multiplicative importance?" [14, p. 34]. His answer is that excess cash reserves obtained by one bank will, upon being lent out, provide another bank with excess cash with which it expands and so on until all cash is employed in supporting deposits at the ratio of one to r.

> The sudden acquisition of a substantial amount of reserve by a representative individual bank...tends to cause that bank to become out of tune with the banks in the system as a whole. As the individual bank increases its loans in order to re-establish its normal reserve-deposits ratio, reserve is lost to other banks and the new reserve, split into small fragments, becomes dispersed among the banks of the system. Through the process of dispersion it comes to constitute the basis of a manifold loan expansion [14, p. 40].

In short,

> Manifold loans are not extended by an individual bank on the basis of a given amount of reserve. Instead, as a consequence of lending, the reserve of the individual bank overflows, leaving only the equivalent of a fractional part of the additional volume of loans extended, the overflow cash finding its way to other and still other banks until it becomes the "residualized," yet shifting, foundation of manifold loans and deposits [14, p. 73].

To emphasize the point, he contrasted the way the banking system and the individual bank reach their desired reserve-deposit ratios—the system by expanding its deposit denominator; the bank by shrinking its reserve numerator.

Third, he was the first to publish algebraic formulas expressing the loan and deposit expansion potential of both the banking system and the individual bank. Then he used the standard mathematical technique of summation of a series to show that aggregation across the individual banks yields the systemwide formulas. His formulas for the banking system are straightforward and need only be summarized here. According to him, a system facing a required reserve ratio r can, upon the receipt of a new cash deposit C, immediately expand its loans L and deposits D by the amounts $L = \left(\dfrac{1}{r} - 1 \right)C$, and $D = (1/r)C$, where the latter parenthesized multiplier is one larger than the former since it takes account of the initial primary deposit as well as deposits created by loan.

His expansion formulas for the individual bank, however, require some explanation. He noted that the expansion power of the individual bank depends not only on its reserve ratio r but also on the fraction k of its loans that remain with it as deposits. This fraction, he argued, depends upon such things as compensating balance requirements, the accumulation of balances in borrowers' accounts in anticipation of loan repayment, and the redeposit of checks in the same bank upon which drawn. Given these factors, it is an easy matter to trace Phillips's derivation of the bank's loan and deposit expansion formulas.

Thus for an individual bank having a reserve ratio r and an initial cash deposit C, let k be the fraction of loan-created deposits retained by the bank, and L the extra loans made. Once the loans are granted and $(1 - k)$ of them withdrawn, final deposits (original plus the retained fraction of those created by loan) of $C + kL$ must, because deposited funds are either held in reserve or lent out, equal loans L plus required reserves $r(C + kL)$ obtained by applying the reserve ratio to deposits. In short, $C + kL = L + r(C + kL)$. Solving this equilibrium condition for loans yields Phillips's loan expansion formula $L = \left[\dfrac{1 - r}{kr + 1 - k} \right] C$, which, when substituted in the preceding definition of final deposits, results in the deposit expansion expression $D = \left[\dfrac{1}{kr + 1 - k} \right] C$, where the bracketed terms are the loan and deposit multipliers.

Using the preceding formulas, Phillips showed that if cash deposits C equal $1,000, and r and k equal 10 and 20 percent, respectively, then the individual bank can expand its loans L and deposits D by $1,097.25 and $1,219.51. These sums are somewhat larger that those of the hypothetical atomistic bank of the textbooks, whose k-factor of zero reduces its loan and deposit multipliers to $(1-r)$ and 1.0, respectively. On the other hand, the loan and deposit sums of Phillips's example are smaller than their counterparts in the case of a single monopoly bank, whose k-factor of 1.0 yields loan and deposit multipliers of $\left(\frac{1-r}{r} \right)$ and $(1/r)$, respectively. Thus Phillips's k-factor, varying as it does between one and zero, essentially indicates the extent to which any one bank can act as a monopoly bank, expanding loans and deposits as if it were the banking system as a whole (see Timberlake [18, pp. 10-12]).

Finally, in a demonstration similar to Marshall's, Phillips showed that the summation of the loan- and deposit-creation series across all individual banks yields the multiple expansion formulas for the system as a whole. Phillips's definitive exposition essentially established the theory once and for all in the form found in economics textbooks today.

The Theory Since Phillips

Since Phillips, at least three innovations have enhanced the theory of deposit expansion. First, economists James Harvey Rogers [15], Procter Thomson [17], and James Angell and Karel Ficek [1] incorporated into the deposit multiplier the public's currency-to-deposit ratio, c, to account for cash drains induced by deposit expansion itself. Using the resulting augmented multiplier expression

$D = \left(\frac{1}{c+r} \right) C$, they showed that *both* the cash and reserve ratios c and r act to limit deposit expansion, which is therefore smaller than it otherwise would be if limited by the reserve ratio alone. Still other writers have incorporated time deposit and excess reserve ratios into the multiplier thus further diminishing its magnitude. Second, James Meade [9], Milton Friedman and Anna Schwartz [5, pp. 784-94] as well as Phillip Cagan [2, p. 12] have extended the idea of the deposit expansion multiplier into the broader concept of the *money multiplier*, m, relating the total money stock (currency plus demand deposits), M, to the so-called high-powered monetary base, B, consisting of bank reserves plus currency held by the public according to the expression $M = mB$. Third, Paul Samuelson [16, p. 283] has observed that the small bank "expands" in symmetry with the system, not by multiplying deposits on a given new reserve but by fractionalizing its reserve on a given new deposit.

But these extensions, important as they are, are merely recent refinements made to the fundamental core of ideas laid down by Pennington and his successors. The key ideas of that core—namely that a fractional reserve banking system multiplies deposits, that the mechanics of multiplication involve the successive lending and redeposit of excess reserves, that some crucial ratio or ratios exist to limit the expansion, and that the individual bank contributes to the expansion process not by multiplying its own deposits but by creating them for others when it makes loans and loses reserves through the clearinghouse—were already enunciated more than a century ago. Even today, one finds these ideas indispensable to a full understanding of how the supply of bank money expands and contracts.

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