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Optimal Public Investment with and without Government Commitment*

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Abstract

We analyze the problem of optimal public investment when government purchases of productive capital assets are financed through income taxes. Virtually all previous work in this literature has prescribed a share of public investment in GDP that is both constant and time consistent. This paper shows that this straightforward prescription derives from specific assumptions relating to preferences and technology. In a more general framework, the optimal policy is neither constant nor time consistent. With full commitment, a policymaker will typically choose a tax rate, or alternatively a share of public investment, that increases over time. He does not exploit the first-period non-distortionary tax on capital but instead delays taxation in order to generate a "take-off" phase with higher consumption and higher private investment. We also show that allowing for discretion in the design of optimal policy does not necessarily result in higher long-run taxes relative to the commitment case. Therefore, the inability to commit to future policy can imply lower taxes and too little public investment in the long run. Finally, in contrast to previous work, the efficient share of public investment in GDP depends importantly on the intertemporal elasticity of substitution, capital depreciation rates, and the growth rates of productivity and population.

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1 Introduction

Over the post-war period, the U.S. share of public investment in GDP has hovered around 4 percent, a share that exceeds that of net exports. At its peak, public investment has represented as much as 6 percent of GDP following increases in military spending associated with the Korean war. The share of public investment was also substantially above average during the construction of the interstate highway system in the 1960s (see Figure 1). Interestingly, Fernald (1999) argues that this endeavor provided a significant one-time increase in productivity. Other forms of public capital that are considered to contribute to economic activity include the provision of water and sewer systems, hospitals, schools, airports, and even public sector R&D. It has been suggested that a one percent increase in public capital contributes as much as 0.39 percent to GDP (Aschauer [1989]). However, many have generally been skeptical of this large elasticity, and estimates ranging from 0.03 (Hulten and Schwab [1991]) to 0.2 (Ai and Cassou [1993], Lynde and Richmond [1993]) are more often cited.

The idea that public capital contributes to private production immediately raises a number of fundamental policy questions: What are the properties of the optimal public investment path? To what level does it converge to in the long run, and how does this level vary with the underlying economic environment? How does the economy inherited by a new government influence the optimal sequence of public investment? In choosing policy, will this government have an incentive to cheat on its promises? If so, what is the nature of the time inconsistency problem?

Barro (1990), Barro and Sala-i-Martin (1992), Glomm and Ravikumar (1994), Cassou and Lansing (1998), Turnovsky (1999), Eicher and Turnovsky (2000), as well as Aschauer (2000), are among the many articles that motivate a productive role for government. In these papers, government expenditures are financed via income taxes and contribute to the economy's productive capacity. The authors find that the welfare-maximizing income tax rate depends on factors such as the output elasticity of public capital, congestion externalities, and the degree of rivalry in public goods. Remarkably, all these articles prescribe an optimal income tax rate that is constant over time.

The present paper reconsiders the problem of optimal public investment when the government uses income taxes to finance its purchases of capital assets. In particular, we first explore the allocations that emerge under the Ramsey optimal plan. Our analysis shows that the time invariance of the optimal income tax rate found in the above articles derives from special assumptions that also make it time consistent. These assumptions involve either specific parameterizations of preferences and technology, or modeling public capital as a flow. Because optimal policy is always time consistent in these papers, the need to study optimal discretionary policy never arises.

In fact, as indicated in the seminal work of Kydland and Prescott (1977), the presence of state variables, namely the stocks of public and private capital, generally imply that optimal Ramsey tax rates are not time consistent. The implementation of Ramsey allocations, therefore, assume that the government is able to commit to future policy actions. Unlike previous literature, our analysis highlights the differences between this scenario and the more pragmatic framework that does not assume commitment. Following Quadrini, Krusell, and Rios-Rull (1997), we consider allocations associated with Markov tax rates.

We find that with full commitment, a benevolent policymaker generally chooses a low first-period tax rate relative to prevailing long-run tax rates. Unlike the model of Chamley (1986), the policymaker does not necessarily take advantage of the initial non-distortionary tax on capital. Instead, he may choose to delay taxation in order to generate higher consumption today as well as higher private investment.[1] This result hinges on the fact that higher levels of consumption in any period reduce the utility-denominated return to investment made in the preceding period through its effect on marginal utility. However, since no history exists prior to period zero, the policymaker can allow for high consumption in the initial period without concern for distortionary effects on past investment decisions. Thus, under full commitment, optimal policy suggests a "take-off" phase in which taxes and public investment are low, and both consumption and private investment are encouraged.

Along a balanced growth path, the optimal tax rate falls short of the elasticity of GDP with respect to public capital. In contrast, the existing literature on productive public spending frequently recommends a tax rate that exactly equals the public capital elasticity of output. This prescription appears in Barro (1990), Barro and Sala-i-Martin (1992), Turnovsky (1999) absent uncertainty, as well as Aschauer (2000) among others, but only maximizes steady state welfare in our framework. It is analogous, therefore, to a policy golden rule and emerges as the fully optimal solution only in situations where the economy lacks transition dynamics to the balanced growth path.[2] In general, the optimal steady state share of public investment in GDP depends importantly on the intertemporal elasticity of substitution, capital depreciation rates, as well as the rates of technical progress and population growth. Moreover, our model nests a special case where the equivalent of a modified policy golden rule is optimal. This rule suggests discounting the public capital elasticity of output by exactly the rate of time preference in the long run. While this solution does not hold generally, we show that the optimal tax rate nevertheless always falls below Barro's (1990) solution. In essence, although the simple policy golden rule leads to more public infrastructure relative to private capital, the impatience reflected in the rate of time preference means that it is not optimal to reduce current consumption through higher taxes to reach this higher ratio.

Without a commitment technology, that is when governments set policy in a discretionary fashion, optimal tax rates emerge to be surprisingly lower than those under full commitment in the long run. Therefore, the inability to abide by past promises eventually leads to too little public

[1] Chamley (1986) considers the problem of the optimal distribution of taxes across factors given an exogenous path for government expenditures. Our analysis instead focuses on the optimal choice of government investment given an exogenous income tax structure.

[2] As Turnovsky (1997) points out, most of the existing literature models productive contributions from government expenditures as a flow. To the degree that it is public infrastructure that increases productive capacity, this formulation, while easily tractable, is questionable in that it is the stock rather than the flow that matters.

investment. Under discretion, successive policymakers take as given the decision rule adopted by their successor. Each government then tends to "cheat" and to mimic the first period behavior of a Ramsey policymaker, which is to set a low tax rate. Thus, steady state discretionary tax rates may ultimately end up lower than those under the Ramsey plan. This type of long-run result also appears in Klein, Krusell and Rios-Rull (2003) but for a partially different reason. In their framework, "an uncommitted government does not take into account that today's taxes increase yesterday's incentives to work", and this effect helps induce a smaller government sector. We shut down this mechanism by fixing labor supply exogenously, and instead focus on the implications of taxation in a given period for consumption and investment in the previous period. In particular, we emphasize the importance of the first period's incentives under commitment in explaining time consistent tax rates.

This paper is organized as follows. Section 2 sets the basic theoretical framework and derives a policy golden rule for public capital. Section 3 addresses optimal policy from the standpoint of a benevolent government that can credibly commit to future policy actions. In section 4, we relax the assumption of full commitment and constrain optimal policy to be time consistent. Section 5 concludes and suggests some directions for future research.

2 Economic Environment

This section sets out the basic environment and derives the steady state welfare-maximizing policy which we treat as a benchmark. We begin with a closed economy where a large number of firms produce a single final good according to the technology,

$$Y_t = K_t^{\alpha}(z_t l_t N_t)^{1-\alpha}\left(\frac{K_{gt}}{N_t}\right)^{\theta}, \tag{1}$$

where $0 < \alpha < 1$, and $0 < \theta < 1 - \alpha$ is the public capital elasticity of output. The variables K_t and K_{gt} denote the stocks of private and public sector capital at date t respectively. The size of the population in the economy is given by N_t. Thus, note that a larger population size reduces the effectiveness of the public capital stock in production. Analogously to Barro and Sala-i-Martin (1992), this feature captures congestion effects in the use of government infrastructure such as highways, water systems, hospitals, etc... Aside from congestion, the technology in (1) is often used to motivate a role for government. This is the case in Baxter and King (1993), for instance, which focuses on the effects of shifts in exogenous policies.

Population grows at rate $\gamma_N > 1$ over time so that

$$N_t = \gamma_N N_{t-1},\ N_0 = 1. \tag{2}$$

The expression $z_t l_t$ in (1) represents the quantity of skill-weighted labor input, where labor-augmenting technical progress allows for increases in z_t at rate $\gamma_Z > 1$,

$$z_t = \gamma_Z z_{t-1},\ z_0 = 1. \tag{3}$$

We treat the per capita stock of public capital as being available to all firms. In other words, we treat K_{gt}/N_t is interpreted as a public good that acts as a common externality with respect to each firm's production.

Public investment, I_{gt}, is financed by a flat tax on income, $0 < \tau_t < 1$, that can vary with time. Hence, $I_{gt} = \tau_t Y_t$ and we can express new outlays of public capital, K_{gt+1}, as

$$\begin{aligned} K_{gt+1} &= \underbrace{\tau_t Y_t}_{\text{Public Investment}} + (1-\delta)K_{gt}, \\ K_{g0} &> 0 \text{ given.} \end{aligned} \tag{4}$$

where $0 < \delta < 1$. Observe that τ_t also corresponds to the share of gross public investment in GDP, I_{gt}/Y_t. As in Klein et. al. (2003), we abstract from sovereign debt considerations. Given the difficulty of imposing a no-Ponzi-game condition on successive governments in the analysis of time consistent policies, the inclusion of debt would require introducing an alternative constraint. While potentially interesting, we leave this consideration to future research.

2.1 Firms

We denote by r_t and W_t respectively the rental price of private capital and the wage at date t. Taking as given the sequence, $\{r_t, W_t\}_{t=0}^{\infty}$, each firm maximizes profits and solves

$$\max_{K_t,\, l_t N_t} \Pi_t = K_t^{\alpha}(z_t l_t N_t)^{1-\alpha}\left(\frac{K_{gt}}{N_t}\right)^{\theta} - r_t K_t - W_t l_t N_t. \tag{5}$$

The corresponding first-order conditions help determine r_t and W_t.

2.2 Households

The economy is inhabited by a large number of identical households that comprise one or more working members of the current generation. Family size is assumed to increase at the rate of population growth. In making plans, households take account of their descendants, and we summarize this intergenerational linkage by envisioning that each generation maximizes utility subject to a budget constraint over an infinite horizon.[3] If C_t represents total consumption at t, then C_t/N_t is consumption per household member. Assuming that preferences are of the Constant Relative Risk Aversion type, each household maximizes

$$\begin{aligned} \mathcal{U} &= \sum_{t=0}^{\infty} \beta^t \gamma_N^t u(C_t/N_t), \\ \text{where } u(C_t/N_t) &= \frac{(C_t/N_t)^{1-\sigma}}{1-\sigma} \text{ when } \sigma \neq 1, \\ u(C_t/N_t) &= \ln(C_t/N_t) \text{ when } \sigma = 1, \end{aligned} \tag{6}$$

[3] See Barro (1974).

and $0 < \beta < 1$ is the subjective discount rate. At each date, households decide how much to consume and save as well as how much capital to rent to firms.

Each household is assumed to be endowed with one unit of time, $l_t = 1$, which they supply inelastically. The representative household's budget constraint is given by

$$C_t + I_t = (1 - \tau_t)\left[W_t N_t l_t + r_t K_t\right], \tag{7}$$

where

$$\begin{aligned} K_{t+1} &= I_t + (1-\delta)K_t, \\ K_0 &> 0 \text{ given.} \end{aligned} \tag{8}$$

Before proceeding with the design of optimal policy, we find it helpful to derive the economy's constant balanced growth rate in the steady state. As shown in King, Plosser, Rebelo (1988), this will eventually allow us to express preferences and technology in terms of transformed variables that are constant in the steady state.

2.3 Balanced Growth

In the following discussion, we denote the long-run growth rate of a given variable X_t by γ_X. Equation (4) and (8) imply $\gamma_{K_g} = \gamma_{I_g}$ and $\gamma_K = \gamma_I$ respectively. It follows from (7) that $\gamma_C = \gamma_I = \gamma_{I_g} = \gamma_Y$. Hence, all variables in the economy eventually grow at the common growth rate γ_Y. From the technology in (1), it immediately transpires that $\gamma_Y = \gamma_Z^{\frac{1-\alpha}{1-\alpha-\theta}}\gamma_N$, so that output growth reflects both labor-augmenting technical progress and population growth. In per capita terms, long-run growth is simply denoted by

$$\gamma = \gamma_Z^{\frac{1-\alpha}{1-\alpha-\theta}}. \tag{9}$$

Given that GDP grows at rate $\gamma\gamma_N$, a sensible normalization for our economy is one which expresses our model's variables in detrended per capita form, which we write below as lower case letters. Thus, we write the detrended per capita counterpart of any variable X_t that grows at rate $\gamma\gamma_N$ in the long-run as $x_t = X_t/(N_t\gamma^t)$. Furthermore, note that this transformed variable will then be constant in the steady state.

2.4 The Transformed Economy: Description of a Policy Golden Rule

We now derive the steady state welfare-maximizing policy for this economy and relate it to earlier work on optimal fiscal policy. Taking the sequence of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, and the sequence of tax rates, $\{\tau_t\}_{t=0}^{\infty}$, as given, the household's problem may be expressed as

$$\max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \mathcal{U} = \sum_{t=0}^{\infty} \underbrace{\left(\beta\gamma_N\gamma^{(1-\sigma)}\right)^t}_{\beta^*} \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{P1}$$

subject to

$$
\begin{aligned}
c_t + \gamma_N \gamma k_{t+1} - (1-\delta)k_t &= (1-\tau_t)\left[w_t l_t + r_t k_t\right] \qquad (10)\\
k_0 &> 0 \text{ given,}
\end{aligned}
$$

where $r_t = \alpha(y_t/k_t)$, $w_t = (1-\alpha)(y_t/l_t)$, and $y_t = k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta}$. The normalized discount rate, $\beta^* = \beta\gamma_N\gamma^{(1-\sigma)}$, reflects the engines of economic growth in the usual fashion. In order for (P1) to be well defined, we assume $\gamma_N\gamma^{(1-\sigma)} < 1/\beta$. This imposes an upper bound on the extent of technical progress and population growth.

The solution to the household's dynamic optimization problem yields the familiar Euler equation,

$$\gamma_N \gamma c_t^{-\sigma} = \beta^* c_{t+1}^{-\sigma}\left[(1-\tau_{t+1})r_{t+1} + 1 - \delta\right], \qquad (11)$$

that describes decentralized allocations over time. In (11), taxes distort private incentives to consume and save, but also induce higher future returns to private investment through the development of public infrastructure. Specifically, the return to private investment, r_{t+1}, depends on k_{gt+1}^{θ}. A question then immediately arises as to where to set the tax rate, or equivalently, the share of public investment in output.

Perhaps the simplest or most common answer to this question, that is the simplest notion of optimal policy, is to set the tax rate so as to maximize steady state welfare. In our framework, this concept of optimal policy replicates Barro's (1990) optimal tax rate and, furthermore, is analogous to a golden rule for public capital.

From equation (11), and the accumulation equation for public capital in detrended form,

$$\gamma_N \gamma k_{gt+1} = \tau_t y_t + (1-\delta)k_{gt}, \qquad (12)$$

the steady state ratio of public to private capital satisfies

$$\frac{k_g}{k} = \frac{\left[\left(\frac{\gamma_N\gamma}{\beta^*}\right) - (1-\delta)\right]\tau}{\alpha\left[\gamma_N\gamma - (1-\delta)\right](1-\tau)}. \qquad (13)$$

More specifically, the higher the tax rate, the lower the after-tax return to private capital and the more government investment takes place. Therefore, the ratio of public infrastructure to private capital rises with τ. Given equation (13), one can easily show that steady state welfare increases monotonically with k, and is otherwise independent of τ. It follows that maximizing steady-state welfare with respect to τ reduces to finding the tax rate that maximizes steady state private capital. As in conventional wisdom, this maximization problem prescribes setting $\tau = \theta$.

The starkness of this standard finding stems from the failure to take into account transition dynamics. In much of the literature on productive government, the outcome $\tau = \theta$ appears as the full welfare maximizing solution precisely because these models lack any dynamics to the balanced growth path. This is the case, for instance, in many of the endogenous growth models inspired by Barro (1990). Given that α lies somewhere between 0.3 and 0.45 in U.S. data, and that $\alpha + \theta = 1$

is required to generate endogenous growth, those models place the optimal share of government investment in GDP between 0.55 and 0.7, which appears excessively large when compared to observed values. To make sense of the optimal solution in that context, one must then argue that the concept of private capital is to be interpreted more broadly to include a human component. This would effectively push α closer to unity and, therefore, reduce θ. However, in departing from the National Income and Product Accounts (NIPA) definition of private capital, we are invariably forced towards a concept that is less tangible and not easily measurable.

Of course, in our economy, long-run growth stems from exogenous advances in labor augmenting technical progress and setting $\tau = \theta$ does not have to be implausible. However, most estimates of the output elasticity of public capital place θ between 0.05 and 0.2 in the U.S., but we also saw earlier in Figure 1 that τ has moved only between 0.03 and 0.06 over the post war period. What then is the nature of the relationship between τ and θ at the optimum? As we now show, the fully optimal steady state share of public investment in GDP depends importantly on both preferences and technology in ways that tend to lower it relative to the public capital elasticity of output.

3 The Ramsey Problem

This section discusses the full welfare-maximizing policy problem from the standpoint of a government that can commit to a sequence of future tax rates and its implied path for public investment. In addition, it also lays out the nature of the time inconsistency problem associated with the optimal path for tax rates.

Consider a benevolent government that, at date zero, is concerned with choosing a sequence of tax rates consistent with the development of public infrastructure that maximizes household welfare. In choosing policy, this government takes as given the decentralized behavior of firms and households. We further assume that at date zero, it can credibly commit to any sequence of policy actions. The problem faced by this government would then be to maximize (P1) subject to equations (10), (11), and (12). The corresponding Lagrangian can be written as

$$\begin{aligned}\max_{\{c_t,\tau_t,k_{t+1},k_{gt+1}\}_{t=0}^{\infty}} \mathcal{L}_t = & \sum_{t=0}^{\infty} \beta^{*t} \frac{c_t^{1-\sigma}}{1-\sigma} \\ & + \sum_{t=0}^{\infty} \beta^{*t} \mu_{1t} \left\{ \beta^* c_{t+1}^{-\sigma} \left[(1-\tau_{t+1}) r_{t+1} + 1 - \delta \right] - \gamma_N \gamma c_t^{-\sigma} \right\} \\ & + \sum_{t=0}^{\infty} \beta^{*t} \mu_{2t} \left\{ \tau_t y_t + (1-\delta) k_{gt} - \gamma_N \gamma k_{gt+1} \right\} \\ & + \sum_{t=0}^{\infty} \beta^{*t} \mu_{3t} \left\{ (1-\tau_t) y_t + (1-\delta) k_t - \gamma_N \gamma k_{t+1} - c_t \right\}. \end{aligned} \tag{P2}$$

The first constraint in (P2) implies that our benevolent planner takes households' consumption-savings behavior as given. He can, however, influence the intertemporal allocations they choose by altering tax policy over time.

The optimal selection of τ_t is governed by the following two equations,

$$\mu_{20} - \mu_{30} = 0 \text{ at } t = 0, \tag{14}$$

and

$$-\mu_{1t-1} c_t^{-\sigma} r_t + \mu_{2t} k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta} - \mu_{3t} k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta} = 0 \; \forall t > 0. \tag{15}$$

The fact that these first-order conditions differ at $t = 0$ and $t > 0$ suggests an incentive to take advantage of initial conditions in the first period with the promise never to do so in the future. It is exactly in this sense that the optimal policy may not be time consistent; once date zero has passed, a planner at date $t > 0$ who re-optimizes would want to start with a tax rate, τ_t, that differs from what was chosen for that date at time zero. For now, therefore, we imagine that the optimization takes place only once, in period zero. Once our benevolent planner has decided on a course of actions, his hands are tied and he is pre-committed to that course of actions.

As noted in early related work by Kydland and Prescott (1980), the choice of τ_t introduces a lagged predetermined variable, $\mu_{1t-1} \geq 0$ in (15). At a purely mechanical level, we can think of this variable as an artificial device that helps make the final system of difference equations that characterizes the optimal solution stationary $\forall t \geq 0$. However, unlike fundamental state variables such as the private or the public capital stock, the appropriate initial condition for μ_{1t-1} is not arbitrary. Instead, since the optimal choice of τ_0 must satisfy equation (14), it must also be the case that $\mu_{1t-1} = 0$ in (15) at $t = 0$. Alternatively, Dennis (2001) points out that the lagged Lagrange multiplier, μ_{1t-1}, may be interpreted as the "current value of promises not to exploit the initial state" and, in particular, to abide by past commitments. However, since no history exists prior to period zero, there are no past commitments on which to assign any value at that date. It is optimal, therefore, to set $\mu_{1,-1} = 0$.

The fact that the optimal policy is chosen once and for all in period zero does not necessarily imply that it is not flexible. On the contrary, the solution to the Ramsey problem provides a description of where to set τ_t in every state of the world. We shall see that this solution is also explicit about how the implied share of public investment in output depends on the economic environment. Therefore, as noted in Dennis (2001), "if a change to one or more parameters takes place, the policy rule automatically reflects this change; (and) there is no need for re-optimization to take place."

3.1 The Ramsey Steady State: Optimal Public Investment and the Modified Policy Golden Rule

The optimal stationary equilibrium is given by a vector $\{c, y, \tau, k, k_g, \mu_1, \mu_2, \mu_3\}$ that solves the four first-order conditions associated with problem (P2), the resource constraint (10), the Euler equation (11), the equation describing the evolution of public capital (12), and the definition of output, $y_t = k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta}$, all without time subscripts. The optimal stationary equilibrium, therefore, simply reduces to a system of eight equations in eight unknowns.

In the long run, households' optimal consumption-savings decisions satisfy

$$\gamma_N \gamma - \beta^* \left[(1-\tau)\alpha \left(\frac{y}{k} \right) + 1 - \delta \right] = 0, \tag{16}$$

from the Euler equation. Furthermore, it is also straightforward to show that the optimal ratio of public capital to output must be such that

$$\gamma_N \gamma - \beta^* \left[\theta \left(\frac{y}{k_g} \right) + 1 - \delta \right] = 0. \tag{17}$$

It follows from equations (16) and (17) that in the steady state, the Ramsey solution equates the after tax return to private investment, $(1-\tau)\alpha(y/k)$, with the marginal return to public investment, $\theta(y/k_g)$. Consequently, we can immediately pin down the ratio of public to private capital as $k_g/k = \theta/\alpha(1-\tau)$. As expected, the higher the tax rate, the lower the after-tax return to private capital and the less private capital is generated relative to public infrastructure.

The idea that, at the optimum, the after-tax return to private investment must equal the marginal return to public investment also helps determine the optimal tax rate and implied share of government investment in output. Note that in this setting, the opportunity cost of 1 unit of resources invested in the public sector is the after tax return this unit would have otherwise earned in the private sector, $(1-\tau)\alpha\,(y/k)$. Equation (16) then tells us that

$$\underbrace{(1-\tau)\alpha \left(\frac{y}{k} \right)}_{\text{return to private investment}} = \frac{\gamma_N \gamma}{\beta^*} - (1-\delta). \tag{18}$$

The marginal benefit of investing 1 unit of resources in the public sector is $\theta(y/k_g)$ and, since public capital accumulates according to the law of motion $k_g[\gamma_N \gamma - (1-\delta)] = \tau y$ in the long run, we have

$$\underbrace{\theta \left(\frac{y}{k_g} \right)}_{\text{return to public investment}} = \theta \left(\frac{\gamma_N \gamma - (1-\delta)}{\tau} \right). \tag{19}$$

Therefore, equating marginal cost and marginal benefit [i.e. the right-hand side of equations (18) and (19)] directly yields the optimal long-run tax rate and share of public investment,

$$\tau = \theta \left\{ \frac{\gamma_N \gamma - (1-\delta)}{\frac{\gamma_N \gamma}{\beta^*} - (1-\delta)} \right\} < \theta. \tag{20}$$

Analogously to the modified golden rule for private capital in the one sector growth model, the share of government investment in output given by (20) falls short of the policy golden rule outlined in the previous section, θ, by an amount that depends importantly on discounting. In fact, observe that when $\delta = 1$ in equation (20), τ is $\beta^* \theta < \theta$ which corresponds exactly to a modified policy golden rule.[4] In addition, from equation (13), it would then be the case that $k_g/k = \theta/\left[\alpha(1-\beta^*\theta)\right]$ which is less than the ratio of public to private capital implied by the policy golden rule, $k_g/k = \theta/\left[\alpha\beta^*(1-\theta)\right]$. In other words, although the policy golden rule eventually leads to more public infrastructure relative to private capital, the impatience reflected in the rate of time preference means that it is not optimal to reduce current consumption through higher taxes to reach this higher ratio.

Relative to most earlier work, equation (20) matters in at least two respects. First, it implies that a high output elasticity of public capital, θ, does not necessarily have to translate into a large share of public investment in output, i_g/y. Recall that in the U.S., estimates of θ generally range up to 0.2 while i_g/y has only hovered around 0.05 since World War II. Second, the optimal share of public investment in GDP now depends on a variety of preference and technology considerations including labor productivity growth, population growth, rates of depreciation, and the coefficient of intertemporal substitution.

Figure 2), panel a), depicts the effects of a rise in labor productivity growth on the optimal ratio of public investment to output. We can see that, at the optimum, an increase in the rate of technical progress from γ to γ' raises both the marginal benefit of public investment, $\theta\left(\frac{\gamma_N\gamma-(1-\delta)}{\tau}\right)$, and its marginal cost, $\frac{\gamma^\sigma}{\beta}-(1-\delta)$ from equation (18). Intuitively, the equilibrium return to private capital rises because it is now more costly to increase future detrended capital; households then save less which reduces the capital output ratio and, therefore, raises the rate of interest. Because this increase depends on the size of σ in Figure 2a), it is not clear whether the initial optimal rate of public investment, τ, should increase to τ', or instead decrease to τ''. For larger values of σ, however, we expect the optimal tax rate to fall with increases in per capita growth.[5]

On a less equivocal note, from equation (20), the impact of a rise in the population growth rate on the steady-state Ramsey tax rate is unambiguously positive,

$$\frac{\partial\tau}{\partial\gamma_N} = \theta\left\{\frac{\gamma}{\frac{\gamma^\sigma}{\beta}-(1-\delta)}\right\} > 0. \tag{21}$$

Observe in Figure 2b) that an increase in population growth from γ_N to γ_N' leaves the return to private investment unchanged. That is, an increase in γ_N raises GDP growth, $\gamma_N\gamma$, but also raises the adjusted discount rate, $\beta^* = \beta\gamma_N\gamma^{1-\sigma}$, since households take account of the greater number of individuals in the future. At the optimum, therefore, the return to public capital must also remain unaffected. From equation (19), it is clear that a rise in γ_N can only leave the return to public capital unaffected if the tax rate also rises. Intuitively, higher population growth erodes the ratio

[4] This nested case appears in Glomm and Ravikummar (1994). The additional assumption of logarithmic preferences in that paper further implies that this solution holds optimally at every date. We explain below why this constant policy turns out to be time consistent in their case.

[5] In the case where $\delta = 1$, $\tau = \beta^*\theta$ so that $\partial\tau/\partial\gamma = (1-\sigma)\beta\gamma_N\gamma^{-\sigma}\theta \lesseqgtr 0 \Leftrightarrow 1/\sigma \lesseqgtr 0$. In other words, when households are relatively less willing to substitute consumption across time, (i.e. $1/\sigma < 1$), an increase in economic growth makes them want to raise present consumption so that the share of public investment must fall.

of public capital to output. Hence, more government investment relative to GDP is now necessary to restore this ratio to its original level.

Figure 3, panel a) plots average per capita growth rates against the flow of government capital expenditures relative to GDP across 106 countries over the period 1976-1997. The data is obtained from the World Development Indicators published by the World Bank in 2000. Capital expenditures in this case include spending on fixed capital assets, land, intangible assets, government stocks, as well as non-military, non-financial assets. The data in Figure 3a) show that the share of capital expenditures in GDP tends to fall with per capita economic growth. This decreasing relationship becomes even more pronounced when the share of public investment is plotted against the level of total factor productivity as in Figure 3, panel b).[6] We shall see that in calibrated versions of our model, the optimal policy indeed implies a declining relationship between the share of public investment in GDP and per capita economic growth. Interestingly, as implied by Figure 2b), Figure 3, panel c), illustrates a positive link between public investment relative to GDP and population growth across nations.

At this point, we find it useful to introduce a numerical example to better highlight the behavior of key economic variables as the rate of technical progress and the population growth rate vary. Furthermore, since the dynamics of optimal policy cannot be characterized analytically, we shall also use this example below in analyzing the optimal sequence of tax rates given initial conditions for the state variables.

3.2 Calibration to U.S. Benchmarks and Steady State Simulations

The U.S. economy has grown at an average annual rate of 2 percent in per capita real terms over the post war period, and we choose γ to match this value in the steady state. We set γ_N to 1.012 to reflect mean population growth since World War II. From the capital accumulation equation, we have that $\gamma\gamma_N = i/k + 1 - \delta$. Given that $\gamma = 1.02$ and that $\gamma_N = 1.012$, we follow Cooley and Prescott (1995) and choose $\delta = 0.044$ to match a value of 0.076 for the ratio of investment to private capital. As in Baxter and King (1993), we set the share of private capital in GDP, α, to 0.42 and choose β to generate an equilibrium real rate of 6.50 percent. This implies $\beta = 0.982$. We set $\sigma = 2$ so as to make the coefficient of intertemporal substitution 1/2. Finally, given values of γ, γ_N, δ, β, and σ, we use equation (20) to make our choice of θ consistent with a public investment share in GDP, τ, of 5 percent. This value of $\tau = i_g/y$, which is taken as a benchmark in Baxter and King (1993), also approximates the average share of public investment in Figure 1) and requires that we set $\theta = 0.068$. Therefore, if the share of public investment is roughly optimal in the U.S., then the public capital elasticity of output implied by the theory lies in the lower range of most empirical estimates.

In principle, the observed U.S. ratio of public investment to GDP does not have to be optimal in a Ramsey sense. Among other considerations, whether government behavior can be approximately

[6] Cross-country data on total factor productivity levels is obtained from Hall and Jones (1998) for the year 1988.

captured by household welfare maximization is still an open question. Alternative reasonable values of θ, however, do not change the substance of our analysis. The parameter values that achieve our calibration targets are summarized in Table 1.

Table 1.

Calibrated Benchmark Parameters		Value
Preferences		
$1/\sigma$	Intertemporal Elasticity of Substitution	1/2
β	Time Discount Rate	0.982
Technology		
δ	Capital Depreciation Rate	0.044
γ	Growth Rate of Output per Capita	1.02
γ_N	Population Growth	1.012
α	Private Capital Elasticity of Output	0.42
θ	Public Capital Elasticity of Output	0.068

Figure 4), panel a), shows that for these parameter values, the optimal share of public investment in GDP falls with increases in per capita output growth. At the same time, note that the consumption-output ratio rises in panel c). In this calibrated example, increases in the rate of technical progress allows households to consume more relative to output and both private and public investment as a fraction of GDP must conversely fall. In Figure 4a), a 1 percent increase in per capita growth is associated with a 0.37 percent fall in capital expenditures relative to output. While this relationship is also negative in the data, a 1 percent rise in economic growth in Figure 3a) corresponds to a smaller 0.1 percent decrease in the share of government investment. Figure 4, panel b) shows an increasing relationship between public capital expenditures as a fraction of GDP and population growth. As we saw earlier, a higher rate of population growth erodes the public capital-output ratio so that greater government investment must be undertaken to keep the returns to public and private capital equalized. In this case, the model suggests that a 1 percent increase in population growth raises public investment relative to GDP by 0.68 percent. In the data (Figure 3c]), this elasticity is actually larger in that a 1 percent rise in population growth is associated with a 1.29 percent increase in the share of capital expenditures.

3.3 Dynamics of Optimal Public Policy with Commitment: First Period Cheating

A standard result in the literature on optimal fiscal policy with commitment stipulates aggressively raising capital taxes in the initial period relative to later dates. Since the capital stock is predetermined in period zero, increases in capital taxation are not distortionary on that date. Taxes on capital *are* distortionary thereafter, however, and it is optimal for a policymaker to commit to low future capital taxes while levying high taxes initially.

It should be clear that these incentives exist in our model economy as well. Recall the difference between equations (14) and (15) that govern the optimal choice of τ_t at dates $t = 0$ and $t > 0$. The additional term in (15),

$$-\mu_{1t-1} c_t^{-\sigma} r_t, \tag{22}$$

originates from the Euler constraint in problem (P2), $\beta^* c_t^{-\sigma} \left[(1-\tau_t) r_t + 1 - \delta\right] = \gamma_N \gamma c_{t-1}^{-\sigma}$, and corresponds to the reduction in the after-tax real return to investment made at date $t-1$ created by an increase in the tax rate at time t. Consequently, in committing to a tax rate in a given period $t > 0$, the government takes into account the implied substitution effect on investment decisions undertaken in the preceding period. Of course, at date $t = 0$, no such distortion exists since history commences on that date with a predetermined capital stock, k_0. In choosing τ_0, therefore, the government is free to ignore its effects on previous investment decisions that can be thought of as "sunk". Thus, it would seem that the optimal sequence of tax rates should indeed begin with a high tax in period zero relative to those at all other dates. However, in our framework, this reasoning is only partially complete and misses an important piece of the analysis that can overturn the standard intuition.

Consider the optimal choice of consumption from the standpoint of our benevolent planner. The first-order conditions associated with $\partial\mathcal{L}/\partial c_t$ in (P2) are

$$c_0^{-\sigma} + \sigma \gamma_N \gamma \mu_{10} c_0^{-\sigma-1} - \mu_{30} = 0 \text{ at } t = 0, \tag{23}$$

and

$$c_t^{-\sigma} - \sigma c_t^{-\sigma-1} \mu_{1t-1} [r_t(1-\tau_t) + 1 - \delta] + \sigma \gamma_N \gamma \mu_{1t} c_t^{-\sigma-1} - \mu_{3t} = 0 \; \forall t > 0. \tag{24}$$

The obvious difference between dates $t = 0$ and $t > 0$ relates to the term $-\sigma c_t^{-\sigma-1} \mu_{1t-1} [r_t(1-\tau_t) + 1 - \delta]$. This expression describes the impact of a change in consumption on the Euler constraint in problem (P2). Specifically, in considering an increase in consumption at any date $t > 0$, the government recognizes the implied wealth effect that reduces marginal utility and, therefore, the utility-denominated return to investment made in the preceding period, $-\sigma c_t^{-\sigma-1} \mu_{1t-1} [r_t(1-\tau_t) + 1 - \delta]$. Put another way, the policymaker understands that $u'(c_t)[r_t(1-\tau_t)+1-\delta]$ falls when present consumption increases. That said, this effect is once again irrelevant at time zero when the capital stock is predetermined. Consequently, equations (23) and (24) suggest that our policymaker is free to allow for high consumption in period zero without concern for prior investment decisions. But achieving high consumption in the first period requires a low initial tax rate, precisely the opposite prescription suggested by (22).

Income vs. Substitution Effects and the Initial Tax Rate

Since the effects we have just described rely crucially on the shape of marginal utility, whether or not the tax rate initially exceeds its long-run value depends importantly on the elasticity of intertemporal substitution, $1/\sigma$. That is, individuals' willingness to smooth consumption across time. Figure 5 illustrates this notion for the case with full depreciation, $\delta = 1$. The initial state of the economy in Figure 5 is given by the private and public capital stocks that prevail in the stationary Ramsey equilibrium. As explained earlier, we set $\mu_{1,-1} = 0$. Thus, starting from this steady state, the figure depicts the optimal sequence of tax rates adopted by a government when it is allowed to abandon, or cheat on, past commitments and to implement a new Ramsey policy.[7]

With $\delta = 1$, next period's private capital is determined only by current investment, $\gamma_N \gamma k_{t+1} = i_t$. Furthermore, in the logarithmic case where $\sigma = 1$, it is well known that in the type of model presented here, investment is itself unrelated to future after-tax returns and depends only on the current tax rate through disposable income. In other words, future taxes have no impact on current investment. Consequently, in implementing a tax rate at any date, the policymaker realizes that this rate never has an effect on past investment decisions. In this sense, period zero is no different than any other period and the government has no incentive to cheat. Starting from the long-run Ramsey tax rate, Figure 5 shows that our policymaker simply stays with this policy at every date. Furthermore, because cheating is not an issue, this Ramsey plan is, in fact, time consistent. Appendix A proves this result formally.

When $\sigma \neq 1$, private investment does depend on the whole stream of current and future taxes. An increase in the tax rate at time $t > 0$ reduces the return to past investments. On the one hand, this creates a negative wealth effect that leads households to want to reduce both present and past consumption. Past investments then tend to rise. On the other hand, the lower return is also associated with a substitution effect that makes past consumption more attractive relative to current consumption. Investments made in prior periods then tend to fall.

When the elasticity of intertemporal substitution is large, $1/\sigma > 1$, the substitution effect dominates as households are easily enticed to reallocate current consumption to previous periods. In this case, a rise in taxes in period $t > 0$ raises past consumption and, more importantly, depresses past investments. Relative to date zero, where this effect is absent, taxes at every date are distortionary and it is optimal for a government to set a high initial tax rate. This argument can simply be reversed for the case where the elasticity of intertemporal substitution is small, $1/\sigma < 1$.

[7] To obtain the transition paths from initial conditions, we linearize the dynamics of our transformed system around its stationary equillibrium. The resulting set of linearized equations possesses a continuum of solutions, but only one of these is consistent with the transversality condition. For transparency, we further set $\gamma_N = \gamma = 1$ so that the optimal long-run tax rate is simply $\beta\theta$ or 6.67 percent.

Here, an increase in the current tax rate lowers past consumption as households are unwilling to shift current consumption to previous periods. That is, the wealth effect dominates and raises past investments. Recognizing this feature, the policymaker then optimally chooses to set higher future tax rates relative to that in the initial period. These cases are shown in Figure 5 for elasticities of intertemporal substitution of 2 and 1/2 respectively.

For our benchmark calibration where $\delta = 0.044$, the same reasoning applies although next period's private capital is no longer just determined by current investment. With undepreciated capital available, the strength of the substitution effect implied by a change in taxes is reduced.[8] The wealth effect, however, remains unchanged. This means that for the case $\sigma = 1$, where the substitution and wealth effect exactly canceled each other with full depreciation, the wealth effect now dominates and the optimal path for taxes starts with a relatively low tax rate at date zero.

Figure 6 shows the transition paths associated with the Ramsey policy for our benchmark economy. The initial state is again given by the private and public capital stocks associated with the Ramsey stationary equilibrium. Therefore, the initial tax rate and corresponding share of public investment, τ_0, reflects *only* the government's first period incentive to disregard past commitments.

Recall that under our benchmark calibration, the share of public investment in GDP and the corresponding tax rate are calibrated to 5 percent in the long run. In Figure 6, panel a), we can see that the incentive to take advantage of period zero motivates the government to set a significantly lower tax rate initially, at approximately 3 percent. Government investment falls (not plotted) and so does the following period's public capital stock in panel e). Thus, with households relatively unwilling to reallocate future consumption to the present ($1/\sigma = 1/2$), a policymaker optimally chooses to generate increases in consumption and private investment in the short run as shown in Figures 6c) and 6d). Observe in panel f) that the increase in investment allows for a build up in private capital which then converges to the steady state from above.

In contrast to what has become standard intuition, we have shown that a policymaker able to commit to future policies may choose a low tax rate in the first period. He does not necessarily take advantage of the initial non-distortionary tax on capital, but may choose instead to delay taxation in order to generate a "take off" phase with higher short-run consumption and private investment.

3.4 Accounting for the Initial State in the Design of Optimal Policy

With both the private and public capital stocks at their Ramsey stationary equilibrium levels, we have seen that a government allowed to disregard past commitments chooses to lower the tax rate in period zero, thus generating a short-run boom in consumption and investment. It is worth noting that this feature of optimal policy does not hold irrespective of the state the policymaker inherits at date zero. In some sense, of course, the concept of date zero seems artificial since, from our current standpoint, this date occurred at some time far in the past. At a deeper level, however,

[8] The substitution effect is governed by changes in the price of past relative to current consumption, $\rho_t = r_t(1 - \tau_t) + 1 - \delta$. In particular, observe that the elasticity of ρ_t with respect to τ_t falls as δ increases.

this concept exactly captures the idea of a new government feeling unbound to commitments made by its predecessors. In formulating policy, we naturally expect that this government would take account of the state of the economy at the time it took office.

Consider the case where a policymaker inherits a public capital stock that is initially below its optimal long-run level. If private capital is at its steady state in period zero, then the return to public investment originally exceeds the after-tax return to private investment. The policymaker might then wish to moderate his desire to lower initial taxes in order to finance greater public investment and bring the two rates of return in line. In other words, inadequate public infrastructure at date zero increases the benefits of postponing consumption and adds to the gains from raising the initial non-distortionary tax on capital. Figure 7 describes precisely this situation. The public capital stock initially lies 4 percent below its steady state level [see Figure 7e)]. Under this scenario, panel a) shows that the optimal tax rate exceeds its long-run equilibrium level at time zero by roughly 0.4 percentage points (see solid line). In fact, its path is virtually opposite to that depicted in Figure 6a). Here, the low stock of initial public capital more than outweighs the government's first period incentives to abandon past commitments.[9] Note in Figures 7c) and 7d) that both consumption and investment are below their steady state values during the transition. Furthermore, aggregate consumption decreases in the short run. This bleak adjustment stems from two reasons. First, with government infrastructure initially falling short of its steady state, and private capital originally at steady state, GDP must necessarily start below its long-run equilibrium [see panel b)]. Second, even with GDP below steady state, the planner nevertheless finds it optimal to raise τ_0 in order to build up the stock of public infrastructure. Put simply, starting with an inadequate stock of public capital makes the transition to the steady state arduous, even under the optimal policy.

In fact, the path for tax rates described in Figure 7a) will be optimal whenever the ratio of private to public capital initially lies significantly above its Ramsey stationary equilibrium value. In Figure 8, the return to public investment once again initially exceeds the after-tax return to private investment, but this time because there is a surplus of private capital. This is shown in panel f) where private capital originally lies 4 percent above its long-run value. Note in Figure 8, panel a), that the optimal path for tax rates is almost identical to that in Figure 7a) we have just discussed. The main difference with Figure 7, of course, is that GDP, consumption, and investment now all lie above their long-run value during the transition. Under this scenario, the inherited surplus of private capital makes it possible for the government to build up its stock of infrastructure while, at the same time, allowing for a comfortable adjustment to the Ramsey stationary equilibrium.

3.5 Modeling Public Contributions to Output as a Flow

We have shown that income and substitution effects, as well as the initial state of the economy, are all relevant in shaping optimal policy. This finding, however, has typically been overlooked in

[9] Under our benchmark calibration, these two effects offset each other when k_{g0} is approximately 3.2 percent below steady state.

earlier literature because of a key assumption on the nature of productive government expenditures. Specifically, public contributions to private output have often been treated as a flow variable. While this modeling strategy has proved tractable, Turnovsky (1997) argues then if it is public infrastructure that expands the production frontier, then the stock of public capital rather than the flow of government spending matters in production. We now show that modeling government input as a flow removes the time consistency problem emphasized above, and the Ramsey solution is then constant through time.

In Barro (1990), Barro and Sala-i-Martin (1992), Turnovsky (1999), and many others, tax revenues are converted into infrastructure within the period. In the context of our model, this implies that

$$k_{gt} = \tau_t y_t. \tag{25}$$

Under this assumption, and in contrast to our earlier set-up, the initial level of public capital is not given but endogenous. The planner, therefore, has an additional instrument that effectively allows him to circumvent the distorting effects of income taxes.

Consider problem (P2) but replace the second constraint in the Lagrangian by

$$\sum_{t=0}^{\infty} \beta^{*t} \mu_{2t} \left\{ \tau_t y_t - k_{gt} \right\}.$$

The first-order conditions corresponding to the optimal choice of τ_t are still given by (14) and(15), but we now have new first-order conditions with respect to k_{gt},

$$\mu_{20}(\tau_0 \theta \frac{y_0}{k_{g0}} - 1) + \mu_{30}(1-\tau_0)\theta \frac{y_0}{k_{g0}} = 0 \text{ at } t = 0 \tag{26}$$

and

$$\mu_{1t-1} c_t^{-\sigma}(1-\tau_t)\frac{\theta \alpha y_t}{k_t k_{gt}} + \mu_{2t}(\tau_t \theta \frac{y_t}{k_{gt}} - 1) + \mu_{3t}(1-\tau_t)\theta \frac{y_t}{k_{gt}} = 0 \ \forall t > 0. \tag{27}$$

Substituting (15) into the first term of (27), with $r_t = \alpha(y_t/k_t)$, immediately yields $k_{gt}/y_t = \tau_t = \theta$ $\forall t > 0$. Moreover, substituting $\mu_{20} = \mu_{30}$ from equation (14) into equation (26) further yields $k_{g0}/y_0 = \tau_0 = \theta$. In this case, therefore, the optimal policy with commitment recommends setting a constant tax rate,

$$\tau_t = \theta \ \forall t, \tag{28}$$

irrespective of the state of the economy, preferences, or other technology parameters. The Ramsey planner behaves at time zero as in any other period, and the optimal policy is time consistent.

As in our benchmark economy, the intuition underlying this result relies on assessing the impact of a change in the tax rate at date t on the utility-denominated return to savings made at date $t-1 > 0$,

$$q_t = \beta^* c_t^{-\sigma} \left[(1-\tau_t) r_t + 1 - \delta \right], \tag{29}$$

where $c_t = (1-\tau_t)k_t^{\alpha} k_{gt}^{\theta} + (1-\delta)k_t - \gamma_N \gamma k_{t+1}$, $k_{gt} = \tau_t k_t^{\alpha} k_{gt}^{\theta}$, and $r_t = \alpha k_t^{\alpha-1} k_{gt}^{\theta}$. Using these expressions to solve for k_{gt}, and substituting the results into (29), allows us to write the utility-denominated return, q_t, as a function of τ_t. It is then straightforward to show that $\partial q_t / \partial \tau_t \mid_{\tau_t = \theta} = 0$.

Hence, distorting public "transfers" offset the distorting nature of income taxes within the period. Put simply, for the choice of τ_t that solves the Ramsey problem, wealth and substitution effects are exactly zero at every date.[10] Every period is then no different than period zero, where past savings decisions are unaffected by current tax decisions. Thus, a Ramsey policymaker never has an incentive to cheat.

4 Optimal Discretionary Policy

We now study the problem of optimal public investment when a commitment technology is no longer available. In such an environment, the solution to the Ramsey problem is generally time inconsistent. A new government coming into office would typically disregard the promises made by its predecessors. Rational households realize that the public sector has an incentive to deviate from the sequence of Ramsey taxes. Hence, setting taxes once and for all at time zero results in policy announcements that are not credible.

Because time consistency can be seen as a minimal requirement for credibility, the objective of this section is to define a maximization problem that yields a policy with this property in equilibrium. The literature on this subject generally follows two approaches.

One approach finds the set of all possible sustainable equilibria, and characterizes the problem using reputational mechanisms relying on trigger strategies that typically involve reversions to the worst possible equilibrium (Chari and Kehoe [1993]). Reputational mechanisms, however, have been criticized for not being renegotiation proof.

A second approach, one that is renegotiation proof, relies on the definition of subgame-perfect Markov equilibria. In this case, the optimal policy rule is a function of the current state of the economy. This function is independent of history and reputation plays no role. While the Ramsey solution captures the best possible equilibrium subject to income taxation, the Markov equilibrium solution delivers the worst equilibrium associated with optimal policies. In adopting the Markov approach in this paper, we compare optimal policy and its implications under these two extremes.

Define the Markov policy rule,

$$\tau^M = \Gamma(k, k_g), \tag{30}$$

that determines the optimal tax rate based on the state of the economy. A Markov-perfect equilibrium can essentially be thought of as a sequence of successive governments, each choosing a single tax rate based on the state it inherits when taking office. In making this choice, each government correctly anticipates the optimal decision rule adopted by its successors. In equilibrium, future policymakers' choices are time consistent if and only if they coincide with the rule that the current policymaker anticipated them to optimally choose. Moreover, the current planner's policy choice of τ must also follow this rule, $\tau = \Gamma(k, k_g)$.[11]

[10] Observe that this property does not necessarily hold with technologies other than Cobb-Douglas.

[11] See Klein et. al. (2002) for a detailed discussion of this notion of optimal discretionary policy.

Since a Markov planner can influence the state inherited by future governments through the interaction of his policy, firm behavior, and households' decisions, he does not take future policy-makers' *actions* as given. In particular, by affecting future states with current policy decisions, a Markov planner possesses some leverage over future governments through (30).

Let x' denote next period's value of any variable x. Unlike the Ramsey problem, where date zero plays a pivotal role, the problem faced by our benevolent government can now be formulated in recursive form,

$$V(k, k_g; \Gamma) = \max_{c,\tau,k',k'_g} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta^* V(k', k'_g; \Gamma) \right\} \tag{P3}$$

subject to the resource constraint,

$$c + i = (1-\tau)[rk + wl],$$

the accumulation equations,

$$\begin{aligned} \gamma_N \gamma k' &= i + (1-\delta)k \\ \gamma_N \gamma k'_g &= \tau y + (1-\delta)k_g \end{aligned}$$

and the Euler relation,

$$\gamma_N \gamma c^{-\sigma} = \beta^* (c')^{-\sigma} [(1 - \Gamma(k', k'_g))r' + 1 - \delta],$$

where the prices r and w reflect firms' profit-maximizing behavior, $r = \alpha(y/k)$ and $w = (1-\alpha)(y/l)$ with $y = k^\alpha l^{1-\alpha} k_g^\theta$.

Markov-perfect Equilibrium:

Let s denote the state variables $\{k, k_g\}$ in (P3), and let f denote the flow variables $\{c, i\}$. A Markov-perfect equilibrium is a set of functions $s' = \Pi(s)$, $f = G(s)$, $\tau = \Gamma(s)$, and $V(s)$ that solve the dynamic program above.

Observe that the steady state and dynamics associated with (P3) have to be determined simultaneously in a Markov-perfect equilibrium. Since steady state allocations depend on the tax rate, the stationary equilibrium cannot be found without knowledge of the policy rule $\Gamma(s)$.[12]

4.1 The Markov Steady State

Because each policymaker treats as given the state of the economy he comes into, the policy rule $\Gamma(s)$ should partly reflect the first period incentives faced by a Ramsey planner. Put differently, in much the same way as a Ramsey planner ignores history at date zero, each government in a Markov-perfect equilibrium considers the past as "sunk".

[12] A companion technical appendix that describes the solution to (P3) using a Linear Quadratic approximation approach is available upon request. This approach allows for a straightforward computation of transition paths.

We have already seen that in the Ramsey problem, the date zero incentives to abandon past commitments were such that the government's choice of initial policy depended on households' willingness to reallocate consumption across time. For small elasticities of intertemporal substitution, the Ramsey government chose to set a low tax rate initially. Since this tendency is reflected by *every* government in a Markov-perfect equilibrium (i.e. at every date), we might then expect Markov tax rates to settle below full commitment tax rates in the long run. The reverse would be true for large intertemporal elasticities of substitution. In fact, Table 2 captures exactly this intuition for the special case, $\delta = \gamma = \gamma_N = 1$.

When $1/\sigma = 2$, the optimal path for tax rates under commitment starts at roughly 6.77 percent, or 0.1 percent above its stationary equilibrium value. In this case, the stationary Markov-perfect tax rate also settles above its Ramsey counterpart at 6.78 percent. Conversely, when $1/\sigma = 1/2$, the date zero Ramsey tax rate falls short of its steady state value. Table 2 indicates that the stationary Markov tax rate now correspondingly lies below the long-run Ramsey tax rate. Finally, when household preferences are logarithmic, Figure 5 indicated that the optimal path for Ramsey tax rates was constant at every date, $\tau_0 = ... = \tau_\infty = \beta\theta = 0.0667$. Specifically, despite the presence of state variables, we argued that this parameterization makes optimal policy under commitment time consistent. Table 2 confirms that the stationary Markov tax rate is indeed also $\beta\theta$ when $\sigma = 1$.

Table 2.

Relation Between Initial Ramsey Policy and Stationary Markov-Perfect Policy, $\delta = \gamma = \gamma_N = 1$

IES Parameter	Initial Ramsey Policy, τ_0, (percent)		Stationary Ramsey Policy, (percent)		Stationary Markov-perfect policy, (percent)
$1/\sigma = 2$	6.77	$>$	6.67	$<$	6.78
$1/\sigma = 1$	6.67	$=$	6.67	$=$	6.67
$1/\sigma = 1/2$	6.58	$<$	6.67	$>$	6.58

The recent macroeconomics literature has seen a number of papers addressing problems of optimal discretionary policy. These include papers by Krusell and Rios-Rull (1999), and Klein et. al. (2003), among others in the context of fiscal policy, as well as Albanesi (2002), Dotsey and Hornstein (2003), and Khan, King and Wolman (2003), in the context of monetary policy. In fact, Table 2 suggests that the nature of the Ramsey problem at date zero plays an important quantitative role in explaining long-run Markov-perfect policy.

Consider the comparison between stationary outcomes under the Markov-perfect equilibrium and the Ramsey equilibrium in Figure 4. As the economic environment changes, we can see that the Markov-perfect solution retains the basic intuition obtained under the Ramsey program. The optimal time-consistent share of public investment in output falls with increases in the rate of technical progress and rises with population growth. In addition, as we have just argued, discretionary government capital expenditures are always lower than those under full commitment in the long run (our benchmark economy adopts the standard assumption, $1/\sigma < 1$). Thus, the inability to commit to future policy eventually implies too little public investment.

While the focus of our analysis has been theoretical in nature, Figure 9 depicts a positive cross-country correlation between the Average Tenure of Executive and Government Capital Expenditures as a percent of GDP. The data on Average Tenure of Executive is obtained from the World Bank's governance database. It answers the question: "How many years has the Chief Executive been in Office?"[13] Thus, to the degree that shorter tenure hinders the executive's ability to abide by past promises, Figure 9 indeed indicates, as in our model, that the share of public investment rises with the ability to commit to future policies.

4.2 Dynamics of Optimal Discretionary Policy

Because the Markov-perfect policy is time consistent, "re-starting" problem (P3) at any date has no effect on the time path of optimal policy. Thus, in contrast to the preceding section, the Markov-perfect policy rule implies that a government that comes into an economy in its stationary equilibrium would simply choose to continue with the corresponding steady state tax rate. However, because policy cannot be changed arbitrarily in the future, the initial state continues to be a key consideration.

Consider the situation in Figure 7 where the government at date zero inherits a public capital stock 4 percent below its long-run level. The private capital stock is initially at it stationary equilibrium value. When policy is discretionary, Figure 7a) shows that the optimal tax rate converges to a level lower than that under full commitment. As we have seen, this result captures the fact that in a Markov-perfect equilibrium, every government ignores the effects of its decisions on the past – (this behavior is itself reflected in a lower period-zero tax rate in the commitment case in Figure 6a]). More importantly, since the government cannot commit to financing the higher steady state Ramsey ratio of public investment to GDP, the Markov-perfect policy suggests considerably increasing the initial tax rate in order to start expanding the stock of infrastructure early. In particular, the date zero tax rate implied by the Markov-perfect policy is roughly 7.2 percent compared to just 5.4 percent for the policy with full commitment. This noticeable increase in initial taxes means larger falls in consumption and investment in the short run relative to the commitment case [see Figure 7, panels c) and d)].

As seen in the preceding section, the time path for Ramsey tax rates depicted in Figure 7a) also emerges when the policymaker inherits a state where the private capital stock originally lies above its long-run value. In this case, if public capital is initially at its steady state level, the rate of return to government capital exceeds the after-tax return to private capital, and a Ramsey policymaker optimally expands the stock of public infrastructure to bring the two rates of return in line. In the Markov-perfect equilibrium, the time path for tax rates follows a similar pattern. As in the previous example, the main difference is that Markov tax rates start above Ramsey tax

[13] See Beck, Clarke, Groff, Keefer, and Walsh (2001) for a detailed discussion of the data series.

rates. Knowing that it cannot commit to the future optimal level of capital expenditures relative to GDP, a Markov government raises taxes more stringently in period zero to make up for the commitment problem. In contrast to Figure 7, Figure 8c) shows that the original surplus of private capital allows for a relatively easy transition to the steady state. In this case, households are able to increase consumption in the short-run under either the Ramsey or Markov policy.

5 Final Comments and Directions for Future Research

We have shown that in models with productive capital financed via income taxes, the optimal policy is generally neither constant through time nor time consistent. This result stands in sharp contrast to most of the existing literature on productive government. Furthermore, contrary to what has become standard intuition, we find that a benevolent policymaker may well choose a low first-period tax rate under full commitment. We also found that allowing for discretion in the design of optimal policy does not necessarily result in higher long-run taxes relative to the commitment case. In our benchmark economy, the inability to abide by past promises eventually leads to lower tax rates and too little public investment.

While this paper extends our understanding of optimal public investment, the present framework abstracts from several features that are worth studying. First, having endogenized the choice of government expenditures, it is then natural to also endogenize the tax structure across factors of production when leisure is valued. Second, we expect that sovereign debt may significantly help the government reallocate resources across time and, therefore, reinforce our results. Specifically, a government seeking to set a low first-period tax rate would nevertheless be able to invest in public infrastructure by borrowing. Finally, the investigation in this paper focuses on two extremes, namely full commitment over the infinite future and no commitment. In practice, different institutional arrangements make it partially costly for governments to simply break past promises. Developing a framework that more closely captures political environments that limit the feasibility of policy change would represent a significant step towards practical policy analysis.

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Appendix A.

Consider the case with logarithmic utility and full depreciation (i.e. $\sigma = 1$ and $\delta = 1$).

Taking the sequence of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, and the sequence of tax rates, $\{\tau_t\}_{t=0}^{\infty}$, as given, the household's problem may be expressed as

$$\max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{*t} \ln(c_t) \tag{P1a}$$

subject to

$$\begin{aligned} c_t &= (1-\tau_t)\left[w_t l_t + r_t k_t\right] - \gamma_N \gamma k_{t+1}, \\ k_0 &> 0 \text{ given,} \end{aligned}$$

where $r_t = \alpha(y_t/k_t)$, $w_t = (1-\alpha)(y_t/l_t)$, and $y_t = k_t^{\alpha} l_t^{1-\alpha} k_{gt}^{\theta}$.

The Euler equation then reads as,

$$c_{t+1} = \frac{\beta^*}{\gamma_N \gamma} c_t (1-\tau_{t+1}) r_{t+1}.$$

Guessing a savings rule of the form $k_{t+1} = \frac{\alpha\beta^*}{\gamma_N\gamma} y_t (1-\tau_t)$, it is straightforward to verify that the individual's Euler equation condition holds under this guess.

Thus, the Ramsey Problem now reads as

$$\max_{\{\tau_t, k_{t+1}, k_{gt+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{*t} \ln((1-\tau_t) y_t - \gamma_N \gamma k_{t+1}) \tag{P2a}$$

subject to

$$\begin{aligned} k_{t+1} &= \frac{\alpha\beta^*}{\gamma_N\gamma} y_t (1-\tau_t), \\ k_{gt+1} &= \frac{1}{\gamma_N\gamma} \tau_t y_t. \end{aligned}$$

The government's optimality conditions $\forall t \geq 0$ are,

$$\lambda_t = \left(\mu_t + \frac{\gamma_N\gamma}{c_t}\right)\left(\frac{1}{\alpha\beta^*}\right), \tag{31}$$

$$\lambda_t - \frac{\gamma_N\gamma}{c_t} - \frac{\alpha\beta^* y_{t+1}}{k_{t+1}}\left[\lambda_{t+1}\frac{\alpha\beta^*}{\gamma_N\gamma}(1-\tau_{t+1}) + \mu_{t+1}\tau_{t+1}\frac{1}{\gamma_N\gamma} - \frac{(1-\tau_{t+1})}{c_{t+1}}\right] = 0, \tag{32}$$

and

$$\mu_t + \frac{\theta\beta^* y_{t+1}}{k_{gt+1}}\left[\frac{(1-\tau_{t+1})}{c_{t+1}} - \lambda_{t+1}\frac{\alpha\beta^*}{\gamma_N\gamma}(1-\tau_{t+1}) - \mu_{t+1}\tau_{t+1}\frac{1}{\gamma_N\gamma}\right] = 0. \tag{33}$$

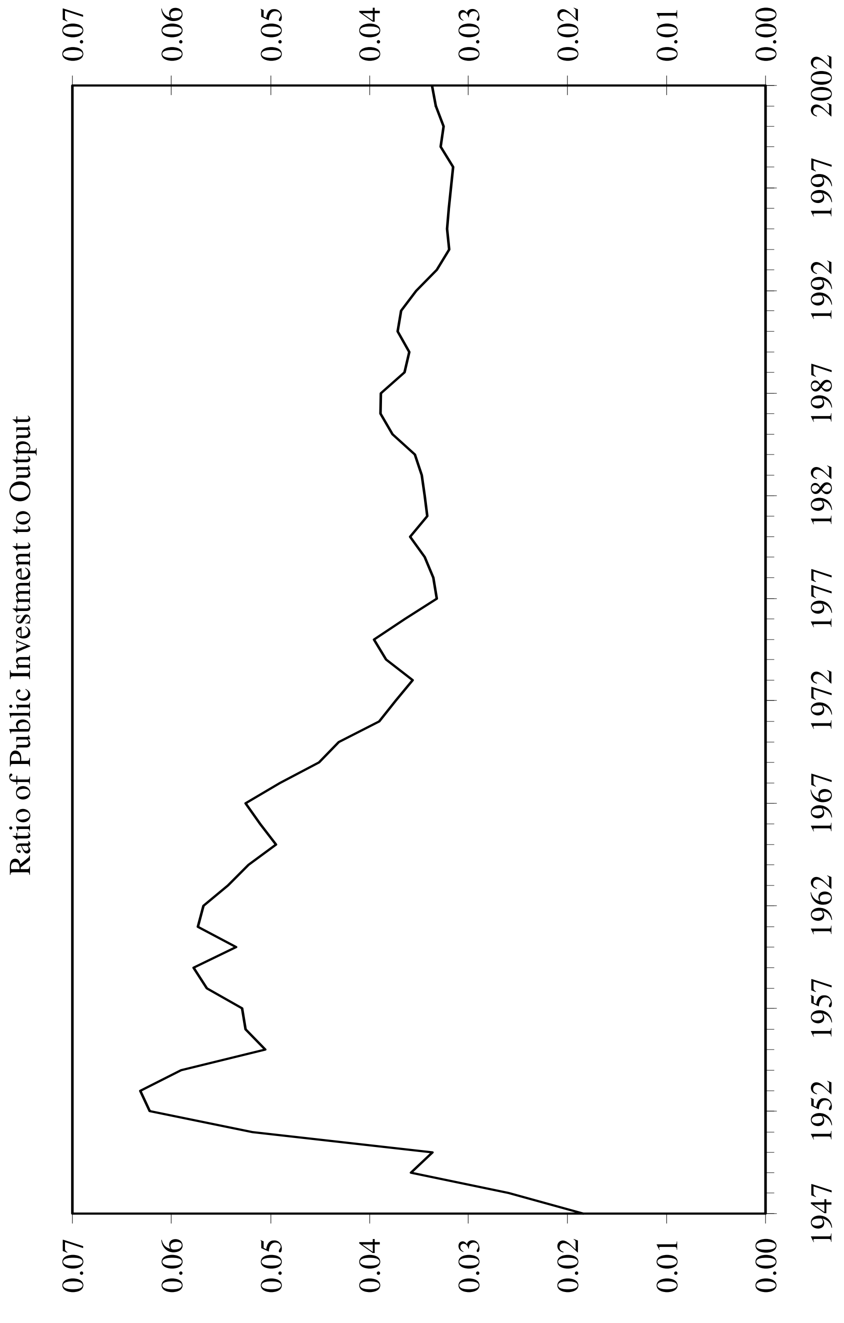
Some algebra helps us get rid of the multipliers,

$$\frac{1}{\alpha\beta^*(1-\tau_t)} + \frac{\theta}{\alpha\beta^*(\alpha\tau_t - \theta(1-\tau_t))} - \frac{\theta}{(1-\tau_t)(\alpha\tau_{t+1} - \theta(1-\tau_{t+1}))} = 0. \tag{34}$$

It is then easy to see that $\tau_t = \tau_{t+1} = \beta^*\theta$ satisfies the above equation.

Since the government faces the same first order conditions at $t = 0$ and $t > 0$, this Ramsey solution is time consistent.

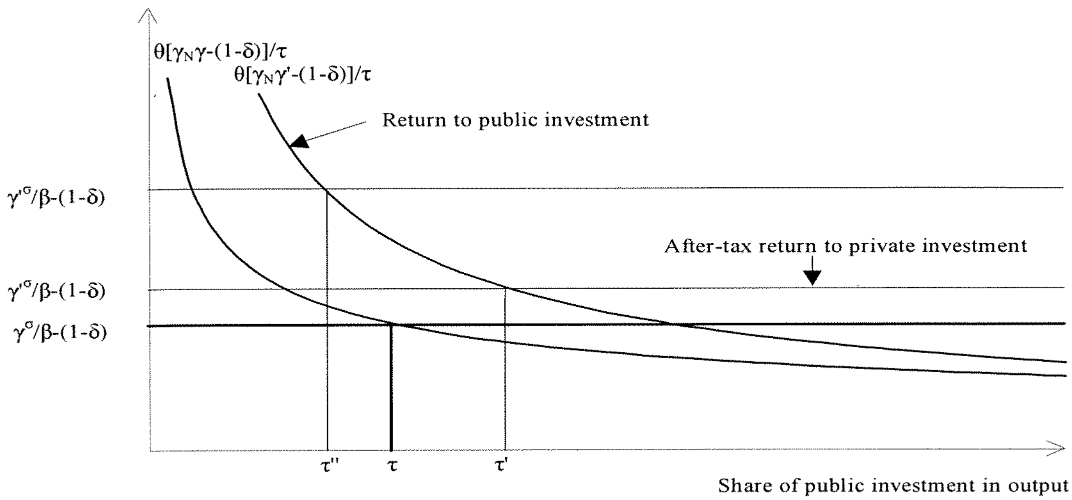
Figure 1.



Source: Bureau of Economic Analysis

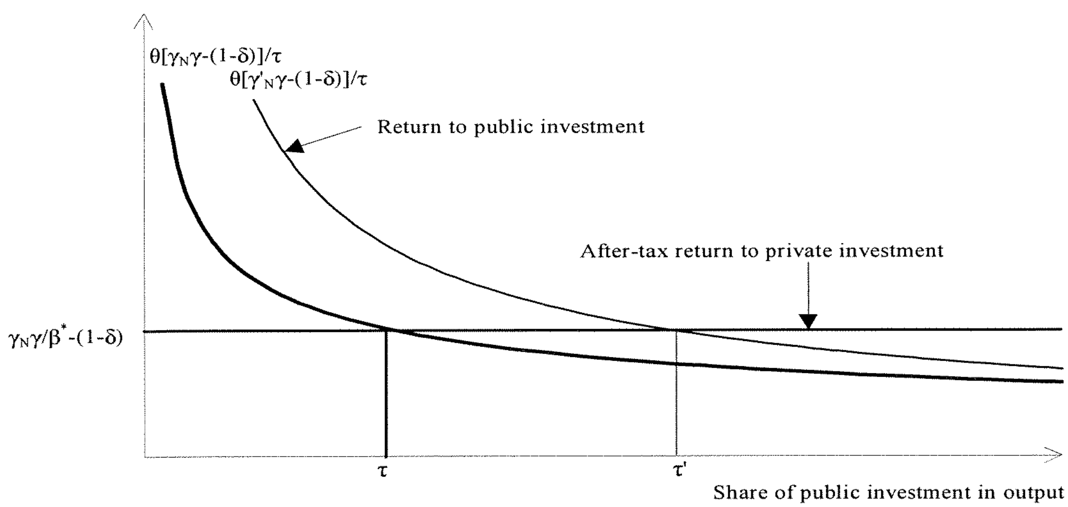
Figure 2.

(a)



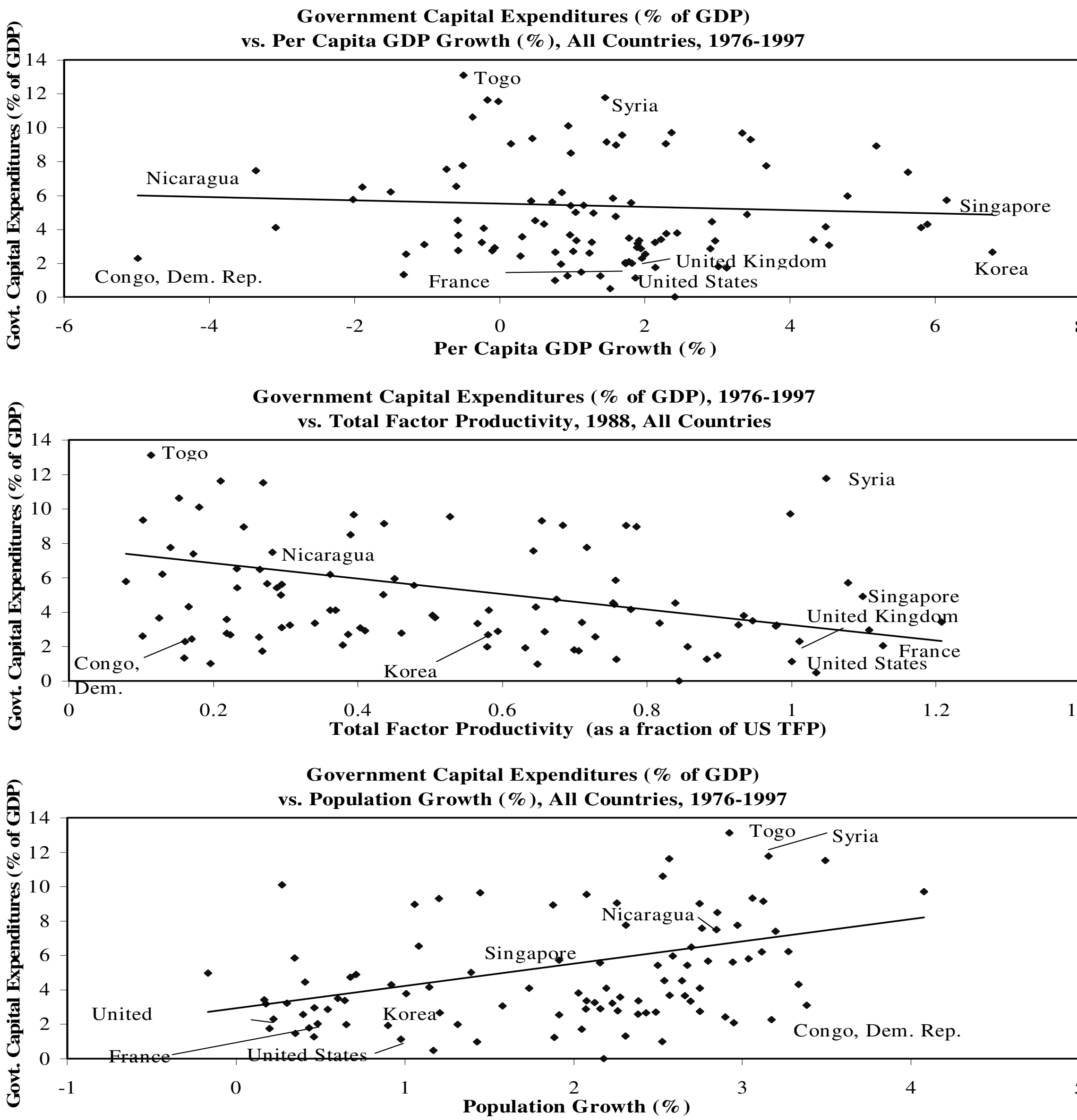
θ[γNγ-(1-δ)]/τ
θ[γNγ'-(1-δ)]/τ
Return to public investment
γ'σ/β-(1-δ)
After-tax return to private investment
γ'σ/β-(1-δ)
γσ/β-(1-δ)
τ''
τ
τ'
Share of public investment in output

(b)



θ[γNγ-(1-δ)]/τ
θ[γ'Nγ-(1-δ)]/τ
Return to public investment
After-tax return to private investment
γNγ/β*-(1-δ)
τ
τ'
Share of public investment in output

Figure 3.



Note: GDP at market prices, constant local currency units

Figure 4.

(—) Ramsey Policy, (- - -) Markov-perfect Policy

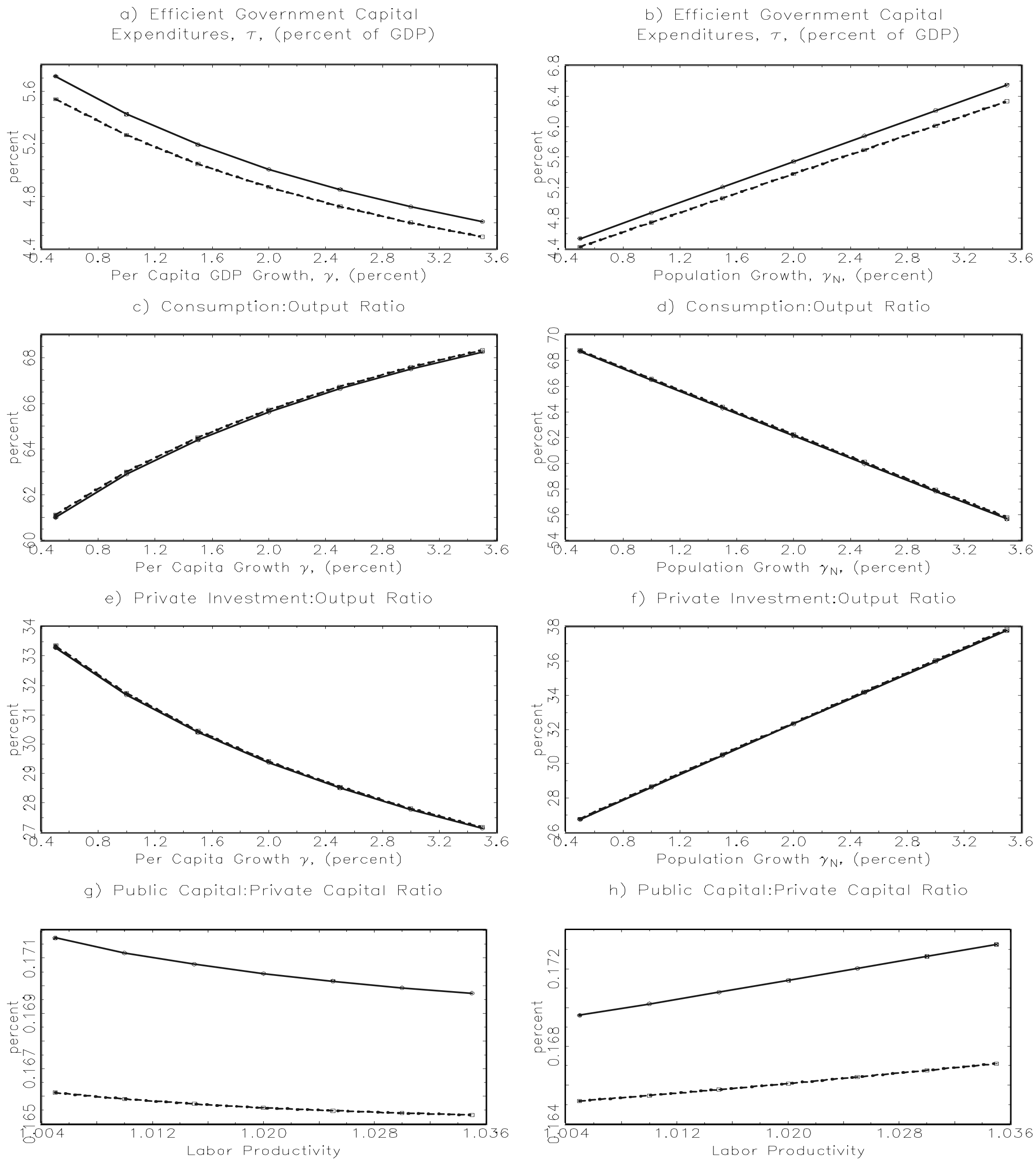


Figure 5.

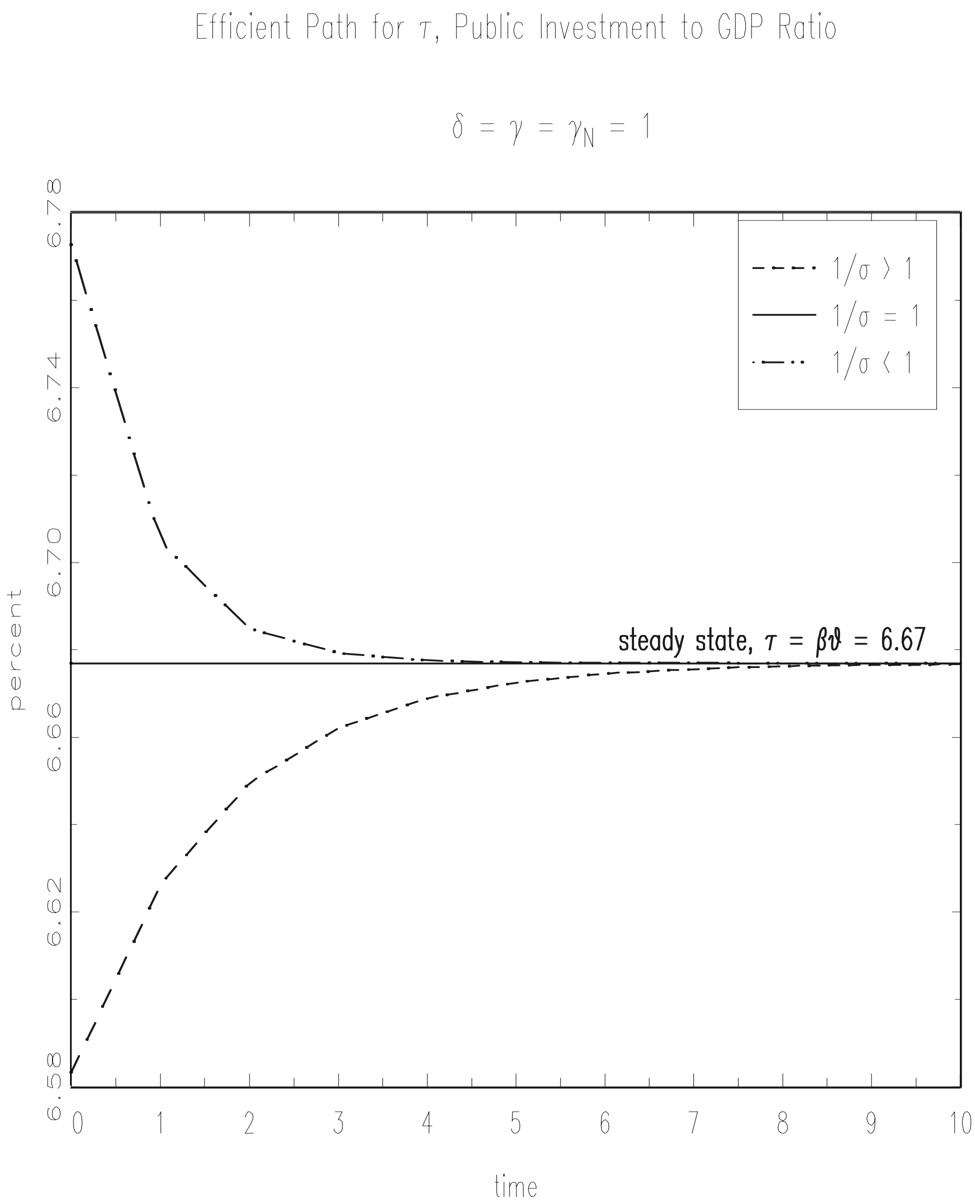
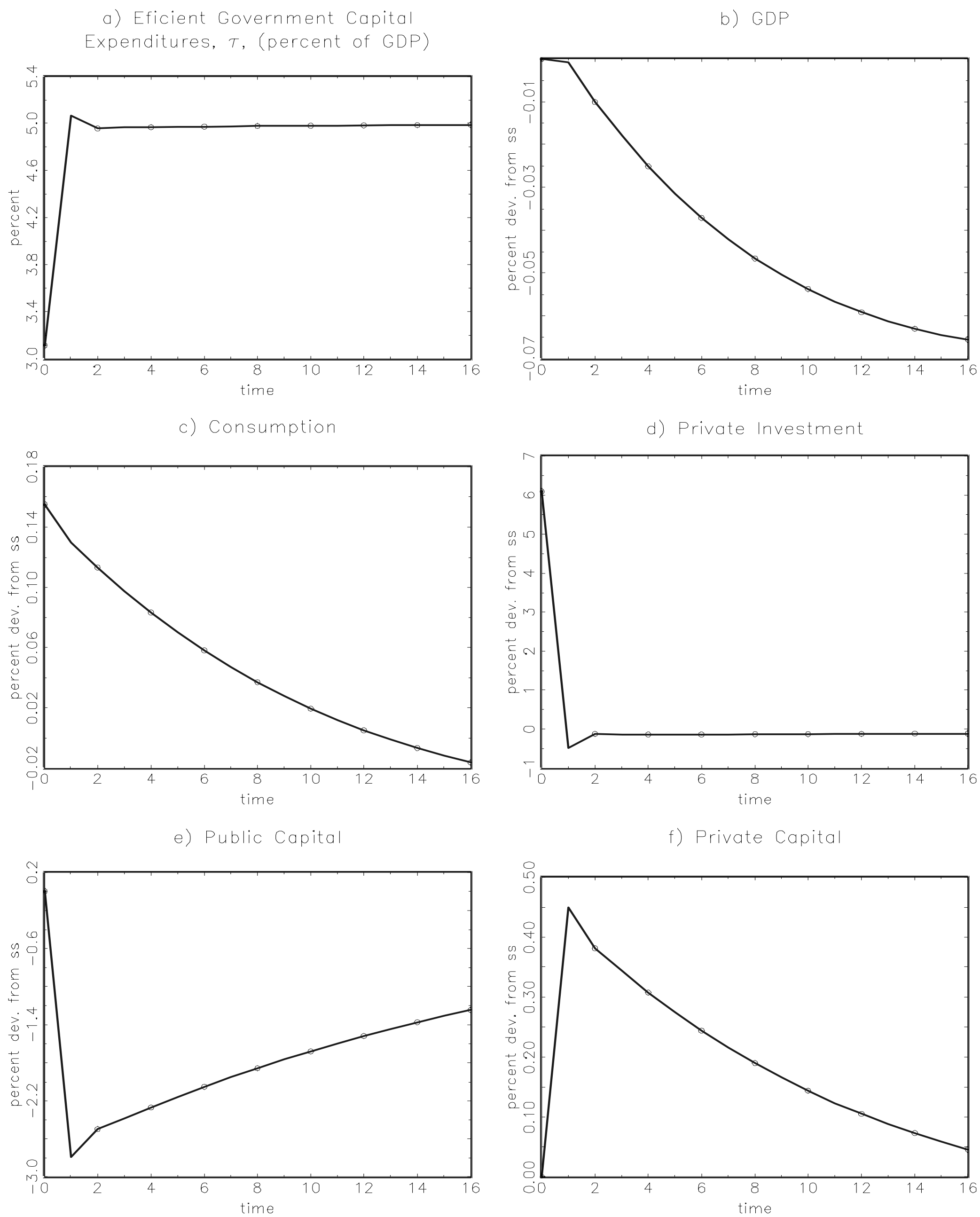

Efficient Path for τ, Public Investment to GDP Ratio
δ = γ = γ_N = 1
1/σ > 1
1/σ = 1
1/σ < 1
steady state, τ = βϑ = 6.67
percent
6.78
6.74
6.70
6.66
6.62
6.58
0
1
2
3
4
5
6
7
8
9
10
time

Figure 6.



a) Eficient Government Capital Expenditures, τ, (percent of GDP)
percent
time
b) GDP
percent dev. from ss
time
c) Consumption
percent dev. from ss
time
d) Private Investment
percent dev. from ss
time
e) Public Capital
percent dev. from ss
time
f) Private Capital
percent dev. from ss
time

Figure 7.

(—) Ramsey Policy, (- - -) Markov-perfect Policy

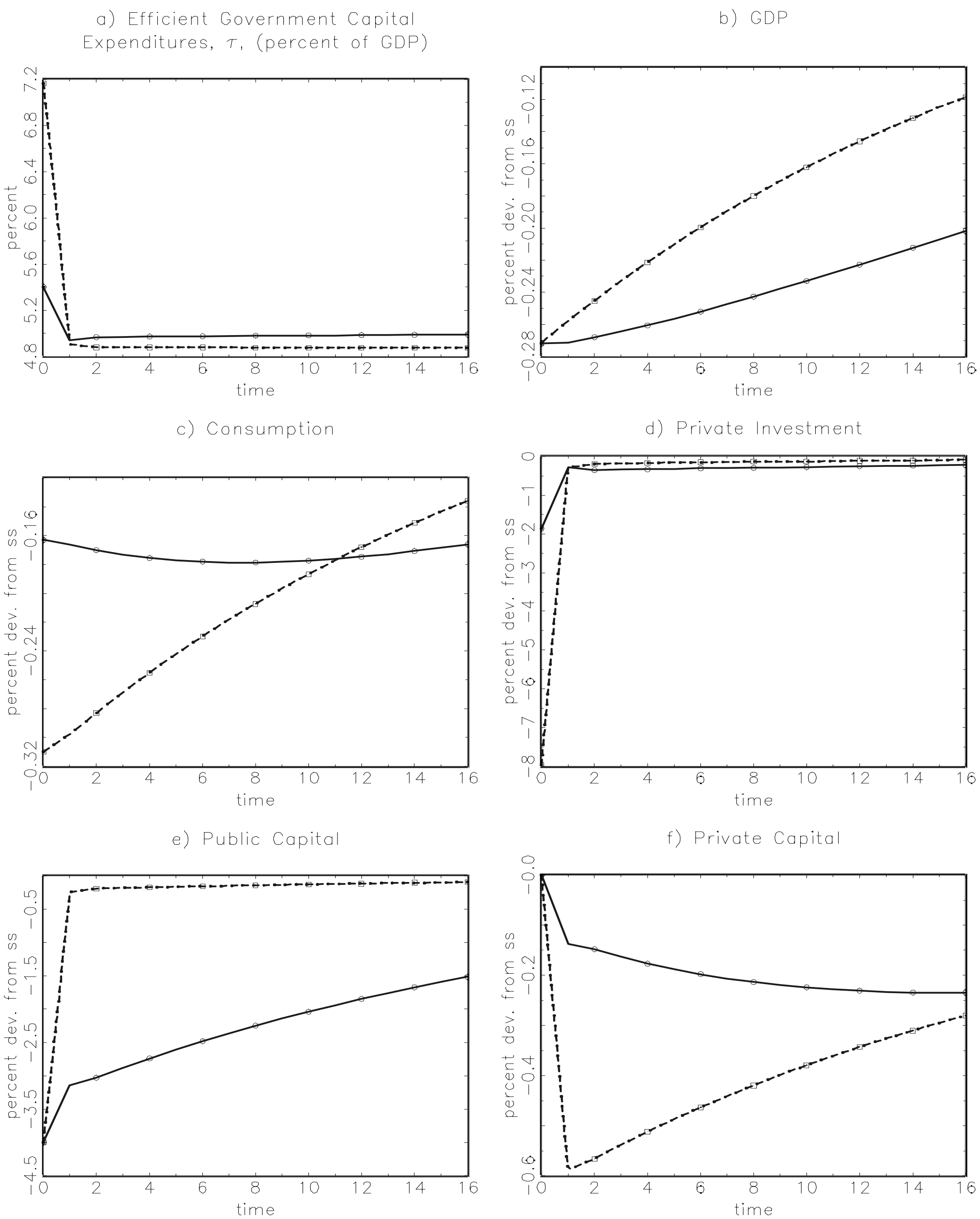


Figure 8.

(—) Ramsey Policy, (- - -) Markov-perfect Policy

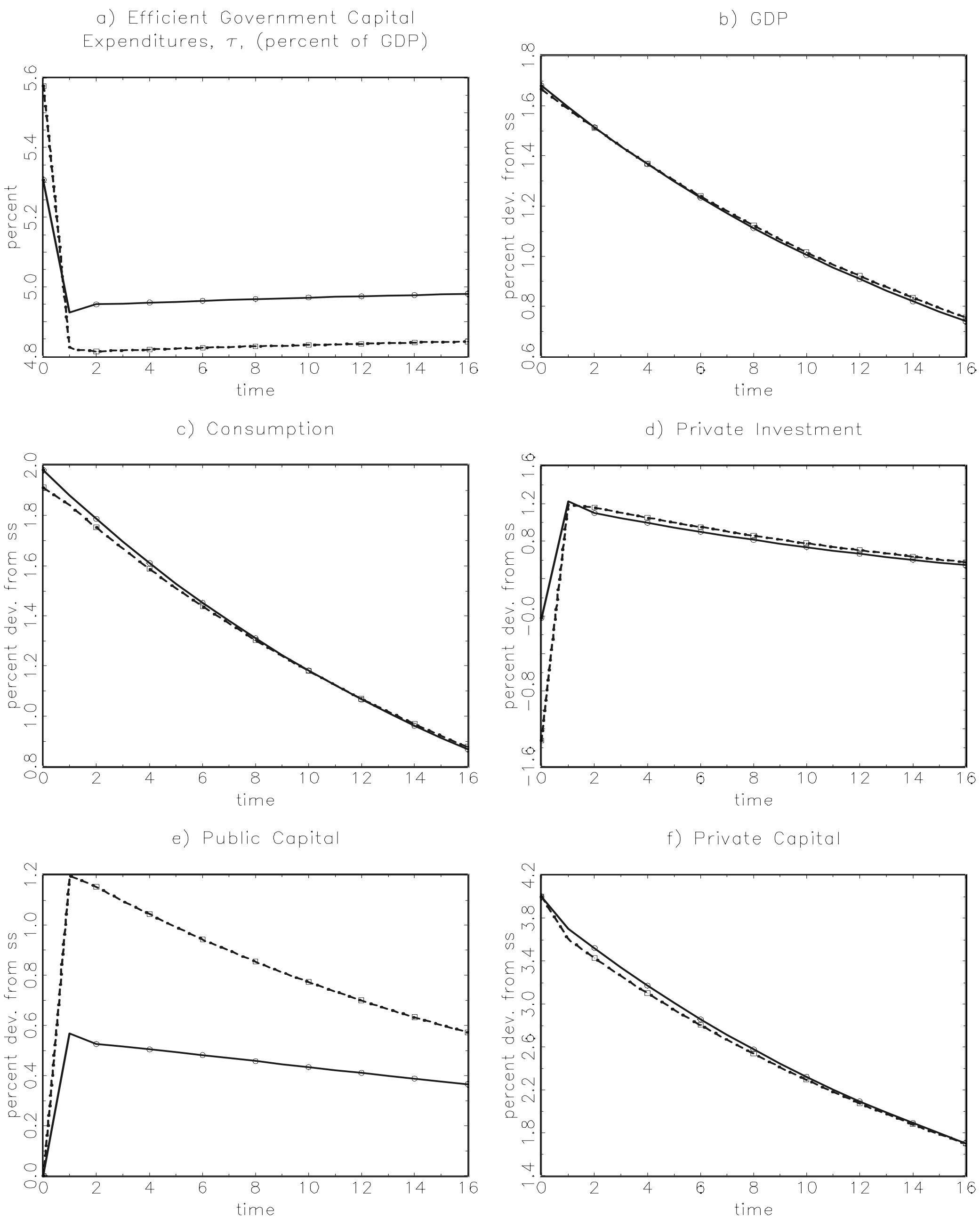
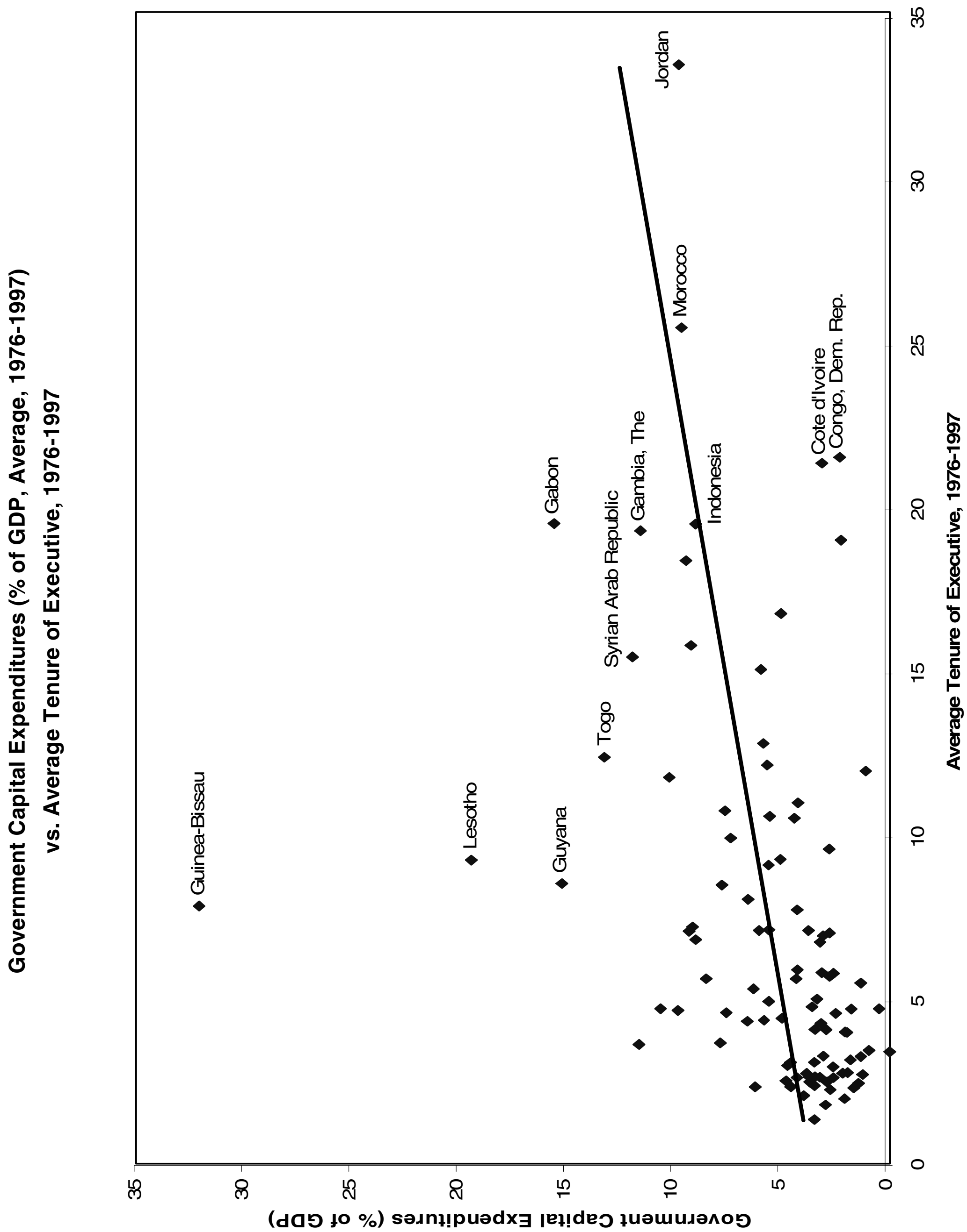


Figure 9.



Government Capital Expenditures (% of GDP, Average, 1976-1997)
vs. Average Tenure of Executive, 1976-1997
Government Capital Expenditures (% of GDP)
Average Tenure of Executive, 1976-1997
0
5
10
15
20
25
30
35
Guinea-Bissau
Lesotho
Guyana
Togo
Syrian Arab Republic
Gabon
Gambia, The
Indonesia
Morocco
Jordan
Cote d'Ivoire
Congo, Dem. Rep.