



# Working Paper Series



This paper can be downloaded without charge from:  
<http://www.richmondfed.org/publications/>



**THE FEDERAL RESERVE BANK OF RICHMOND**  
RICHMOND ■ BALTIMORE ■ CHARLOTTE

# The Pitfalls of Monetary Discretion\*

Aubhik Khan<sup>†</sup>      Robert G. King<sup>‡</sup>      Alexander L. Wolman<sup>§</sup>

Federal Reserve Bank of Richmond Working Paper No. 01-08

October 2001

JEL Nos. E52, E51, E58, E31, E42

Keywords: discretion, time-consistency problem, optimal monetary policy,  
sticky prices, multiple equilibria

## Abstract

In a canonical staggered pricing model, monetary discretion leads to multiple private sector equilibria. The basis for multiplicity is a form of policy complementarity. Specifically, prices set in the current period embed expectations about future policy, and actual future policy responds to these same prices. For a range of values of the fundamental state variable – a ratio of predetermined prices – there is complementarity between actual and expected policy, and multiple equilibria occur. Moreover, this multiplicity is not associated with reputational considerations: it occurs in a two-period model.

---

\*We thank Huberto Ennis, Andreas Hornstein and Per Krusell for helpful discussions, Luca Guerrieri and Eric Swanson for comments on a previous draft, and participants in the 2001 NBER Summer Institute and seminars at the European Central Bank and the Federal Reserve Board for their comments. They bear no responsibility for the contents of this paper. The views expressed here are the authors' and not necessarily those of the Federal Reserve Banks of Philadelphia or Richmond or the Federal Reserve System.

<sup>†</sup>Federal Reserve Bank of Philadelphia, [Aubhik.Khan@phil.frb.org](mailto:Aubhik.Khan@phil.frb.org).

<sup>‡</sup>Boston University, Federal Reserve Bank of Richmond, and NBER, [rking@bu.edu](mailto:rking@bu.edu).

<sup>§</sup>Federal Reserve Bank of Richmond, [Alexander.Wolman@rich.frb.org](mailto:Alexander.Wolman@rich.frb.org).

# 1 Introduction

A discretionary monetary authority responds optimally to the economy's state. The state includes prices set by firms in the past, and those prices were set based on expectations of what the future monetary authority would do. Implicitly then, the monetary policy action optimally responds to the expected monetary policy action. Under commitment, this channel is non-existent, because the entire *sequence* of policy actions was determined in the initial period. We study discretionary monetary policy in a canonical staggered price-setting model, and show that the endogeneity of policy under discretion can lead to multiple private-sector equilibria. There can be more than one set of beliefs about future monetary policy that, when incorporated into current prices, induces the future monetary authority to rationalize these beliefs.

The multiple equilibria we describe are not associated with an infinite horizon: they occur even in a two-period model, which is the case we study in this paper. The two-period case lets us highlight two different situations for the monetary authority. In the final period it is straightforward to describe the nature of private sector equilibrium and optimal discretionary monetary policy; there is a unique optimum which varies smoothly with the single fundamental state variable. In the initial period, matters are much more complicated.

Just as in the literature on coordination failure in macroeconomics (Cooper and John [1988]), multiple equilibria under discretionary monetary policy reflect a form of complementarity. But in contrast to many examples in that literature, here the complementarity is intrinsically dynamic. The staggered price-setting model contains two mechanisms that drive the dynamic complementarity. First, a monopolistically competitive firm will set a higher nominal price in the current period if it expects a higher future price level, because it will continue to charge that price in the future. Second, the firm understands that the future monetary authority will respond to the state of the economy, which varies with the prices set by all firms in the current period. In some circumstances, it is optimal for the future monetary authority to aggressively increase the money stock in response to the state, thereby making the future price level increase with the future state. This policy endogeneity can create complementarity among price-setters, and multiple equilibria can arise.

When certain policy actions can lead to multiple equilibria, the initial

period monetary authority must have beliefs about the likelihood that each equilibrium will occur in order to solve its policy problem. We explore two assumptions about equilibrium selection: under the first assumption it is common knowledge that all equilibria are equally likely (uniform selection) and under the second it is common knowledge that the best equilibrium will always occur (Pareto selection). Under each of these assumptions, we find that the presence of multiple equilibria considerably complicates the nature of optimal policy. Even though maximized welfare is a smooth function of the state, optimal policy exhibits nonstandard behavior. The policy function is nonmonotonic and has one or more discontinuities, depending on the selection assumption. These discontinuities arise because of multiple local maxima to the policymaker's objective function, with the global maximum flipping from one local maximum to the other as the state variable changes.

Like other recent work discussed in section 6, we study a modern version of the Kydland-Prescott and Barro-Gordon time consistency problem for monetary policy. A form of the time consistency problem they described naturally arises in the sticky price models that are now commonly used for monetary policy analysis. Unlike the original models however, with explicit staggered price-setting there is a natural state vector (here it is a scalar) representing predetermined relative prices. The existence of a natural state drives our results. These results, however, are not familiar from recent work. Despite the fact that we study an ordinary model of staggered pricing with optimizing behavior, as in King and Wolman (1999), little has been known about the nature of discretionary monetary policy in this setting.<sup>1</sup> The work that has been done, for example by Dotsey and Hornstein (forthcoming) in a model almost identical to ours, and by Clarida, Galí and Gertler (1999), Woodford (1999) and Svensson and Woodford (1999) in models with Calvo-style pricing, has employed linear-quadratic approximation methods and a primal approach to optimal policy.<sup>2</sup> Those methods have not uncovered the multiple equilibria we describe.<sup>3</sup>

---

<sup>1</sup>The model in this paper is close to that of Chari, Kehoe and McGrattan (2000). Here there is no capital, however, and firms that adjust their prices do so after current period information is revealed.

<sup>2</sup>We should also note that the central topic of Dotsey and Hornstein's paper is how different information structures affect the monetary policy problem.

<sup>3</sup>Currie and Levine (1993) contains references to an earlier literature that derived Markov-perfect discretionary equilibria in linear models with quadratic objectives. Oudiz and Sachs (1984) is perhaps the first example.

The paper proceeds as follows. Section 2 lays out the model. Section 3 describes private sector equilibrium under arbitrary monetary policy. We use backward induction to describe and compute equilibrium. Section 4 describes the channels through which monetary policy can affect real outcomes in the model. These channels are summarized by two distortions: a relative price distortion across symmetric firms, and the markup of price over marginal cost. Section 5 contains the results on optimal policy. In section 6 we place our results in the context of other recent literature that emphasizes multiple equilibria under discretion in related models. Section 7 concludes.

## 2 The model

The model has three sets of agents: a representative household, a continuum of monopolistically competitive firms, each producing a differentiated consumption good, and a government that supplies money and levies lump-sum taxes. There is three-period staggered pricing: firms set their prices for three periods, and each period one-third of all firms readjust their prices. The model does not have any life-cycle features. While it is natural to assume an infinite horizon for households, in order to study the issue of multiple equilibria we find it useful to analyze a finite-horizon model.

We make the following assumptions about timing of actions in a given period. At the beginning of period  $t$ , the prices set by firms in previous periods are predetermined. A ratio of these nominal prices serves as a real state variable. The monetary authority chooses the money supply as a function of the state, given its beliefs about the future evolution of the economy (that is, given its beliefs about future monetary policy). Finally, those firms adjusting their prices in period  $t$  choose their prices, and all other period- $t$  variables are simultaneously determined. The state for period  $t + 1$  is known at the end of period  $t$ .

### 2.1 Households

Our model has  $T+1$  periods, where  $T \in [0, \infty)$ , and households have standard preferences over consumption ( $c$ ) and leisure ( $l$ ):

$$U_t = \sum_{j=0}^T \beta^j u(c_{t+j}, l_{t+j}), \quad T \geq 0. \quad (1)$$

Note that households live for the duration of the model. Consumption is a Dixit-Stiglitz aggregate of the differentiated products,

$$c_t = \left( \int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (2)$$

and households supply labor ( $n_t = 1 - l_t$ ) in a competitive labor market for wage  $w_t$ . The household's budget constraint is

$$P_t c_t = P_t w_t n_t + D_t,$$

where  $P_t$  is the price index for the aggregate consumption good and  $D_t$  is dividend payments from firms. Efficient behavior dictates that consumption and leisure are chosen so that the real wage equals the marginal rate of substitution between consumption and leisure:

$$w_t = u_l(c_t, l_t) / u_c(c_t, l_t). \quad (3)$$

We do not derive money demand from microfoundations, but instead assume that velocity is constant and equal to unity:

$$M_t = P_t c_t. \quad (4)$$

By abstracting from any distortions associated with money demand, we isolate phenomena associated with sticky prices. In other work (Khan, King and Wolman [2000]) we have found that the distortions associated with money demand appear to be small. It would of course be interesting to study a version of the model in which money demand is generated by money-in-the-utility-function, for example, or the costly credit framework in Khan, King and Wolman (2000).

Denoting by  $P_t(z)$  the nominal price of good  $z$ , individuals' demands for the differentiated consumption goods are

$$c_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} c_t, \quad (5)$$

and the price index is

$$P_t = \left( \int_0^1 P_t(z)^{1-\varepsilon} dz \right)^{1/(1-\varepsilon)}. \quad (6)$$

Both the demand functions and the price index are implied by optimal allocation of a given level of consumption across the differentiated products, taking the price of each product as given.

## 2.2 Firms

Each of the differentiated product manufacturers ( $z \in [0, 1]$ ) has access to a deterministic technology that is linear in labor:

$$y_t(z) = n_t(z). \quad (7)$$

Thus, firm  $z$ 's real profits in period  $t$  are given by

$$\pi_t(z) = y_t(z) \cdot \left( \frac{P_t(z)}{P_t} - w_t \right). \quad (8)$$

By substituting from the demand functions (i.e.- using  $y_t(z) = c_t(z)$ ), we can write profits as a function of relative price, the wage, and aggregate demand:

$$\pi \left( \frac{P_t(z)}{P_t}, w_t, c_t \right) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} \cdot \left( \frac{P_t(z)}{P_t} - w_t \right) \cdot c_t. \quad (9)$$

A firm adjusting its price in period  $t$  will charge the same price in periods  $t+1$  and  $t+2$ . Given that firms are owned by households, when a firm adjusts its price it solves the following problem

$$\max_{P_{0,t}} \sum_{j=0}^{\min\{2, T-t\}} \beta^j \cdot \frac{u_c(c_{t+j}, l_{t+j})}{u_c(c_t, l_t)} \cdot \pi \left( \frac{P_{0,t}}{P_{t+j}}, w_{t+j}, c_{t+j} \right), \quad (10)$$

the solution to which is

$$\frac{P_{0,t}}{P_t} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{\sum_{j=0}^{\min\{J-1, T-t\}} \beta^j \cdot u_c(c_{t+j}, l_{t+j}) \cdot c_{t+j} \cdot w_{t+j} \cdot (P_{t+j}/P_t)^\varepsilon}{\sum_{j=0}^{\min\{J-1, T-t\}} \beta^j \cdot u_c(c_{t+j}, l_{t+j}) \cdot c_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon-1}}. \quad (11)$$

Note that because of the nature of price setting, the continuum of firms sorts naturally into three groups. Let  $c_{j,t}$  denote consumption in period  $t$  of a good whose price was set in period  $t-j$ , and let  $P_{j,t}$  denote the price of such a good. Then the consumption aggregator is

$$c_t = \left( \sum_{j=0}^2 \frac{1}{3} \cdot c_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (12)$$

and total labor input is

$$n_t = \sum_{j=0}^2 \frac{1}{3} \cdot n_{j,t} = \sum_{j=0}^2 \frac{1}{3} \cdot c_{j,t}. \quad (13)$$

The price index is given by

$$P_t = \left( \sum_{j=0}^2 \frac{1}{3} \cdot P_{j,t}^{1-\varepsilon} \right)^{1/(1-\varepsilon)}. \quad (14)$$

Because individual prices remain fixed for three periods,

$$P_{j,t-1} = P_{j+1,t}, \quad j = 0, 1. \quad (15)$$

That is, a newly set price in period  $t - 1$  becomes a one-period old price in period  $t$ , etc.

### 2.3 The government, and the natural state variable

The government and the monetary authority are synonymous in our analysis. The government chooses the level of the money supply, and changes in the money supply are accomplished via lump-sum transfers,  $F_t$  (or lump-sum taxes, in the case of decreases in the money supply):

$$M_t - M_{t-1} = F_t. \quad (16)$$

This equation is the government budget constraint for an economy where there is no interest-bearing government debt, and no distortionary taxes or government spending.

Henceforth, we will normalize the money supply and all other nominal variables by  $P_{D,t}$ , an index of the prices charged in period  $t$  by firms that set their prices in periods  $t - 1$  and  $t - 2$ :

$$P_{D,t} \equiv \left( \frac{1}{2} \cdot P_{1,t}^{1-\varepsilon} + \frac{1}{2} \cdot P_{2,t}^{1-\varepsilon} \right)^{1/(1-\varepsilon)}. \quad (17)$$

Note that the aggregate price index can be written as

$$P_t = \left( \frac{1}{3} \cdot P_{0,t}^{1-\varepsilon} + \frac{2}{3} \cdot P_{D,t}^{1-\varepsilon} \right)^{1/(1-\varepsilon)}. \quad (18)$$

We define the following normalized variables:

$$m_t \equiv M_t/P_{D,t} \quad (19)$$

$$p_{j,t} \equiv P_{j,t}/P_{D,t}, \quad j = 0, 1, 2, \quad (20)$$

$$\text{and} \quad (21)$$

$$p_t \equiv P_t/P_{D,t}. \quad (22)$$



We will assume that the government chooses  $m_t$  as a function of the economy's single natural state variable, which indexes the deviation between the two predetermined nominal prices. Without loss of generality, we will define the state variable to be the ratio of the price set by firms in the previous period to the index of predetermined prices:

$$s_t \equiv \frac{P_{1,t}}{P_{D,t}} = p_{1,t}. \quad (23)$$

One might think that with two predetermined prices there would be two state variables. However, we can normalize by one of those prices, or by an index of them: the level of past prices is irrelevant, because the monetary authority can choose the nominal level of  $M_t$  in an unconstrained way. Our purpose is to study optimal monetary policy without commitment, so we are mainly concerned with optimal choice of the function  $m_t(s)$ .<sup>4</sup> However, part of the analysis will involve studying how the economy behaves under exogenous policy in period  $t$ , given that policy will be optimally chosen in the future.

The above definitions and normalizations imply

$$p_t = \left( \frac{1}{3} \cdot p_{0,t}^{1-\varepsilon} + \frac{2}{3} \right)^{1/(1-\varepsilon)}. \quad (24)$$

The normalized price level is uniquely determined by the normalized price set by adjusting firms.

### 3 Private Sector Equilibrium with Arbitrary Policy

In order to determine optimal monetary policy, we need to be able to construct a private sector equilibrium for arbitrary monetary policy. We do this using backward induction. This methodology makes it relatively straightforward to determine optimal policy, again using backward induction. Here, policy in period  $t$  will be represented by a policy function  $m_t(s_t)$ . Under optimal policy these functions will be chosen to maximize welfare.

---

<sup>4</sup>Typically, analysis of equilibrium under discretionary optimization would involve finding a time-invariant function  $m(s)$ . In the finite-horizon cases we examine, policy functions will depend on the horizon, hence the subscript  $t$  in  $m_t(s)$ .

Because the horizon is finite, we will not be constructing a recursive equilibrium, nor should we expect to find a steady state equilibrium. However, there are three time-invariant functions that will be important ingredients in computing an equilibrium. The appendix (A.2) shows how one can use the money demand equation and the firm-level technologies to express consumption, leisure and the future state as time-invariant functions of  $p_{0,t}$ ,  $m_t$  and  $s_t$  :

$$c_t = c(p_{0,t}; m_t), \quad (25)$$

$$l_t = l(p_{0,t}; m_t, s_t), \quad (26)$$

and

$$s_{t+1} = \sigma(p_{0,t}, s_t). \quad (27)$$

Under arbitrary policy, the only endogenous variable in these expressions is the price set by adjusting firms in the current period ( $p_{0,t}$ ). Note also that these expressions can be combined with the labor supply equation (3) to determine the real wage – and hence marginal cost – as a function of  $p_{0,t}$ ,  $m_t$  and  $s_t$ . All endogenous variables have been eliminated except for  $p_{0,t}$ .

The remaining equation, needed to determine  $p_{0,t}$ , is the optimal pricing condition. We reproduce that equation here for convenience, substituting out for the real wage as the marginal rate of substitution between consumption and leisure, and normalizing the nominal variables on the left hand side:

$$\frac{p_{0,t}}{p_t} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{\sum_{j=0}^{\min\{2, T-t\}} \beta^j u_l(p_{0,t+j}; m_{t+j}, s_{t+j}) c(p_{0,t+j}; m_{t+j}) \cdot \left( \frac{P_{t+j}}{P_t} \right)^\varepsilon}{\sum_{j=0}^{\min\{2, T-t\}} \beta^j u_c(p_{0,t+j}; m_{t+j}, s_{t+j}) c(p_{0,t+j}; m_{t+j}) \cdot \left( \frac{P_{t+j}}{P_t} \right)^{\varepsilon-1}} \quad (28)$$

(note that we use shorthand to directly express marginal utilities as functions of  $p_{0,t+j}; m_{t+j}, s_{t+j}$ , instead of indirectly through consumption and leisure). The normalized price level on the left-hand side is a known function of  $p_{0,t}$  (24), and the appendix (A.3) shows how to express future inflation rates on the right-hand side in terms of current and future  $p_0$ ,  $m$  and  $s$  :

$$\frac{P_{t+j+1}}{P_{t+j}} = \Pi(p_{0,t+j+1}, m_{t+j+1}, s_{t+j}, s_{t+j+1}).$$

Note that  $\frac{P_{t+2}}{P_t} = \frac{P_{t+1}}{P_t} \cdot \frac{P_{t+2}}{P_{t+1}}$ .

Proceeding further means looking for equilibrium values of  $p_{0,t}$  that solve the pricing equation (28). Although the details of solving the pricing equation depend on the horizon (that is,  $T - t$ ), in general knowledge of  $p_{0,t}$  and  $m_{t+j}(s_{t+j})$ ,  $j = 0, 1, \dots, T - t$  is sufficient to pin down all current and future aggregate variables, which in turn pins down the optimal price for any individual firm. That is, (28) can be shown to depend only on  $p_{0,t}$ , and thus in principle it is straightforward to determine the set of equilibria.

### 3.1 The final period

In period  $T$ , the model is static. This simplifies the search for equilibrium dramatically, because no future policy needs to be taken into account. From (28), the optimal pricing equation in the final period is

$$\frac{p_{0,T}}{p_T} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{u_l(c(p_{0,T}; m_T), l(p_{0,T}; m_T, s_T))}{u_c(c(p_{0,T}; m_T), l(p_{0,T}; m_T, s_T))}.$$

Recall that the normalized price level is determined by  $p_{0,T}$  (from (24)). Then given the state and the policy action, both the left- and right-hand sides of this equation vary only with  $p_{0,T}$ . A more intuitive version of this pricing equation results from viewing it as a best response function for an individual firm. Let  $\hat{p}_{0,T}$  denote the price chosen by an individual firm, and  $p_{0,T}$  denote the price chosen by all firms. The latter pins down all aggregates, so that an individual firm's optimal price is a function of the price chosen by all other firms:

$$\hat{p}_{0,T} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \cdot \left( \frac{1}{3} \cdot p_{0,t}^{1-\varepsilon} + \frac{2}{3} \right)^{1/(1-\varepsilon)} \cdot \frac{u_l(c(p_{0,T}; m_T), l(p_{0,T}; m_T, s_T))}{u_c(c(p_{0,T}; m_T), l(p_{0,T}; m_T, s_T))}. \quad (29)$$

Any fixed point of the best response function is an equilibrium value of  $p_{0,T}$ , for given values of  $m_T$  and  $s_T$ . Knowing  $p_{0,T}$ , all other endogenous variables can be computed using expressions derived earlier. For an arbitrary function  $m_T(s_T)$ , we can then compute the equilibrium mappings  $p_{0,T}(s_T; m_T)$ ,  $c_T(s_T; m_T)$ ,  $l_T(s_T; m_T)$ . Without imposing restrictions on the utility function, there is no guarantee that (29) will have a unique fixed point, so that these mappings will be functions. However, the potential for multiple equilibria at this stage is not our focus. Thus, we will proceed assuming that (29) does have a unique fixed point. In the example we focus on below it is

easy to show uniqueness. If preferences are given by  $u(c, l) = \ln c + \chi l$ , then (29) simplifies to

$$\widehat{p}_{0,T} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \cdot \chi \cdot m_T,$$

which is a flat best response function, implying  $p_{0,T} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \cdot \chi \cdot m_T$ .

### 3.2 Period $T - 1$

In period  $T - 1$ , equilibrium for a given state and a given policy action depends on the policy function in the final period,  $m_T(s_T)$ . Taking that policy function as given and arbitrary, we again compute equilibrium by analyzing the optimal pricing condition. Distinguishing between the prices chosen by an individual firm and all other firms, the optimal pricing condition can be interpreted as a best-response function:

$$\widehat{p}_{0,T-1} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \left( \frac{1}{3} \cdot p_{0,T-1}^{1-\varepsilon} + \frac{2}{3} \right)^{1/(1-\varepsilon)} \times \quad (30)$$

$$\frac{\sum_{j=0}^1 \beta^j u_l(p_{0,T-1+j}; m_{T-1+j}, s_{T-1+j}) c(p_{0,T-1+j}; m_{T-1+j}) \left( \frac{P_{T-1+j}}{P_{T-1}} \right)^\varepsilon}{\sum_{j=0}^1 \beta^j u_c(p_{0,T-1+j}; m_{T-1+j}, s_{T-1+j}) c(p_{0,T-1+j}; m_{T-1+j}) \left( \frac{P_{T-1+j}}{P_{T-1}} \right)^{\varepsilon-1}}.$$

The right-hand side can be expressed as a function of only  $p_{0,T-1}$ , given  $s_{T-1}$  and  $m_{T-1}$ . The first terms in the denominator and numerator, as well as the period  $T - 1$  price index factor, depend only on  $p_{0,T-1}$ ,  $m_{T-1}$  and  $s_{T-1}$ . The second terms in the numerator and denominator depend also on period  $T$  variables. However, the policy function for period  $T$  is known and the equilibrium correspondences for period  $T$  are known, so all period  $T$  variables can be determined as functions of  $m_{T-1}$ ,  $s_{T-1}$  and  $p_{0,T-1}$ . Mechanically, this works as follows.

1. Given  $m_{T-1}$ ,  $s_{T-1}$  and  $p_{0,T-1}$ , the law of motion for the state variable (27) determines  $s_T$ .
2. Given  $s_T$  and an assumed policy function  $m_T(s_T)$  for the final period,  $m_T$  is known.
3. Given  $s_T$  and  $m_T$ , the set of equilibrium  $p_{0,T}$  is the set of fixed points of the best response function for the final period (29).

Following these steps allows the right-hand side of the best response function for period  $T - 1$  to be expressed as a function of only  $p_{0,T-1}$ , given  $s_{T-1}$  and  $m_{T-1}$ . The set of equilibrium  $p_{0,T-1}$  is the set of fixed points of the best response function (30), and the equilibrium values of other variables can be computed from (25) - (27) and steps 1 - 3. If one specifies exogenous policy functions for period  $T - 1$  and  $T$ , one can compute equilibrium correspondences for all  $T - 1$  and  $T$  variables by varying  $s_{T-1}$  and following the procedure just described.

This procedure is the natural extension of that used above for the final period, but the distinction is important. In the final period, equilibrium is determined entirely by the state and the policy action. One period earlier, in order to solve the optimal pricing equation for  $p_{0,T-1}$ , we also need to know what the policy *function* will be in the final period. That future policy function determines how  $p_{0,T-1}$  will map – via  $s_T$  – into policy actions in the future. Those future policy actions affect future demand, marginal cost and inflation, and therefore the optimal price for a firm in the current period ( $\hat{p}_{0,T-1}$ ). Optimal policy in the final period generates a particular policy function which will be taken as given by firms – and the monetary authority – in period  $T - 1$ .

## 4 Monetary Policy and Real Activity

Monetary policy has the potential to affect real variables in this model because some prices are predetermined. Monetary policy has an incentive to affect real variables because monopolistic competition makes the flexible price level of output inefficiently low. The specific channels through which monetary policy works involve relative price distortions and the markup. With complete discounting, optimal discretionary policy would simply involve balancing these two distortions. With  $\beta > 0$ , the policymaker also considers the implications of her actions for the future state, which affects future utility.

### 4.1 Relative price distortions

The consumption aggregate is a concave, symmetric function of each good, and each good is produced using identical technology. For a given amount of total labor input then, consumption is maximized by producing the same quantity of each good. More formally, we will define the relative price

distortion in period  $t$  ( $\rho_t$ ) as the ratio of input to output:

$$\rho_t \equiv \frac{1}{3} \cdot \frac{\sum_{j=0}^2 n_{j,t}}{c_t} = \sum_{j=0}^2 \frac{1}{3} \cdot (c_{j,t}/c_t) \geq 1. \quad (31)$$

In an appendix (A.1), we derive an expression for the relative price distortion in terms of  $p_{0,t}$  and  $s_t$ :

$$\rho_t = \rho(p_{0,t}, s_t). \quad (32)$$

If all relative prices are unity, then  $\rho_t = 1$ , which corresponds to the relative price distortion being minimized. A sufficient condition for all relative prices to be unity is that the economy be in a steady state with a constant price level, or that prices be perfectly flexible. Furthermore, if  $J > 1$ , as it will be throughout the paper, the only steady state with  $\rho_t = 1$  is a steady state with a constant price level. In general, if the price level varies over time, the relative price distortion will exceed unity.

## 4.2 The markup distortion

With a competitive, economy-wide labor market, and technology that is linear in labor, the real marginal cost of all firms is simply given by

$$\psi_t = w_t. \quad (33)$$

The markup ( $\mu_t$ ) of the price index over nominal marginal cost is the inverse of real marginal cost:

$$\mu_t = 1/\psi_t = 1/w_t. \quad (34)$$

In a flexible price economy, all firms charge the same price, and the markup is  $\bar{\mu} = \frac{\varepsilon}{\varepsilon-1}$ . With staggered pricing, the markup differs from  $\bar{\mu}$  even in a steady state, unless the steady state involves zero inflation. See King and Wolman (1999, pp. 363-364) for details.

Output is inefficiently low when the markup exceeds unity, as it will in any steady state. By choosing a higher  $m_t$ , the policymaker can decrease the markup and raise output. For high enough  $m_t$  though, the increase in output comes at the expense of an increase in the relative price distortion. Furthermore, the policymaker also cares about the state with which she endows the policymaker next period.

## 5 Equilibrium with Discretionary Policy

A discretionary equilibrium is a private sector equilibrium in which the monetary authority chooses an action that maximizes welfare, given that future monetary authorities will also act with discretion. In terms of the earlier description of equilibrium with arbitrary policy, the monetary authority chooses  $m_t(s_t)$  to maximize welfare for every  $s_t$ , given that  $m_{t+j}(s_{t+j})$  will be chosen to maximize welfare for  $j = 1, \dots, T - t$ . One can view the policymaker as implicitly computing the equilibria associated with all arbitrary policy actions, and choosing the action that generates the equilibrium yielding highest welfare. If equilibrium is unique for every arbitrary policy action, then this assessment is straightforward. If some policy actions would generate multiple equilibria, then the policymaker needs to assign probabilities to the equilibria in order to evaluate the welfare associated with each action. In the final period we find no evidence that any policy actions generate multiple equilibria, whereas in the previous period we find multiple equilibria even under optimal policy for a range of the state variable.

### 5.1 Period $T$

The policy problem in the final period is

$$v_T(s_T) = \max_{m_T} (u(c_T, l_T)) \quad (35)$$

subject to (25)-(26) for  $t = T$ , which express consumption and leisure as functions of the state, the policy action, and  $p_{0,T}$ , and subject to  $p_{0,T}$  being determined as the (assumed unique) solution to the optimal pricing condition (29). The three panels of figure 1 illustrate the policy problem in period  $T$ , for three different values of the state variable, and the three panels of figure 2 illustrate the solution as a function of the state variable.

From figure 1 it is clear that the policy problem in the final period amounts to trading off the markup against the relative price distortion. At low values of  $m_T$ , both the markup and the relative price distortion can be brought down by choosing a higher  $m_T$ . Above a level that depends on  $s_T$ , further increases in the money supply continue to bring about lower values of the markup, but at the cost of a greater relative price distortion. At an optimum, the marginal cost of a higher relative price distortion is exactly offset by the marginal benefit of a lower markup. The nature of the trade-off

between the relative price distortion and the markup depends on the state variable, because the state variable represents an inherited component of the relative price distortion. If  $s_T = 1.0$ , the policymaker in period  $T$  does not inherit any relative price distortion, and it is feasible – though not optimal – to eliminate the relative price distortion entirely.

One notable feature of figure 1 is that the markup is determined entirely by the money supply: it is insensitive to the state variable. This is not a general result, but occurs because Hansen-Rogerson preferences make the real wage proportional to consumption. Since the markup is the inverse of the real wage in this model, the markup is determined entirely by consumption. From (25), consumption depends on  $p_{0,T}$  and  $m_T$ , but we saw in the previous section that these preferences make  $p_{0,T}$  proportional to  $m_T$ . Therefore consumption and the markup are determined entirely by  $m_T$ .

Figure 2 summarizes final period outcomes under optimal policy, as a function of the state. Welfare ( $v_T$ ), in panel A, and the policy action ( $m_T$ ), in panel B, are both quasiconcave functions that are maximized at  $s_T = 1.0$ . The markup and relative price distortions, plotted in panel C, are both quasiconvex functions that are minimized at  $s_T = 1.0$ . The key to understanding these panels is to realize that there is no inherited relative price distortion if and only if  $s_T = 1.0$ . When  $s_T = 1.0$ , the policymaker can eliminate the relative price distortion and achieve the flexible price outcome by setting  $m_T = 1.0$ . This choice is suboptimal, however, because a marginal increase in  $m_T$  from 1.0 generates a lower markup and the same relative price distortion. Optimally then, even when it is feasible to eliminate the relative price distortion, in the final period the policymaker chooses to accept a small relative price distortion, because the high value of  $m_T$  generates a relatively low markup. For states that involve a high inherited relative price distortion, bringing down the markup is quite costly in terms of the relative price distortion, so optimal policy chooses a low value of the money supply, thereby accepting a higher markup.

The final period is obviously special, in that the policymaker faces no intertemporal trade-off. In the initial period the policymaker cares about the state with which she endows the future policymaker, because this state affects future welfare. The final period is special in another way as well. In the final period, firms that set their prices one and two periods earlier are differentiated only by the prices they charge.<sup>5</sup> In the initial period, these firms

---

<sup>5</sup>It is clearly awkward to talk about firms setting their prices for three periods in the



influence the policy problem in different ways, because they will be resetting their prices at different times. This final period equivalence between the two types of firms implies that if we plotted the panels of figure 2 using  $P_2/P_D$  on the horizontal axis instead of  $s_T = P_1/P_D$ , the figures would look identical.

## 5.2 Period $T - 1$

With the monetary policy action chosen optimally in the final period, *arbitrary* policy in the previous period lead to multiple equilibria. The nature of optimal policy in period  $T - 1$  therefore depends on the equilibrium selection mechanism. Figure 3 displays period  $T - 1$  optimal policy – the value function (panel A) and the policy function (panel B) – under two assumptions about equilibrium selection.<sup>6</sup> The circles represent optimal policy under what we call uniform selection: if a particular value of  $m_{T-1}$  generates multiple equilibria, then it is assumed to be common knowledge that each equilibrium has equal probability of occurring. The plus signs represent optimal policy under Pareto selection: it is assumed to be common knowledge that the equilibrium yielding highest welfare will occur with probability one. Under both selection assumptions, the value functions are well-behaved (i.e.-quasiconcave). However, the policy functions are *discontinuous*: under uniform selection there are two discontinuities, and under Pareto selection there is one discontinuity. For  $s_{T-1}$  less than approximately 1.25, there is a unique private sector equilibrium under optimal policy, and therefore ex-ante welfare and the policy functions are identical across the two selection criteria. For higher values of the state, the functions differ according to the selection criterion, indicating multiple equilibria under optimal policy.<sup>7</sup>

The subsections below are devoted to explaining in some detail multiple equilibria and discontinuities in the (optimal) policy function. Before diving into that detail, it will be helpful to have an intuitive sense of where these results are coming from. Both the multiple equilibria and the policy

---

context of a two-period model. One should think about our results as describing the early stages of solving a longer horizon model through backward induction. See the end of section 5 for more on this issue.

<sup>6</sup>The probability distribution across equilibria is taken as exogenous, and we consider two distributions. Ennis and Keister (2001) discuss endogenizing the distribution. We have not yet studied how to incorporate their approach into our model.

<sup>7</sup>When there are multiple equilibria under optimal policy, the value function measures ex-ante value, where the weighting criterion is applied to welfare in each of the equilibria.

discontinuities stem from the fact that the current policy action determines a state variable that affects the incentives and opportunities faced by the future policymaker.

Multiple equilibria occur because the future policy action is endogenous with respect to current pricing behavior. A single current price setter bases its pricing decision on expectations about the future policy action. In turn, the future policymaker bases her policy action on the future state, which is determined by the behavior of all current price setters (27). Intuitively this relationship can be characterized by complementarity: if current price setters set a higher price, the future policymaker responds to the higher state with a higher money supply, and in turn the higher future money supply raises the future price level, making it optimal for individual price-setters to set a high price in the initial period. Figure 2 suggests that complementarity will not be omnipresent, because for high values of the state variable increases in the state lead to a lower money supply.

Discontinuities in the policy function are associated with flips from one local maxima of the policymaker's objective function to another. Typically, one local maximum involves a low current relative price distortion and correspondingly high current utility, whereas the other local maximum involves a high current relative price distortion and low current utility, but leaves the future policymaker with a favorable value of the state, leading to high future utility.

Multiple equilibria occur because the current policy action affects the incentives facing the future policymaker. In contrast, policy discontinuities stem from the fact that the set of feasible current and future utilities is not convex. Both of these features would be absent were there no state variable linking the present to the future.

### 5.2.1 The Policy Problem

For period  $T - 1$ , Figures 4.I (uniform selection) and 4.II (Pareto selection) illustrate the objective function, the two distortions and the future state as functions of the policy action, for three levels of the current state ( $s_{T-1}$ ). Recall that we solve the model using backward induction: optimal future policy is taken into account in determining equilibrium in period  $T - 1$ . Our purpose in including these figures is to emphasize the existence of multiple equilibria, and to shed some light on the discontinuities that we have already seen exist under optimal policy. In panels B through D of figure 4, multiple

equilibria are reflected in there being more than one level of the distortions and the future state for certain policy actions. In panel A, but not in the other panels, we have imposed the relevant selection criterion in order to generate expected welfare. Thus, discontinuities in panel A indicate points where the number of equilibria switches from one to three or three to one. A graphical explanation for the multiplicity will follow the discussion of figure 4.

If the current state is high enough, figure 4 shows that there are multiple local maxima to the objective function. The policy discontinuities seen in figure 3 are associated with the global maximum switching from one local maximum to another. In some cases (e.g.  $s_{T-1} = 1.2$ , Pareto selection) two local maxima occur with a smooth objective function, whereas in other cases (e.g.  $s_{T-1} = 1.25$ , Pareto selection) multiple local maxima are associated with jumps in the objective function. These possibilities raise two questions. First, why are there multiple local maxima, and second, why are there discontinuities in the objective function (as opposed to the policy function)?

Figure 4 illustrates that multiple local maxima to the policymaker's objective function occur because increasing the current policy action ( $m_{T-1}$ ) can have differential effects on current and future utility. Focusing on the case of Pareto selection with  $s_{T-1} = 1.2$  (this is the set of circles in figure 4.B), there is a local maximum with a low level of  $m_{T-1}$ , which corresponds to a low current relative price distortion but a relatively bad future state (that is, a future state far from 1.0). There is another local maximum with a high level of  $m_{T-1}$ , in which the current relative price distortion is high, but the future state is much closer to 1.0. Referring back to figure 2, the closer is the future state to 1.0, the higher is welfare in the final period. Thus, the first local maximum has high first-period welfare and low final-period welfare, whereas the second local maximum has low first-period welfare and high final-period welfare. Multiple local maxima can also be understood in terms of a utility frontier: from the standpoint of the initial period monetary authority (who cares about current and future utility, but takes as given the behavior of future policy), the feasible set of current and future utility levels is not convex.<sup>8</sup>

Discontinuities in the policymaker's objective function occur as  $m_T$  moves in or out of regions where there are multiple equilibria, for either selection

---

<sup>8</sup>Given that the relevant indifference curves have constant slope, nonconvexity naturally leads to the possibility of policy discontinuities, which we do find.

criterion that we consider. In fact, given the regions of multiple equilibria shown in figure 4, it would take a highly contrived selection criterion to avoid discontinuities in the objective function.

### 5.2.2 Multiple Equilibria

Multiple equilibria occur for a range of  $T - 1$  policy actions; this can be seen in figure 4 for two values of the state variable (for a third –  $s_{T-1} = 1.1$  – multiple equilibria do not occur under any policy action). That multiple equilibria occur under *optimal* policy for a wide range of values of  $s_{T-1}$  can be seen from figure 3.A: optimal policy and welfare depend on equilibrium selection, which indicates multiple private-sector equilibria under optimal policy. The multiplicity of equilibria implies some form of complementarity, and as is often the case there is more than one way to think about that complementarity.

Figure 5 describes the multiple equilibria in terms of what we call *policy complementarity*. Holding fixed the current state and policy action, panel A displays the future policy action as a function of the expected future policy action. This relationship is characterized by an s-shaped function that crosses the 45° line three times, indicating three equilibria. The complementarity illustrated in figure 5.A is that the future policymaker chooses a higher level of  $M_T$  in response to expectations of higher  $M_T$ . This is a reduced form relationship, and panel B displays optimal policy and optimal pricing, the two more fundamental objects which underlie the relationship. On the horizontal axis of panel B is the future state variable, and on the vertical axis are the actual and expected future nominal money supply.<sup>9</sup> The dashed line is the final period optimal policy function  $m_T(s_T)$  (figure 2.B), except that the money supply is normalized by  $P_{D,T-1}$  rather than  $P_{D,T}$ . The solid line is derived from the optimal pricing condition (30), and describes private sector equilibrium. To understand the solid line, it is helpful to view  $E_{T-1}M_T$  as the “right-hand side” variable. As  $E_{T-1}M_T$  varies, the corresponding equilibrium price  $p_{0,T-1}$  can be determined as the solution to (30). And from  $p_{0,T-1}$ , the future state  $s_T$  can be determined using (27). Summarizing panel B, the dashed line describes how the future state determines the future policy

---

<sup>9</sup>We refer here to the nominal money supply ( $M_T$ ). What is crucial though is that we are referring to the final period money supply normalized by initial period predetermined prices rather than final period predetermined prices. Thus it is equivalent to refer to  $M_T$  or  $M_T/P_{D,T-1}$ .

action, and the solid line describes how the expected future policy action determines the future state. Panel A combines these elements to express the optimal policy action as a function of the expected policy action. If firms expect the future money supply to be high, they set a high price, which translates into a high value of the future state. Encountering a high value of the state variable, the future policymaker responds with a high value of the money supply, validating expectations. Figures 3 and 4 show that multiple equilibria occur under optimal policy only for high values of the state variable. When the state variable is low, at the optimal policy action there is a unique fixed point to the policy response function illustrated in figure 5.

One can also interpret the multiple equilibria in terms of a more conventional notion of complementarity among price-setting firms. Complementarity in pricing is induced by the nature of policy. Figure 6.A displays the best-response pricing function of an individual firm in period  $T - 1$ , conditional on the same initial-period state and policy action used in figure 5 (the best-response function is given by (30)). As one would expect from figure 5, this best-response function is s-shaped and has three fixed points. Panel A illustrates the multiplicity in terms of  $p_{0,T-1}$ . Panel B plots the relationship between  $s_T$  and  $p_{0,T-1}$  given by (27), allowing one to verify that the equilibria illustrated in figure 6 for  $p_{0,T-1}$  correspond to those illustrated in figure 5 for  $s_T$ . Obviously there is a connection between the forms of complementarity represented in figures 5 and 6. Figure 6 shows that an individual firm responds to higher prices of all other firms by setting a higher price. This occurs because the higher price set by all other firms raises the future state variable, inducing the monetary authority to choose a higher money supply in the future. Figure 5 emphasizes the role of policy: expectations of a higher future money supply raise the future state variable, inducing the monetary authority to validate those expectations.

### 5.3 Optimal Policy Under Commitment

The multiple equilibria illustrated in figures 5 and 6 are inherently related to discretion. Because there is no exogenous uncertainty in the model, optimal policy under commitment involves choosing in the initial period the best *path* for the nominal money supply. Thus, under commitment the final period nominal money supply (normalized by  $P_{D,T-1}$ ) is only a function of the initial state ( $s_{T-1}$ ). Referring to figure 5.B, this means that under commitment the optimal policy curve (dashed) would be a horizontal line. The optimal pricing

curve would be unchanged, so there would be only one possible equilibrium. The essence of commitment is that in the initial period the policymaker promises that in the final period she will not react to  $s_T$ , the endogenous state variable. A credible promise not to react to the final period state eliminates the mechanism that under discretion leads to multiple equilibria.

## 5.4 Robustness

There are of course a number of robustness exercises it would be interesting to conduct. We have already undertaken some of them. As stated earlier, our results on multiple equilibria and discontinuities under discretion are inherently related to the existence of a state variable, representing a pre-determined relative price. When prices are set for two periods rather than three, there is no state variable. Wolman (forthcoming) describes the unique steady state under discretion with two-period staggered pricing.

In this paper results are presented for preferences with an infinite labor supply elasticity. It is natural to wonder whether the results extend to other preference specifications. We have studied examples of Cobb-Douglas preferences and found qualitatively similar results.

Given that we assume three-period staggered price-setting, it would be desirable to extend the *horizon* to more than two periods. We have done this, and found that similar multiplicities and discontinuities occur in earlier periods. However, these very features make it difficult to compute solutions and to explain results.

We intend to pursue at least two other robustness exercises. First, with respect to money demand, do the results carry over to settings where the money demand function is derived from a standard assumption such as money-in-the-utility function? Based on our work on optimal policy with commitment, we are inclined to believe the answer is yes. Second, although the staggered price-setting framework we use is more appealing at a micro level, the literature on monetary policy has leaned more towards Calvo pricing. It would therefore be interesting to know whether our results carry over to the Calvo setting. Because there will still be a state variable, it seems likely that similar results will obtain with Calvo pricing.<sup>10</sup>

---

<sup>10</sup>Articles mentioned in the introduction that describe discretionary policy under Calvo pricing use linear quadratic approximations and a primal approach.

## 6 Related Literature

The nature of equilibria under discretion in sticky-price models is sensitive to specific assumptions about price setting. In a wide class of models, monopolistic competition makes the level of output inefficiently low, and with some prices predetermined, a surprise monetary expansion will produce a real expansion as well. When *all* prices are predetermined, as in Ireland (1997), there is no cost to the monetary authority of a surprise expansion. The only Markov-perfect equilibrium in that environment involves the highest possible inflation rate.<sup>11</sup> In Albanesi, Chari and Christiano (2000), all period  $t$  prices are set after period  $t - 1$  variables have been observed, but a fraction of firms set their prices before the monetary authority moves. There is thus a cost to surprise inflation: it creates a relative price distortion. However, this relative price distortion is invariant to the expected rate of inflation.<sup>12</sup> In our model, some firms choose their prices contemporaneously with (or before) period  $t - 1$  variables. This feature leads to a relative price distortion even in steady state (as long as the price level is not constant), and the magnitude of this distortion varies with the steady state rate of inflation. The monetary authority thus faces the following trade-off in our model. With output inefficiently low, the discretionary policymaker would like to create a surprise expansion. The cost of a surprise is that it can exacerbate relative price distortions. In equilibrium there can be no surprises, so in equilibrium the marginal benefit of a surprise in terms of increased output (reduced markup) must be exactly offset by the marginal cost in terms of increased relative price distortion. Because steady inflation raises relative price distortions, intuitively there ought to be a constant rate of inflation at which the discretionary policymaker is content not to create surprises. Wolman (forthcoming) shows that such a steady state does exist when prices are set for two periods. With three-period pricing however, a state variable is introduced

---

<sup>11</sup>Ireland's model also contains nontrivial distortions associated with money demand, whereas our model does not. However, the discussion here would also apply to a version of his model without money demand distortions. Benigno and Benigno (2001) discuss a two-country version of such a model. They show that strategic considerations make price stability a discretionary equilibrium under certain conditions.

<sup>12</sup>Albanesi, Chari and Christiano study an environment with nontrivial money demand distortions, and these play an important role in determining the Markov-perfect equilibria of their model. We conjecture that absent money demand considerations, their model would have no interior Markov-perfect equilibrium – the monetary authority would always want to create a surprise expansion.

that leads to the unusual behavior we have described.

Albanesi, Chari and Christiano also stress the existence of multiple equilibria without reputational considerations. The nature of multiplicity that we are describing is different from that in Albanesi, Chari and Christiano.<sup>13</sup> There, multiplicity appears in the current optimal policy action: there are generally two current policy actions which can be equilibria (i.e.-optimal), and which one occurs is indeterminate. Given the policy action, however, all other variables are uniquely determined. Here, there is generally a continuum of policy actions that can be equilibria, depending on the probabilities associated with the three equilibria under exogenous policy. These probabilities are not given by fundamentals. Given some common set of beliefs about the probabilities of different outcomes under exogenous policy, the policymaker has a unique optimal action, but that action does not uniquely pin down current outcomes. Practically speaking, neither the inflation rate nor output is pinned down by the choice of money growth. In addition, the mechanism behind multiplicity is quite different than that in Albanesi, Chari and Christiano. There, multiple equilibria are related to the nature of money demand (in particular, multiplicity occurs when money demand is sufficiently interest elastic), and there are no endogenous state variables. Here money demand is completely interest inelastic, and multiple equilibria are due to the presence of an endogenous state variable.

Dedola (1999) studies discretionary policy in a Rotemberg-style model of pricing, and finds multiple equilibria. Dedola models money demand using a cash-in-advance constraint, and like Albanesi, Chari and Christiano, the multiple equilibria are related to the money demand specification.

The introduction referred to several papers that do not find multiple equilibria under discretion, in models quite similar to ours.<sup>14</sup> Two features are important in explaining the different conclusions we reach. First, those papers take a primal approach, meaning that the policymaker is assumed to be able to select allocations (we assume the policymaker is able to select only the money supply). This approach would seem to directly eliminate the multiplicity described here. Second, they approximate the constraints with linear functions and the objective with a quadratic function. In the infinite

---

<sup>13</sup>As in this paper, the multiple equilibria in Albanesi, Chari and Christiano do not derive from reputational considerations. In contrast, the multiplicity in Chari, Christiano and Eichenbaum (1998) is reputation-driven.

<sup>14</sup>Clarida, Gali and Gertler (1999), Woodford (1999) and Svensson and Woodford (1999), Dotsey and Hornstein (forthcoming).



horizon settings of those papers, it is possible for this L-Q approach to find multiple steady states. However, for a particular steady state the dynamics will necessarily be unique. In the finite horizon, non-primal framework of this paper, an L-Q approximation would eliminate the multiplicity: from figure 5 it is clear that multiple equilibria are related to a nonlinearity in the constraints.

Krusell and Smith (2001) study a model of time-inconsistent preferences, and show that there are a continuum of Markov-perfect equilibria to the game that represents the consumption-savings problem of an infinitely lived agent who cannot commit to future actions. They also suggest that multiplicity may be endemic to a large class of problems in which there is no commitment. Our results support their suggestion, and can even be thought of as indicating that things are worse than they suggested. For we find multiple equilibria in a finite horizon model, whereas the multiplicity they discuss arises with an infinite horizon.

## 7 Conclusion

In a simple staggered price-setting model we find that discretionary monetary policy leads to multiple equilibria. Inflation and real allocations are not pinned down by fundamentals alone. The nature of the multiplicity is such that even for a given value of the money supply, the outcome can be indeterminate. From a positive perspective, we conclude that lack of commitment may help to explain fluctuations in inflation and real activity. From a normative perspective, multiplicity under discretion provides an additional argument for a monetary commitment mechanism. Our results also suggest that caution should be used in applying popular approximation methods to staggered-pricing models. These methods have not uncovered the multiple equilibria that we find.

Multiple equilibria occur under discretion because there is a form of policy complementarity. Expected future policy influences current pricing decisions, which determine a state variable to which future policy responds. In effect then, policy responds to expected policy. For a nontrivial range of values of the initial state, this response involves complementarity and multiple equilibria: if firms expect a high money supply in the future, they set a high price in the current period. The monetary policymaker in the future then finds it optimal to validate this high price with a high money supply. Likewise,

expectations of a low money supply are also self-fulfilling. Feedback from expected policy to policy itself is of course present in a large class of discretionary policy environments. The likelihood of such mechanisms leading to multiple equilibria is an open question.

## References

- [1] Albanesi, Stefania, V.V. Chari and Lawrence Christiano (2000), “Expectation Traps and Monetary Policy,” manuscript, <ftp://ftp.igier.unibocconi.it/wp/2001/198.pdf>.
- [2] Barro, Robert and David B. Gordon (1983), “A Positive Theory of Monetary Policy in a Natural Rate Model,” *Journal of Political Economy* 91, 589-610.
- [3] Benigno, Gianluca and Pierpaolo Benigno (2001), “Price Stability in Open Economies,” <http://homepages.nyu.edu/~pb50/FNash4.pdf>.
- [4] Chari, V.V., Lawrence Christiano and Martin Eichenbaum (1998), “Expectation Traps and Discretion,” *Journal of Economic Theory* 81, 462-492.
- [5] Chari, V.V., Patrick Kehoe and Ellen McGrattan (2000), “Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?” *Econometrica* 68, 1151-1179.
- [6] Clarida, Richard, Jordi Galí and Mark Gertler (1999), “The Science of Monetary Policy,” *Journal of Economic Literature* 37, 1661-1707.
- [7] Cooper, Russell and A. Andrew John (1988), “Coordinating Coordination Failures in Keynesian Models,” *Quarterly Journal of Economics* 103, 441-463.
- [8] Currie, David and Paul Levine (1993), *Rules, reputation and macroeconomic policy coordination*, Cambridge University Press.
- [9] Dedola, Luca (2000), “Credibility and the Gains from Commitment in a General Equilibrium Model,” University of Rochester Ph.D. thesis.
- [10] Dotsey, Michael and Andreas Hornstein, forthcoming, “Is the Behavior of Money Useful to a Discretionary Policymaker?” *Journal of Monetary Economics*.
- [11] Ennis, Huberto and Todd Keister (2001), “Optimal Policy with Probabilistic Equilibrium Selection,” Federal Reserve Bank of Richmond Working Paper 01-03, <http://www.rich.frb.org/pubs/wpapers/>.

- [12] Ireland, Peter (1997), "Sustainable Monetary Policies," *Journal of Economic Dynamics and Control* 22, 87-108.
- [13] Khan, Aubhik, Robert G. King and Alexander L. Wolman (2000), "Optimal Monetary Policy," Federal Reserve Bank of Richmond Working Paper 00-10, <http://www.rich.frb.org/pubs/wpapers/>.
- [14] King, Robert G. and Alexander L. Wolman (1999), "What Should the Monetary Authority Do When Prices Are Sticky?" in *Monetary Policy Rules*, John Taylor, ed.
- [15] Krusell, Per and Anthony A. Smith Jr. (2001), "Consumption-Savings Decisions with Quasi-Geometric Discounting," <http://fasttone.gsia.cmu.edu/tony/quasi.pdf>.
- [16] Kydland, Finn E. and Edward C. Prescott (1977), "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* 85, 473-492.
- [17] Oudiz, G. and J. Sachs (1984), "Macroeconomic Policy Coordination Among the Industrial Countries," *Brookings Papers on Economic Activity* 1, 1-64.
- [18] Svensson, Lars E.O. and Michael Woodford (1999), "Implementing Optimal Policy through Inflation-Forecast Targeting," <http://www.princeton.edu/~svensson/papers/swiftabs.htm>.
- [19] Wolman, Alexander L. (1999), "Sticky Prices, Marginal Cost, and the Behavior of Inflation," Federal Reserve Bank of Richmond *Economic Quarterly*, <http://www.rich.frb.org/eq/abstract.cfm?articleID=64>
- [20] \_\_\_\_\_, forthcoming "A Primer on Optimal Monetary with Sticky Prices," Federal Reserve Bank of Richmond *Economic Quarterly*.
- [21] Woodford, Michael (1999), "Optimal Monetary Policy Inertia," NBER working paper 7261.

## A Appendix: Derivations

In the appendix we allow for more general staggered pricing behavior;  $\omega_j$  will denote the fraction of firms in period  $t$  charging a price set in period  $t - j$ . In the body of the paper we impose  $\omega_0 = \omega_1 = \omega_2 = 1/3$ .

### A.1 Relative price distortion as function of $p_{0,t}$ and $s_t$

Starting from (31), using the demand functions, we can rewrite the relative price distortion in terms of relative prices instead of quantities:

$$\begin{aligned}
 \rho_t &= p_t^\varepsilon \sum_{j=0}^{J-1} \omega_j \cdot p_{j,t}^{-\varepsilon} \\
 &= (\omega_0 p_{0,t}^{1-\varepsilon} + (1 - \omega_0))^{\varepsilon/(1-\varepsilon)} (\omega_0 p_{0,t}^{-\varepsilon} + \omega_1 p_{1,t}^{-\varepsilon} + \omega_2 p_{2,t}^{-\varepsilon}) \\
 &= (\omega_0 p_{0,t}^{1-\varepsilon} + (1 - \omega_0))^{\frac{\varepsilon}{1-\varepsilon}} \left( \omega_0 p_{0,t}^{-\varepsilon} + \omega_1 s_t^{-\varepsilon} + \omega_2 \left( \frac{P_{2,t}}{P_{1,t}} \right)^{-\varepsilon} s_t^{-\varepsilon} \right). \quad (36)
 \end{aligned}$$

Above we have written the relative price distortion as a function of  $p_{0,t}$ ,  $s_t$  and  $\frac{P_{2,t}}{P_{1,t}}$ . By manipulating the definition of  $s_t$ , we can express  $\frac{P_{2,t}}{P_{1,t}}$  as a function of  $s_t$ , and thus express the relative price distortion solely as a function of  $p_{0,t}$  and  $s_t$ :

$$\begin{aligned}
 s_t &= \left( \left( \frac{\omega_1}{1 - \omega_0} \right) + \left( 1 - \frac{\omega_1}{1 - \omega_0} \right) \cdot \left( \frac{P_{2,t}}{P_{1,t}} \right)^{1-\varepsilon} \right)^{1/(\varepsilon-1)} \quad \text{so} \\
 s_t^{\varepsilon-1} &= \left( \frac{\omega_1}{1 - \omega_0} \right) + \left( 1 - \frac{\omega_1}{1 - \omega_0} \right) \cdot \left( \frac{P_{2,t}}{P_{1,t}} \right)^{1-\varepsilon} \\
 s_t^{\varepsilon-1} - \left( \frac{\omega_1}{1 - \omega_0} \right) &= \left( 1 - \frac{\omega_1}{1 - \omega_0} \right) \cdot \left( \frac{P_{2,t}}{P_{1,t}} \right)^{1-\varepsilon} \\
 \left( \frac{P_{2,t}}{P_{1,t}} \right)^{1-\varepsilon} &= \left( \frac{1 - \omega_0}{1 - (\omega_0 + \omega_1)} \right) \left( s_t^{\varepsilon-1} - \left( \frac{\omega_1}{1 - \omega_0} \right) \right) \\
 \left( \frac{P_{2,t}}{P_{1,t}} \right) &= \left[ \left( \frac{1 - \omega_0}{1 - (\omega_0 + \omega_1)} \right) \left( s_t^{\varepsilon-1} - \left( \frac{\omega_1}{1 - \omega_0} \right) \right) \right]^{\frac{1}{1-\varepsilon}}. \quad (37)
 \end{aligned}$$

Now sub this last expression into (36):

$$\begin{aligned} \rho_t &= \rho(p_{0,t}, s_t), \text{ where} \\ \rho(p_{0,t}, s_t) &= (\omega_0 p_{0,t}^{1-\varepsilon} + (1 - \omega_0))^{\varepsilon/(1-\varepsilon)} \times \\ &\left( \omega_0 p_{0,t}^{-\varepsilon} + s_t^{-\varepsilon} \left( \omega_1 + \omega_2 \left[ \left( \frac{1 - \omega_0}{\omega_2} \right) \left( s_t^{\varepsilon-1} - \left( \frac{\omega_1}{1 - \omega_0} \right) \right) \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) \right). \end{aligned} \quad (38)$$

## A.2 Time-invariant functions for consumption, leisure, and evolution of the state variable

For consumption, substitute the definition of the normalized price index into the money demand equation:

$$\begin{aligned} c(m_t, p_{0,t}) &= m_t / p_t \\ &= m_t \cdot (\omega_0 p_{0,t}^{1-\varepsilon} + (1 - \omega_0))^{1/(\varepsilon-1)}. \end{aligned}$$

For leisure, use the firm-level technology and the definition of the relative price distortion:

$$\begin{aligned} l(m_t, p_{0,t}; s_t) &= 1 - n_t \\ &= 1 - c_t \cdot \left( \sum \omega_j (c_{j,t} / c_t) \right) \\ &= 1 - c(m_t, p_{0,t}) \cdot \rho(p_{0,t}, s_t). \end{aligned}$$

The state variable in period  $t + 1$  can be written as a function of period  $t$  values of the state variable and the price chosen by adjusting firms:

$$\begin{aligned}
s_{t+1} &= \frac{P_{1,t+1}}{P_{D,t+1}} \\
&= \frac{P_{0,t}}{P_{D,t+1}} \\
&= \frac{P_{0,t}}{\left( \frac{\omega_1}{1-\omega_0} \cdot P_{0,t}^{1-\varepsilon} + \left(1 - \frac{\omega_1}{1-\omega_0}\right) P_{1,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}} \\
&= \left( \frac{\omega_1}{1-\omega_0} + \left(1 - \frac{\omega_1}{1-\omega_0}\right) \left(\frac{P_{1,t}}{P_{0,t}}\right)^{1-\varepsilon} \right)^{\frac{1}{\varepsilon-1}} \\
s_{t+1} &= \left( \frac{\omega_1}{1-\omega_0} + \left(1 - \frac{\omega_1}{1-\omega_0}\right) \left(\frac{P_{0,t}}{s_t}\right)^{\varepsilon-1} \right)^{\frac{1}{\varepsilon-1}}. \tag{39}
\end{aligned}$$

In the text, we refer to the right hand side of (39) as the function  $\sigma(p_{0,t}, s_t)$ .

### A.3 Cumulative inflation rate

The cumulative inflation rate between periods  $t$  and  $t + j$  is relevant because it determines the real deterioration of a nominal price that was set in period  $t$ . It is easiest to work with the cumulative inflation rate if we view it as the product of one-period inflation rates ( $\frac{P_{t+1}}{P_t}$ ). The first step is to use the money demand equation for  $t + 1$  to eliminate  $P_{t+1}$ :

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{c_{t+1}} \cdot \frac{1}{P_t}.$$

Next, multiply and divide by  $P_{D,t+1}$ , and use the definition of the normalized money supply:

$$\begin{aligned}
\frac{P_{t+1}}{P_t} &= \frac{M_{t+1}}{P_{D,t+1}} \frac{1}{c_{t+1}} \frac{P_{D,t+1}}{P_t} \\
&= \frac{m_{t+1}}{c_{t+1}} \cdot \frac{P_{D,t+1}}{P_t}.
\end{aligned}$$

Now replace  $P_t$  with its definition, and divide the numerator and denominator by  $P_{D,t+1}$ , recalling the definition of  $s_{t+1}$  :

$$\begin{aligned}\frac{P_{t+1}}{P_t} &= \frac{m_{t+1}}{c_{t+1}} \cdot \frac{P_{D,t+1}}{(\omega_0 P_{1,t+1}^{1-\varepsilon} + (1-\omega_0) P_{D,t}^{1-\varepsilon})^{1/(1-\varepsilon)}} \\ &= \frac{m_{t+1}}{c_{t+1}} \cdot (\omega_0 s_{t+1}^{1-\varepsilon} + (1-\omega_0) (P_{D,t}/P_{D,t+1})^{1-\varepsilon})^{1/(\varepsilon-1)}. \quad (40)\end{aligned}$$

To proceed further, express  $P_{D,t}/P_{D,t+1}$  as a function of  $s_t$  and  $s_{t+1}$  :

$$\begin{aligned}\frac{P_{D,t}}{P_{D,t+1}} &= \frac{P_{D,t}/P_{1,t}}{P_{D,t+1}/P_{1,t}} \\ &= \frac{P_{D,t}/P_{1,t}}{P_{D,t+1}/P_{2,t+1}} \\ &= \frac{(1/s_t)}{(P_{D,t+1}/P_{1,t+1})(P_{1,t+1}/P_{2,t+1})} \\ &= \frac{s_{t+1}}{s_t} \cdot \frac{P_{2,t+1}}{P_{1,t+1}}; \text{ then use (37) to get} \\ &= \frac{s_{t+1}}{s_t} \cdot \left[ \left( \frac{1-\omega_0}{1-(\omega_0+\omega_1)} \right) \left( s_{t+1}^{\varepsilon-1} - \left( \frac{\omega_1}{1-\omega_0} \right) \right) \right]^{\frac{1}{1-\varepsilon}}, \quad (41)\end{aligned}$$

and substitute this last expression into (40):

$$\begin{aligned}\frac{P_{t+1}}{P_t} &= \frac{m_{t+1}}{c_{t+1}} \cdot \left( \omega_0 s_{t+1}^{1-\varepsilon} + (1-\omega_0) \left( \frac{s_{t+1}}{s_t} \cdot h(s) \right)^{1-\varepsilon} \right)^{1/(\varepsilon-1)} \\ &= \frac{m_{t+1}}{c_{t+1}} \cdot \left( \omega_0 s_{t+1}^{1-\varepsilon} + (1-\omega_0) \left[ \left( \frac{s_{t+1}}{s_t} \right) h(s) \right]^{1-\varepsilon} \right)^{1/(\varepsilon-1)}.\end{aligned}$$

where

$$h(s) = \left[ \left( \frac{1-\omega_0}{1-(\omega_0+\omega_1)} \right) \left( s^{\varepsilon-1} - \left( \frac{\omega_1}{1-\omega_0} \right) \right) \right]^{\frac{1}{1-\varepsilon}}.$$

Thus, we have

$$\frac{P_{t+1}}{P_t} = \Pi(p_{0,t+1}, m_{t+1}, s_t, s_{t+1}), \quad (42)$$

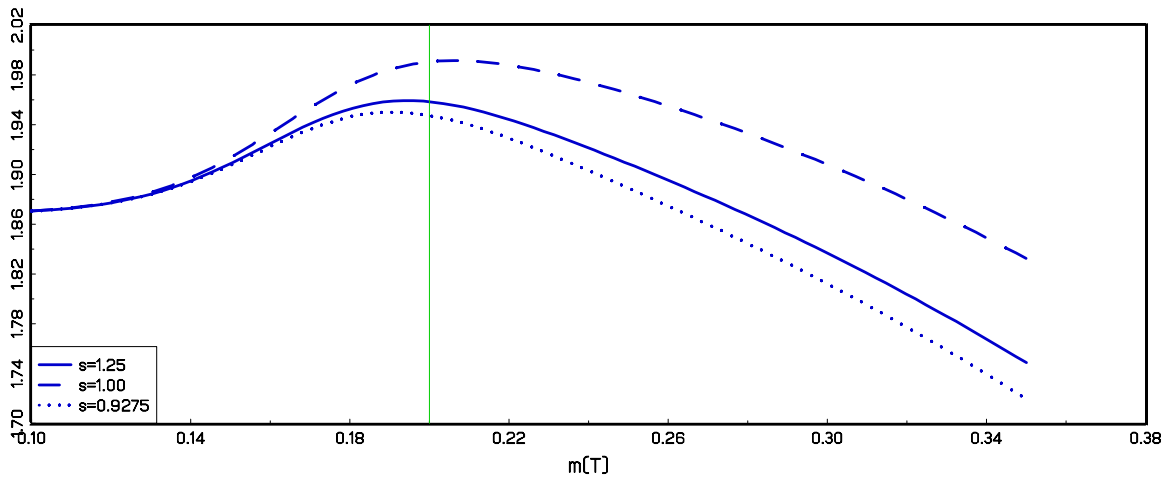


where

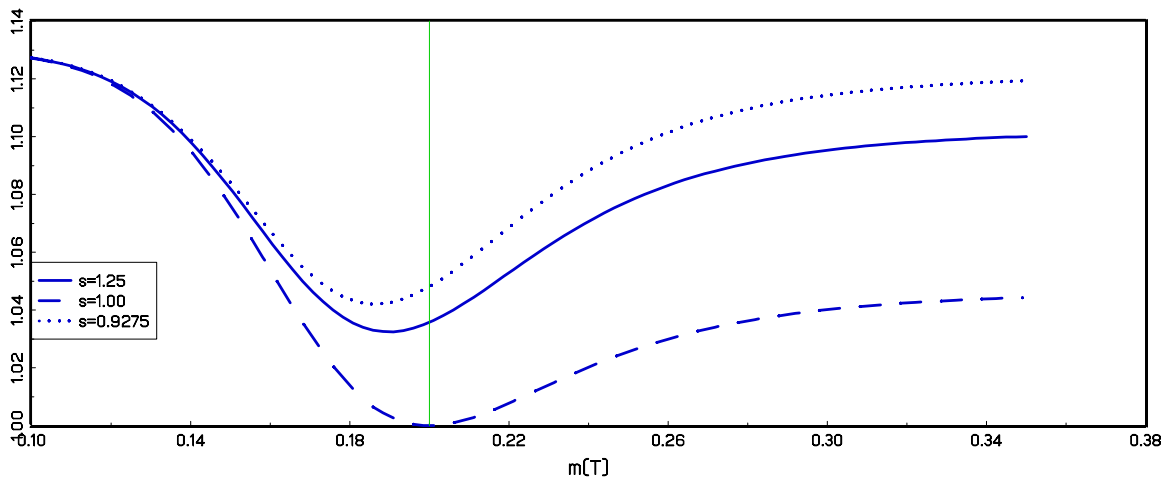
$$\begin{aligned} \Pi(p_{0,t+1}, m_{t+1}, s_t, s_{t+1}) \equiv & \quad (43) \\ & \frac{m_{t+1}}{c(p_{0,t+1}; m_{t+1})} \cdot \frac{1}{s_{t+1}} \cdot \left[ \omega_0 + \right. \\ & \left. s_t^{\varepsilon-1} \left( \frac{1 - \omega_0}{1 - (\omega_0 + \omega_1)} \right) ((1 - \omega_0) s_{t+1}^{\varepsilon-1} - \omega_1) \right]^{1/(\varepsilon-1)}. \end{aligned}$$

Figure 1: The final period policy problem

A. objective function



B. relative price distortion



C. markup

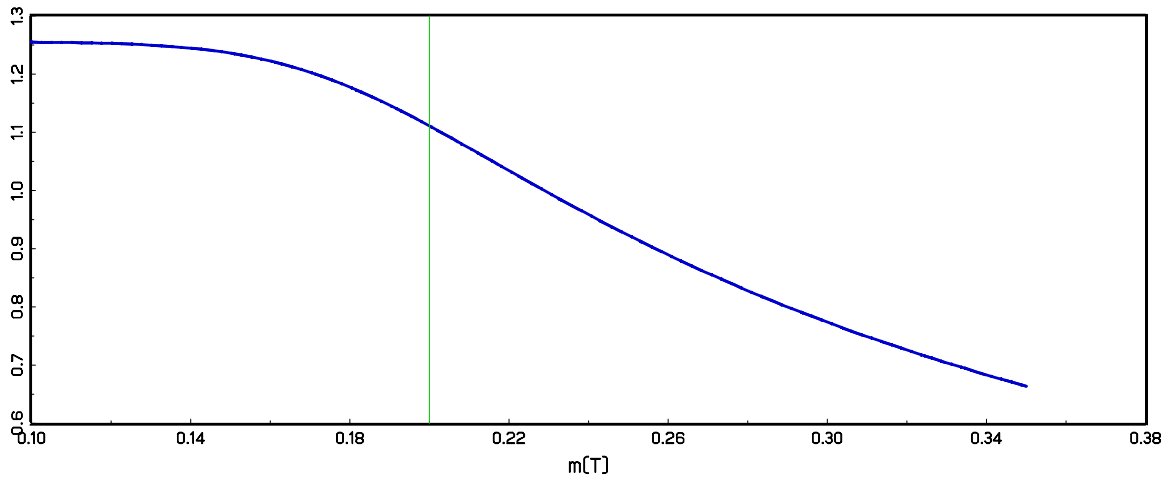
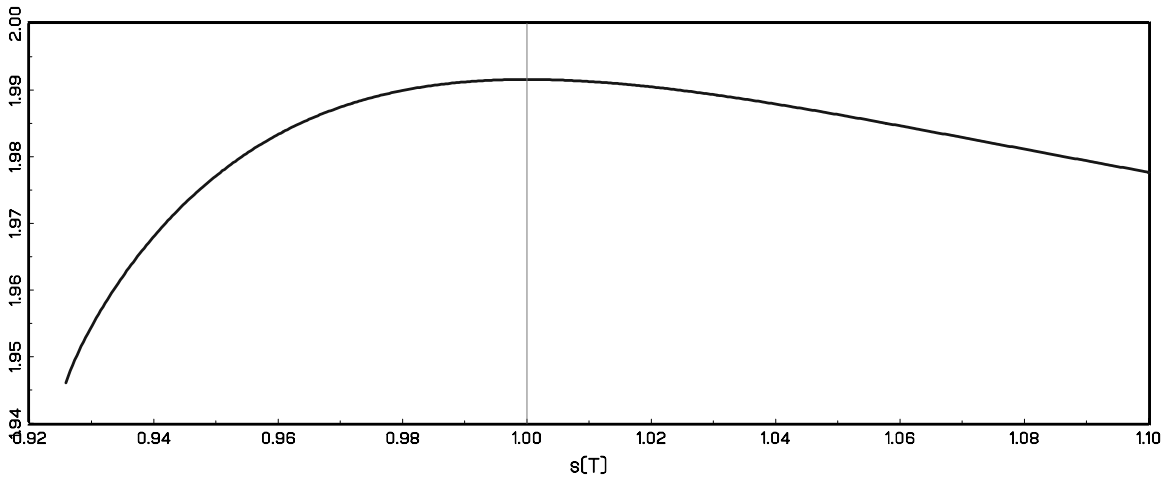
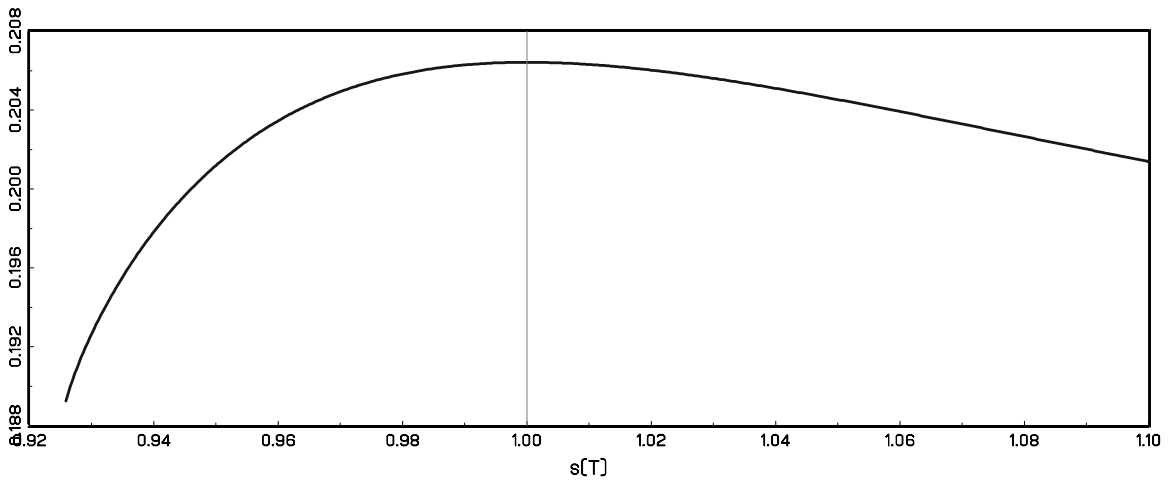


Figure 2: Optimal policy in the final period

A. value function



B. policy function



C. relative price distortion and markup

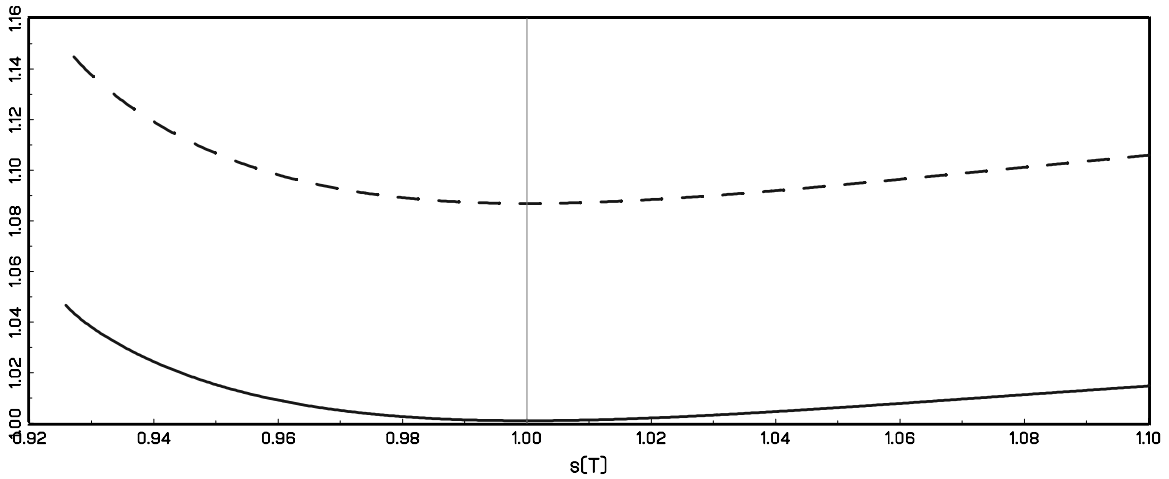
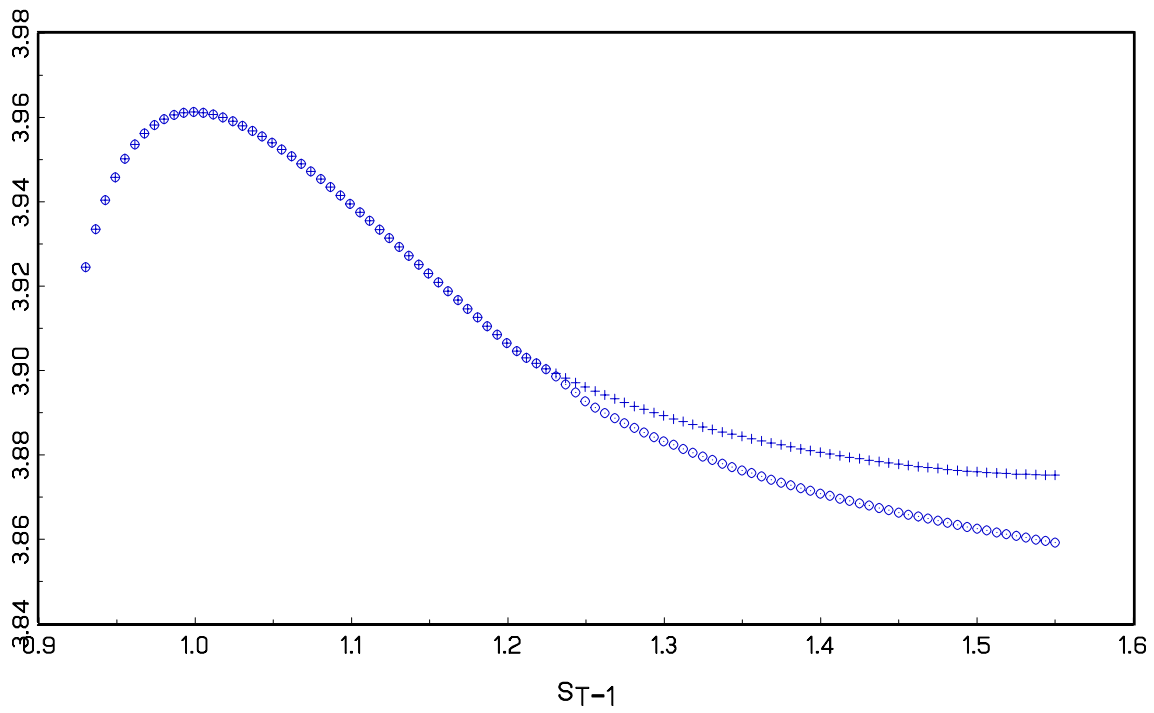


Figure 3. Optimal policy in period T-1  
uniform selection [o] Pareto selection [+]

A. Value function [ $v_{T-1}$ ]



B. Policy function [ $m_{T-1}$ ]

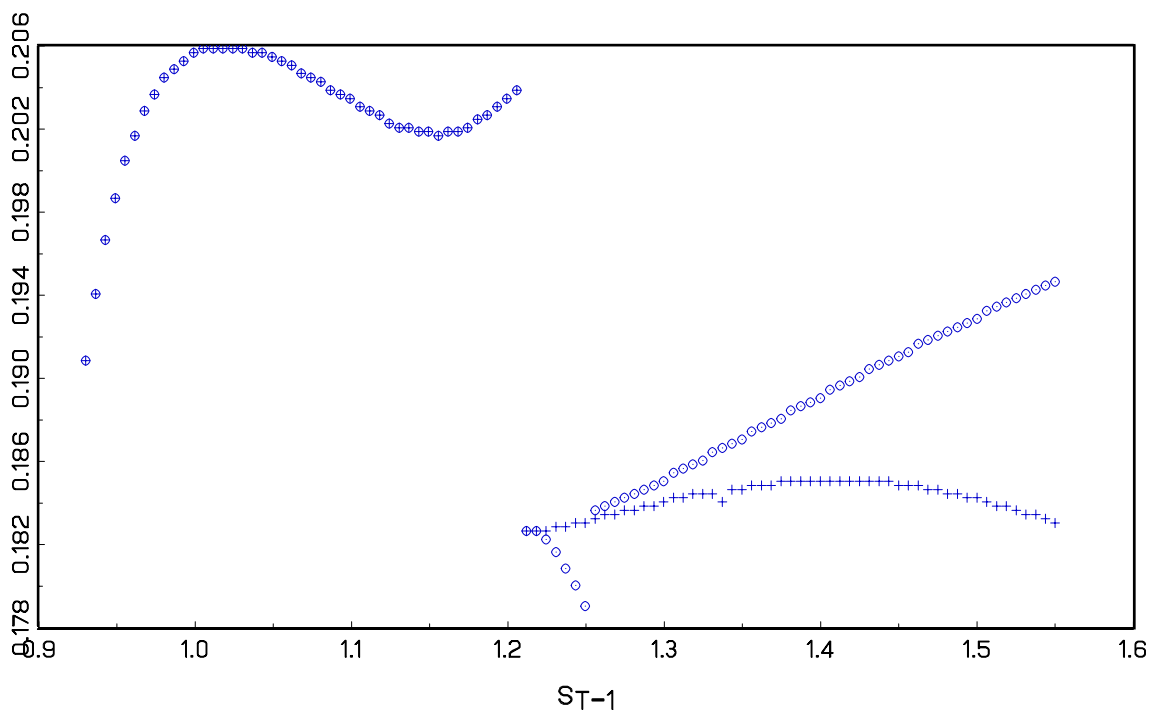
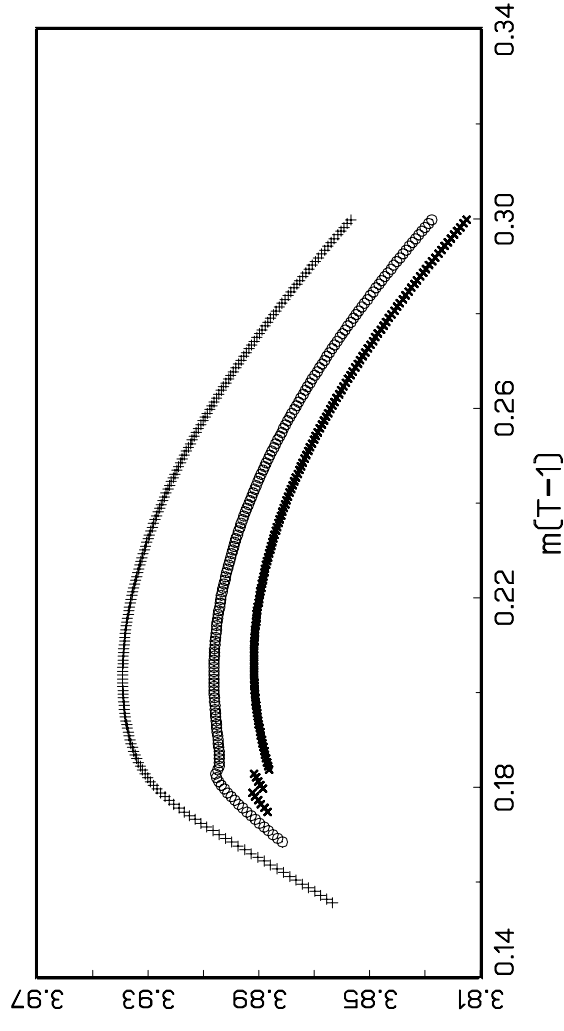


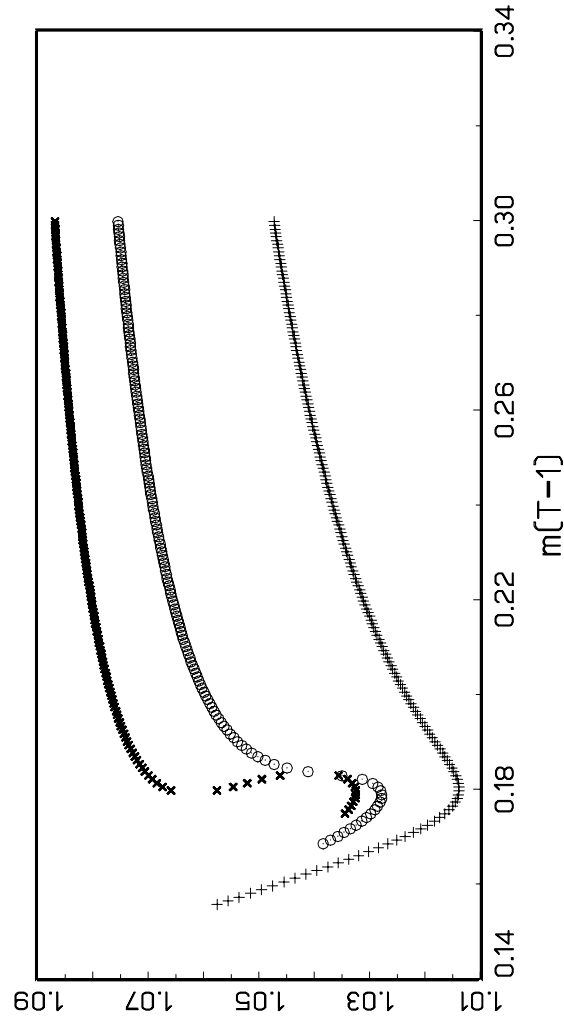
Figure 4.1: The Period T-1 policy problem for uniform selection

ST-1= 1.1[+] 1.2[o] 1.25[\*]

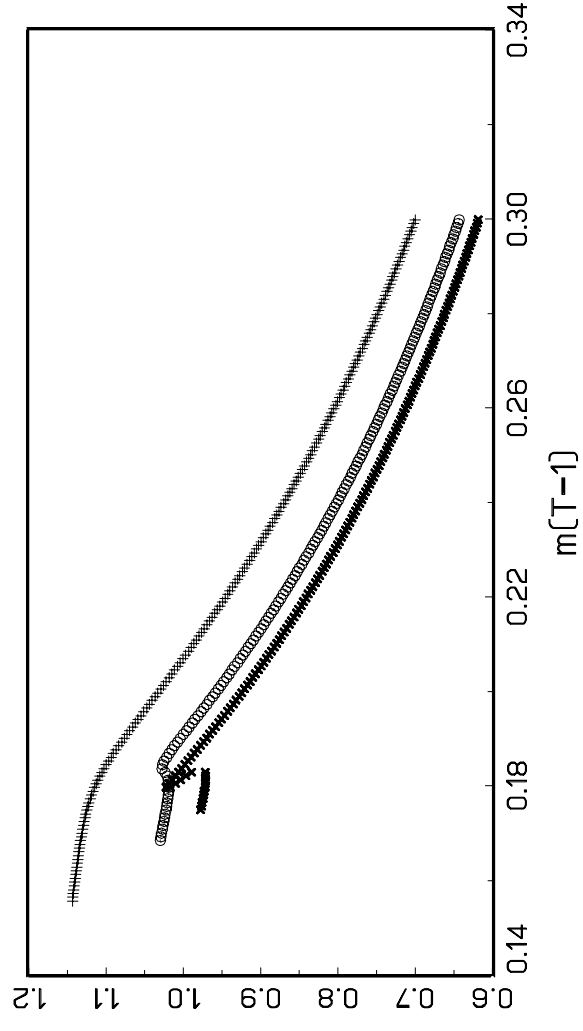
A. objective function



B. relative price distortion



C. markup



D. ST

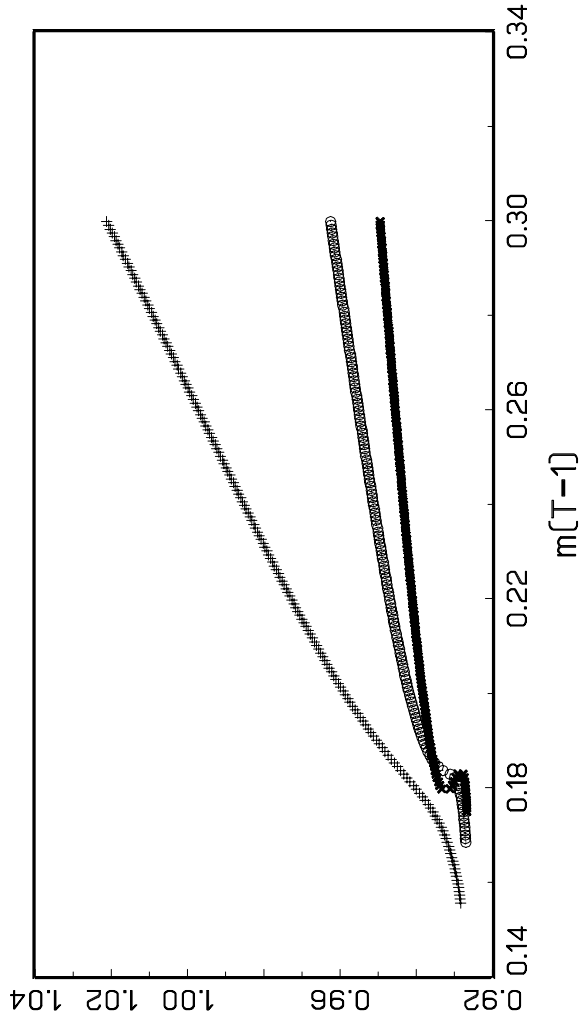
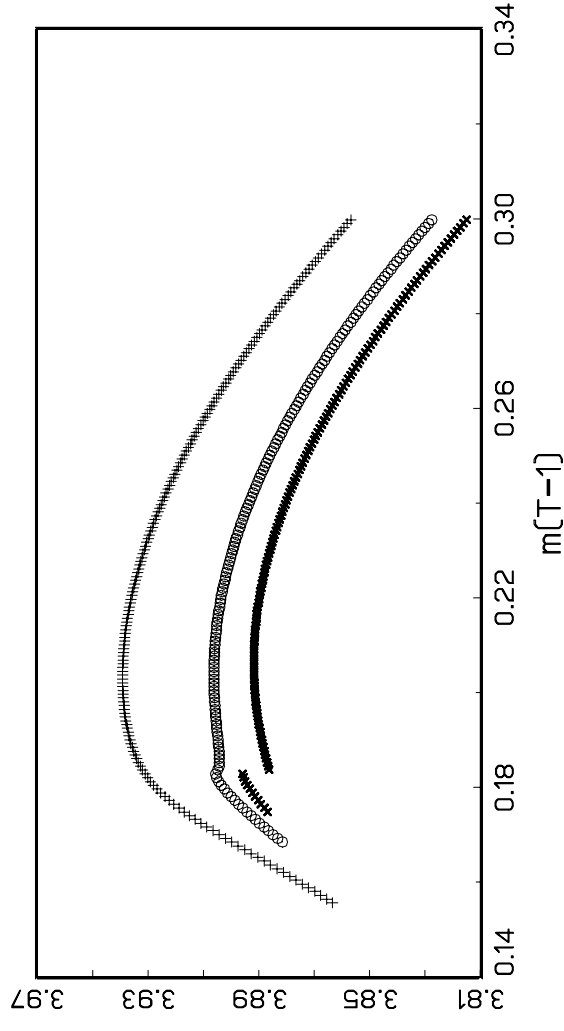


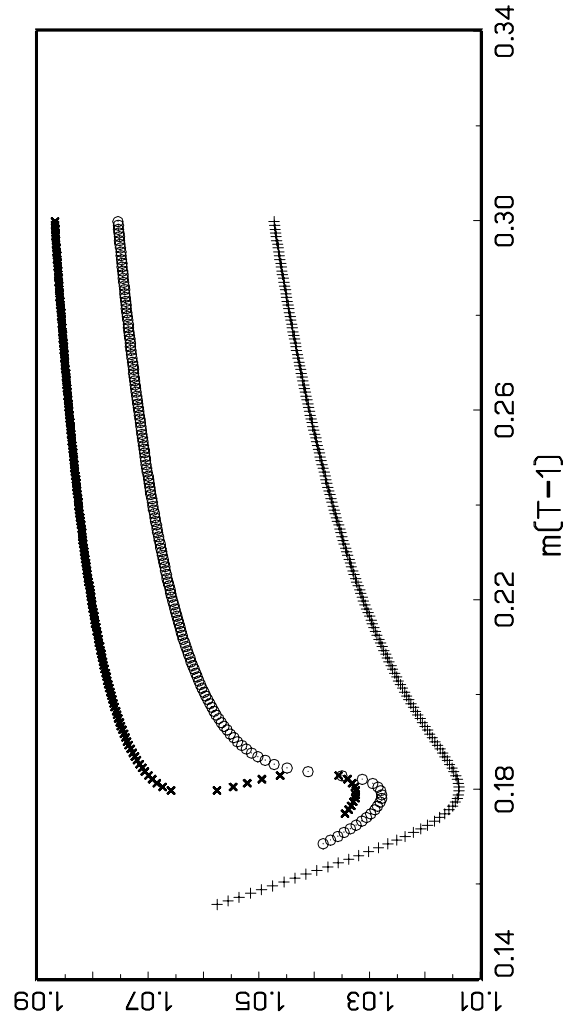
Figure 4.II: The Period T-1 policy problem for pareto selection

ST-1= 1.1[+] 1.2[o] 1.25[\*]

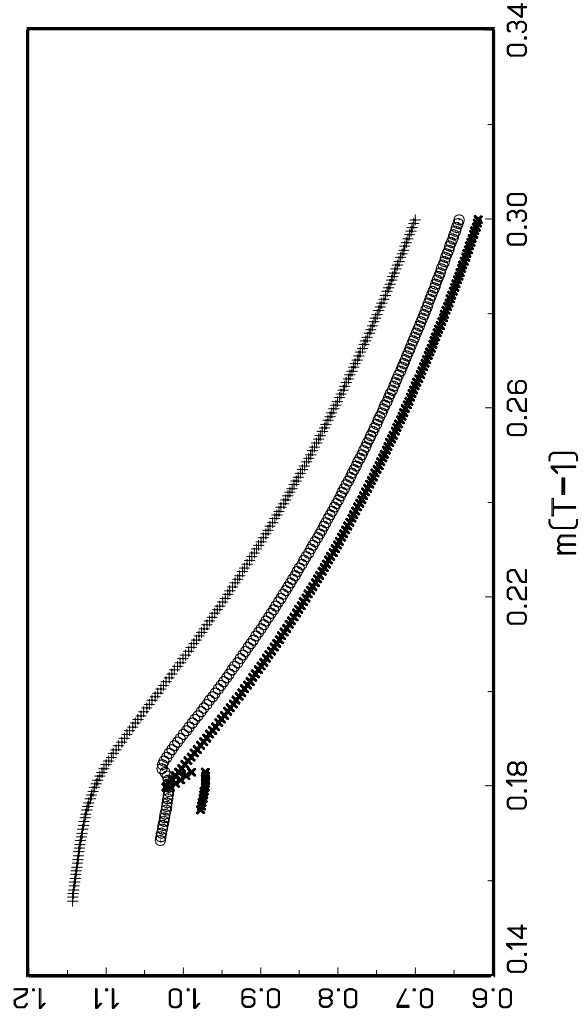
A. objective function



B. relative price distortion



C. markup



D. ST

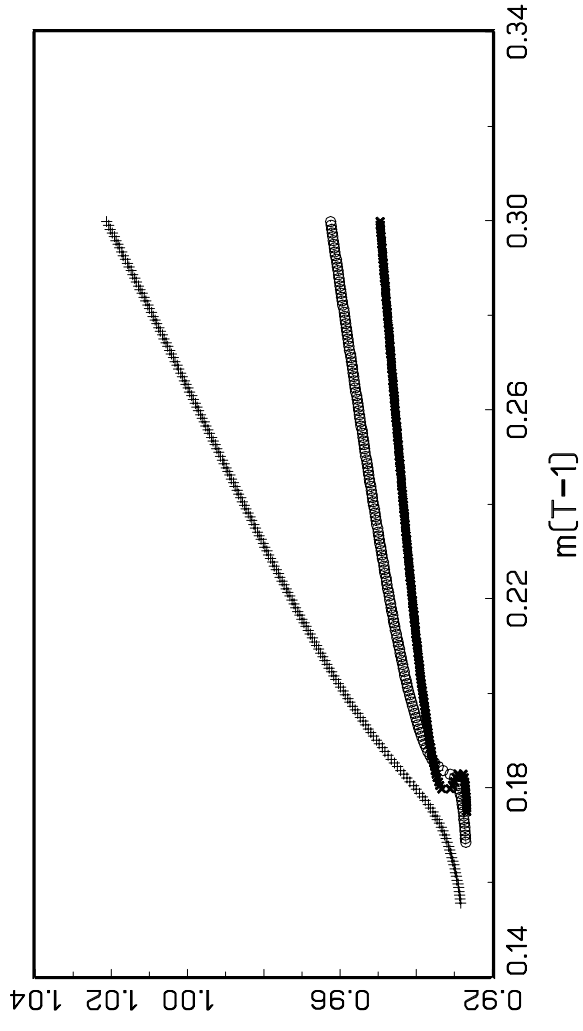
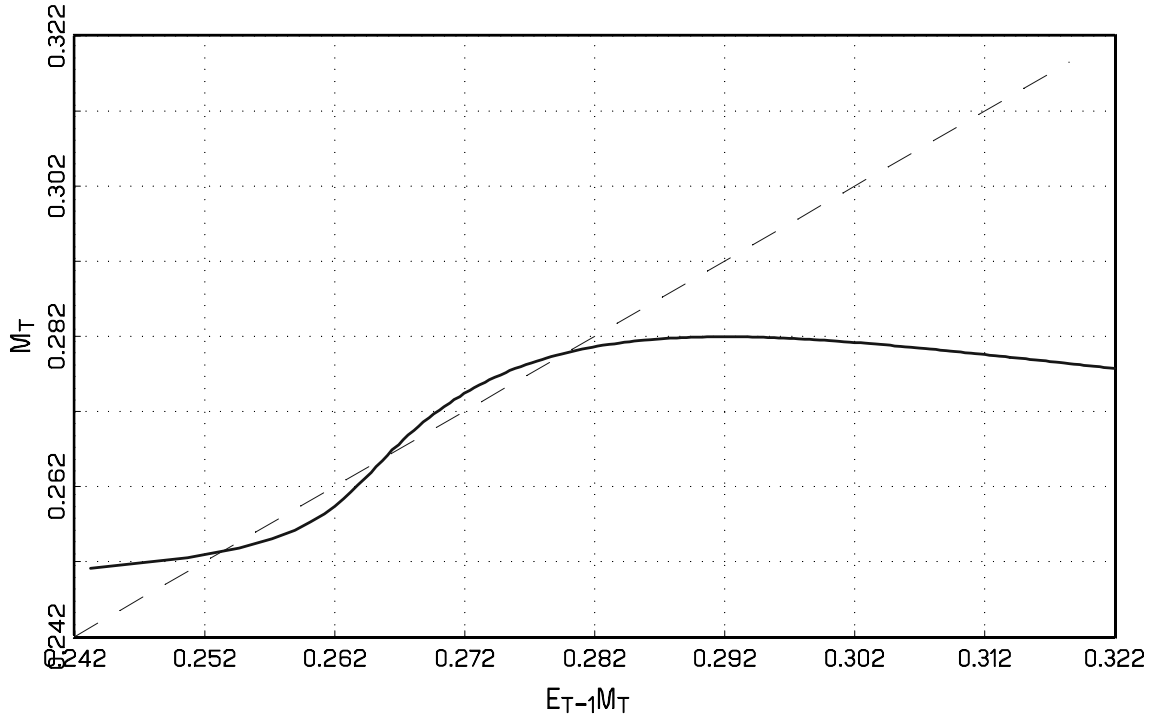


Figure 5. Equilibrium in T for  
 $s_{T-1}=1.300$  and  $m_{T-1}=0.180$

A. Optimal policy action vs. Expected policy action



B. Optimal Policy [ $M_T(s_T)$ ] and  
 Optimal Pricing [ $s_T(E_{T-1}M_T)$ ]

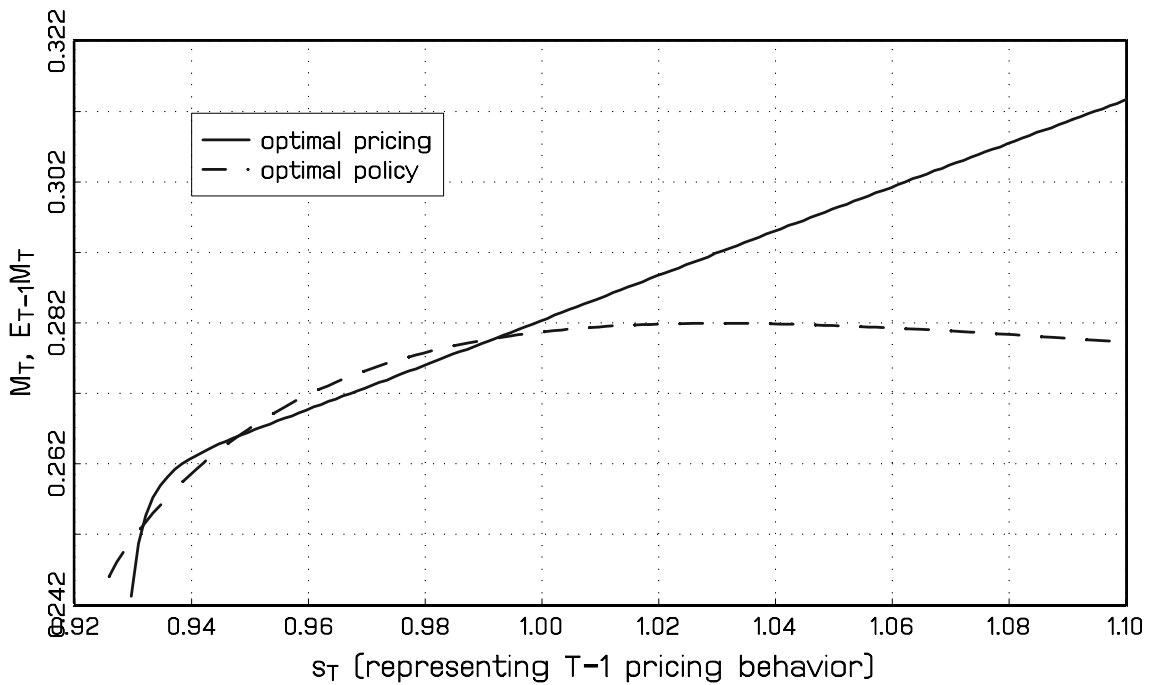
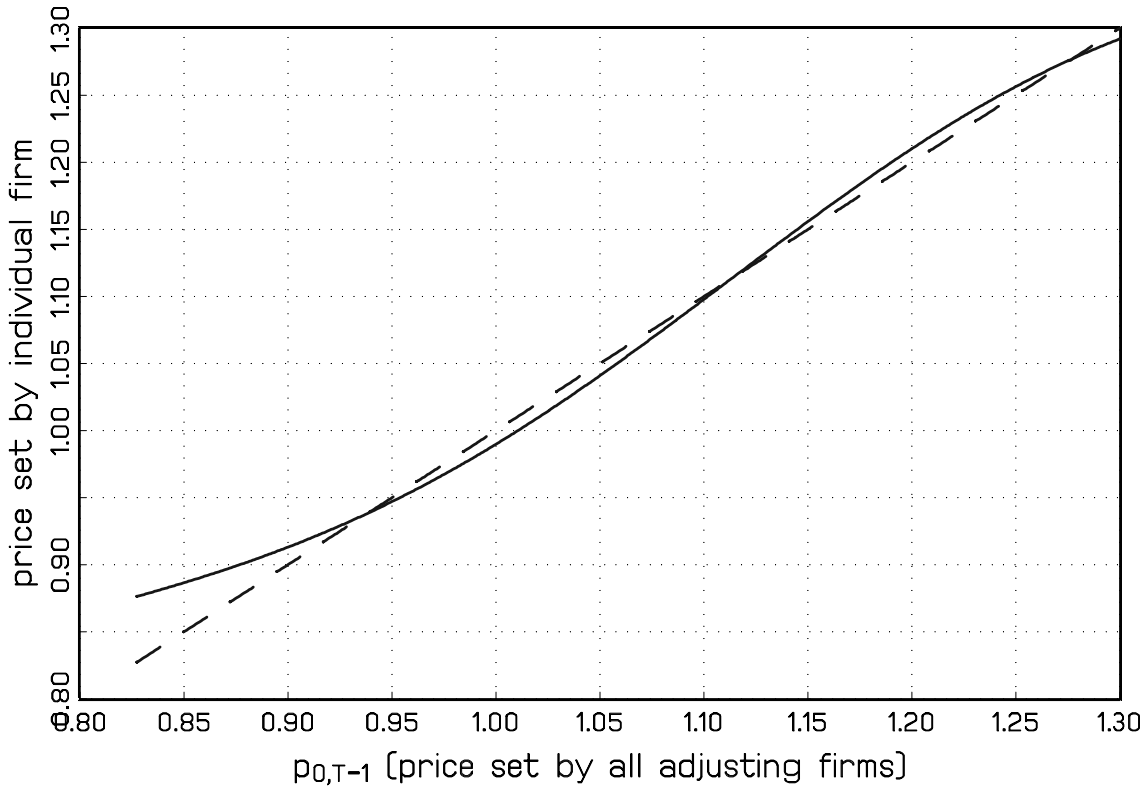


Figure 6. Multiple equilibria as fixed points of firm's best response function, for  $s_{T-1}=1.30$ ,  $m_{T-1}=0.18$

A. Individual  $p_{0,T-1}$  as function of aggregate  $p_{0,T-1}$



B.  $s_T(p_{0,T-1}|s_{T-1}=1.3)$  [equation 39]

