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Abstract

A discretionary policymaker responds to the state of the economy each period. Private agents' current behavior determines the future state based on expectations of future policy. Discretionary policy thus can lead to dynamic complementarity between private agents and a policymaker, which in turn can generate multiple equilibria. Working in a simple new Keynesian model with two-period staggered pricing – in which equilibrium is unique under commitment – we illustrate this interaction: if firms expect a high future money supply, (i) they will set a high current price and (ii) the future monetary authority will accommodate with a higher money supply, so as not to distort relative prices. We show that there are two point-in-time equilibria under discretion and we construct a related stochastic sunspot equilibrium.

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1 Introduction

A discretionary policymaker takes into account, each period, only the current state of the economy and the effect of the current policy action on current and future economic conditions. Private agents' behavior determines the future state of the economy and rationally incorporates expectations of future policy. The effects of future policy on current behavior can be direct–as in the case of a tax levy–or indirect, influencing the decisions of other private agents that are important for the individual's rewards. In either case, discretionary policy creates the possibility of dynamic complementarity and multiple equilibria: future policy responds to a state that was determined based on forecasts of the future policy. Accordingly, discretionary policymaking may open the door to welfare-reducing levels of economic activity or welfare-damaging economic fluctuations.

The economics of discretionary policymaking has been most studied in the context of monetary policy and that is the context in which we make explicit the general mechanism described above. To illustrate how interaction between discretionary policy and forward-looking private behavior can lead to multiple equilibria, we focus on a simple dynamic macroeconomic model with standard New Keynesian features: monopolistic competition and two-period staggered price setting.

Firms adopt *forward-looking pricing rules* because their nominal prices are held fixed for two periods. In choosing a price, firms in the current period need to form expectations about the behavior of the monetary authority–and other firms–in the next period. For example, a higher future money supply leads to a higher future price level and higher future nominal marginal cost, which raises the optimal price for a firm in the current period. Under discretion, the monetary authority chooses a money stock that is proportional to the price set by firms in the previous period; we call this a *homogenous money stock rule.* The combination of forward-looking pricing with discretionary policy leads to *complementarity* between the price-setting actions of firms: if all other firms set a higher price in the current period, the monetary authority will set a higher money supply in the subsequent period, raising the desired price for a single firm in the current period.

We show that this policy-induced complementarity implies that there are typically two private-sector equilibria which can prevail at any point in time. In general, there is one equilibrium in which firms make small adjustments and the newly set price is relatively close to the price that firms set in the previous period. But there is another in which the adjusting firms make a much larger adjustment. This second equilibrium is individually rational because each firm knows that (i) other firms are also making large adjustments and that (ii) these collective price increases will be ratified by the monetary authority in the future. A description of discretionary equilibrium includes a description of the distribution over the point-in-time equilibria. This distribution describes beliefs of agents and the monetary authority; the nature of these beliefs affects the equilibrium real outcomes.

The central implication of the immense literature on discretionary monetary policy–originating with Kydland and Prescott [1977] and Barro and Gordon [1983] (KDBG)–is inflation bias, specifically the idea that there would be a positive inflation rate in a discretionary equilibrium even though zero inflation is optimal under commitment. This result was a striking one, since it suggested that discretionary monetary policy could explain average rates of inflation such as those experienced in the United States and other OECD countries. In our model, there are two steady-state inflation rates consistent with the requirements of perfect foresight discretionary equilibrium, one that is modest and one that is extreme. Consequently, our model can explain why countries with similar fundamentals could display widely varying average rates of inflation. Within the KPBG model, since all inflation is expected in discretionary equilibrium, there are no effects of discretion on the level or variability of output. In our model, welfare-reducing real variability can arise from nonfundamental sources under discretion. Thus, our model suggests that discretion could be a source of cross-country and cross-time period differences in economic volatility.

The organization of the paper is as follows. After describing the model in section 2, section 3 shows that there are multiple point-in-time equilibria for arbitrary homogeneous policies. Section 4 analyzes discretionary equilibrium with perfect foresight and section 5 introduces sunspot fluctuations. In section 6 we relate our analyses to three branches of the existing literature on discretionary monetary policy. In section 7 we consider generalizations of our analysis to richer staggered pricing environments and empirical implications for the study of inflation and real activity. We also discuss other economic settings in which interaction between discretionary policy and forward-looking private behavior may be important. Section 8 summarizes the paper and concludes.

2 Model

The model economy that we study is a plain vanilla "New Keynesian" framework, featuring monopolistic competition and nominal prices which are fixed for two periods. There is staggered pricing, with one-half of a continuum of firms adjusting price in each period. Since all of the firms have the same technology and face the same demand conditions, it is natural to think of all adjusting firms as choosing the same price. We impose this symmetry condition in our analysis.

There are many different types of New Keynesian models, which differ in terms of their implications for the extent of complementarity in price-setting. Our model assumes that (i) there is a constant elasticity demand structure originating from a Dixit-Stiglitz aggregator of differentiated products; (ii) there is a centralized labor market so that the common marginal cost for all firms is powerfully affected by aggregate demand; and (iii) preferences for goods and leisure display exactly offsetting income and substitution effects of wage changes, as is common in the literature on real business cycles. Kimball [1995] and Woodford [2002] have stressed that these assumptions make it difficult to generate complementarity between price-setters when there is an exogenous money stock. As we will see, our model has exactly zero complementarity in this situation. From our perspective, this is a virtue because it highlights the importance of the policy-based complementarity that arises from monetary policy under discretion.

2.1 Households

There is a representative household, which values consumption (c_t) and leisure (l_t) according to a standard time separable expected utility objective,

$$E_t\{\sum_{j=0}^{\infty}\beta^j u(c_{t+j}, l_{t+j})\} \tag{1}$$

with β being the discount factor. We assume that the momentary utility function takes the form

$$u(c_t, l_t) = \log(c_t) + \chi l_t \tag{2}$$

which has the properties stated above and also has some other convenient implications that we describe later.

As is standard in the analyses of imperfect competition macro models that follow Blanchard and Kiyotaki [1987] and Rotemberg [1987], we assume that consumption is an aggregate of a continuum of individual goods, $c_t = [\int_0^1 c_t(z)^{(\varepsilon-1)/\varepsilon} dz]^{\varepsilon/(\varepsilon-1)}$. Households distribute their expenditure efficiently across these goods, resulting in constant-elasticity demands for individual products from each of the two types of firms which they will encounter in the equilibrium below:

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} c_t,\ j = 0, 1. \tag{3}$$

The subscript j in (3) denotes the age of the nominal price, so that $P_{0,t}$ is the price set by firms in period t and $P_{1,t}$ is the price set by firms one period earlier ($P_{1,t} = P_{0,t-1}$). Likewise,

$c_{j,t}$ is the period-t demand for goods produced by a firm that set its price in period $t-j$. The price level which enters in these demands takes the form

$$P_t = [\frac{1}{2}P_{0,t}^{1-\varepsilon} + \frac{1}{2}P_{1,t}^{1-\varepsilon}]^{1/(1-\varepsilon)}. \tag{4}$$

We assume that households also hold money to finance expenditure, according to

$$M_t = \int_0^1 P_t(z)c_t(z)\,dz \tag{5}$$

so that our model imposes a constant, unit velocity, in common with many macroeconomic analyses. We adopt this specification because it allows us to abstract from all the wealth and substitution effects that normally arise in optimizing models of money demand, so as to focus on the consequences of price-stickiness. With constant-elasticity demands for each good, the money-demand specification in (5) implies

$$M_t = P_t c_t. \tag{6}$$

Since this is a representative agent model and since no real accumulation is possible given the technologies described below, we are not too explicit about the consumption-saving aspect of the household's problem. We do assume that there is a Lagrange multiplier which represents the shadow value of wealth,

$$\lambda_t = \frac{\partial u(c_t, l_t)}{\partial c_t} = \frac{1}{c_t}, \tag{7}$$

and that households equate the marginal rate of substitution between leisure and consumption to the real wage rate prevailing in the competitive labor market, i.e.,

$$w_t = \frac{\partial u(c_t, l_t)/\partial l_t}{\partial u(c_t, l_t)/\partial c_t} = \chi c_t. \tag{8}$$

In each case, the second equality indicates the implications of the specific utility function introduced above.

2.2 Firms

Firms produce output according to a linear technology, where for convenience we set the marginal product of labor to one. So, for each type of firm, the production function is

$$c_{j,t} = n_{j,t}. \tag{9}$$

This implies that real marginal cost is unrelated to the scale of the firm or its type and is simply

$$\psi_t = w_t$$

and that nominal marginal cost is $\Psi_t = P_t\psi_t = P_t w_t$.

Much of our analysis will focus on the implications of efficient price-setting by one of the continuum of monopolistically competitive firms, which is assumed to be owned by the representative household. The adjusting firms in period t set prices so as to maximize the expected present discounted value of their profits, using the household's marginal utility as a (possibly stochastic) discount factor. That is, they choose $P_{0,t}$ to maximize their market value,

$$[\lambda_t(P_{0,t} - \Psi_t)c_{0,t} + \beta E_t \lambda_{t+1}\frac{P_t}{P_{t+1}}(P_{0,t} - \Psi_{t+1})c_{1,t+1}].$$

As monopolistic competitors, firms understand that $c_{0,t} = (P_{0,t}/P_t)^{-\varepsilon}c_t$ and that $c_{1,t+1} = (P_{0,t}/P_{t+1})^{-\varepsilon}c_{t+1}$, but take c_t, P_t, c_{t+1} and P_{t+1} as not affected by their pricing decisions. The efficient price must accordingly satisfy

$$P_{0,t} = \frac{\varepsilon}{\varepsilon - 1}\frac{P_t^{\varepsilon-1}\Psi_t + \beta E_t\left(P_{t+1}^{\varepsilon-1}\Psi_{t+1}\right)}{P_t^{\varepsilon-1} + \beta E_t\left(P_{t+1}^{\varepsilon-1}\right)}, \tag{10}$$

where we again give the result under the specific momentary utility function. In fact, this reveals one motivation for the form of the particular utility function chosen. In general, both aggregate demand (c_t) and the discount factor (λ_t) would appear in (10), but our choice of a utility function that is logarithmic in consumption means that these two effects exactly cancel out. With perfect foresight, the pricing equation can be written compactly as

$$P_{0,t} = \frac{\varepsilon}{\varepsilon - 1}[(1 - \theta_{t,t+1})\,\Psi_t + \theta_{t,t+1}\Psi_{t+1}]. \tag{11}$$

The optimal price is a constant markup $(\varepsilon/(\varepsilon-1))$ over a weighted average of nominal marginal cost over the two periods, where the weight on the future is

$$\theta_{t,t+1} = \frac{\beta\lambda_{t+1}c_{t+1}P_{t+1}^{\varepsilon-1}}{\lambda_t c_t P_t^{\varepsilon-1} + \beta\lambda_{t+1}c_{t+1}P_{t+1}^{\varepsilon-1}} = \frac{\beta P_{t+1}^{\varepsilon-1}}{P_t^{\varepsilon-1} + \beta P_{t+1}^{\varepsilon-1}}. \tag{12}$$

The weights on current and future nominal marginal cost represent the shares of marginal revenue associated with the current and future periods.

2.3 Defining Complementarity in Price Setting

The standard definition of complementarity – contained, for example, in Cooper and John [1988] – is that the optimal strategy of one decision-maker is increasing in the strategies of other similar decision-makers. In our context, we are interested in the complementarity in price-setting implied by equation (11). The left-hand side of this expression is the strategy of the particular decision-maker under study: the optimal price of an individual monopolistically competitive firm that is currently making a price adjustment. Other monopolistically competitive firms are also simultaneously adjusting prices: these firms take an action $P_{0,t}$ that influences the right-hand of (11). The common price chosen by adjusting firms influences the price level directly because $P_t = [\frac{1}{2}P_{0,t}^{1-\varepsilon} + \frac{1}{2}P_{1,t}^{1-\varepsilon}]^{1/(1-\varepsilon)}$ and may also affect current nominal marginal cost. Given that prices are sticky, there can be real effects of variations in the price level, so that these could influence nominal marginal cost. Finally, the weights on the present and the future in (11) also depend on the price level. To determine whether there is complementarity, we must work through these mechanisms and determine the sign of the relevant partial derivative. The extent of complementarity will depend on the behavior of the monetary authority.

2.4 Timing

The sequence of actions within a period is as follows. In the first stage, the monetary authority chooses the money stock, M_t, taking as given the price set by firms in the previous period, $P_{1,t}$. In the second stage, adjusting firms set prices ($P_{0,t}$). Simultaneously, wages are determined and exchange occurs in labor and goods markets.

There are two important consequences of these timing assumptions. First, since price-setters move after the monetary authority, they cannot be surprised by the monetary authority during the initial period that their price is in effect. Accordingly, the monetary authority faces an economy in which it can surprise some agents (those with pre-set prices) but not others (those adjusting prices) within a period. This gives rise to a relative price distortion across firms in the discretionary equilibrium that we will construct, which in turn means that there is an interior solution for the monetary authority's choice problem. If we reversed the timing order so that the monetary authority moved last, we conjecture that there would not be a discretionary equilibrium unless some other aspect of the economy were modified, such as allowing firms to reset their prices after paying an adjustment cost.[1] Second, the fact that the price-setters move after the monetary authority means that there is the potential for more than one equilibrium price to correspond to a given monetary policy action.

2.5 Complementarity with exogenous and constant money

We now consider a situation in which $M_t = M_{t+1} = M$. Under the assumptions of our model, it turns out to be easy to investigate the influence of other adjusting firm's actions, i.e., to compute the effect of $P_{0,t}$ on the right-hand side of (11). The constant velocity assumption ($P_t c_t = M_t$) and the particular utility function together imply $\Psi_t = P_t w_t = P_t(\chi c_t)$. Hence,

[1]The nonexistence of a discretionary equilibrium is a feature of Ireland's [1997] analysis of a model in which all prices are set simultaneously, before the monetary authority determines the current money supply.

equilibrium nominal marginal cost is exactly proportional to the money stock, $\Psi_t = \chi M_t$. Since the nominal money stock is assumed constant over time, nominal marginal cost is also constant over time and (11) becomes

$$P_{0,t} = \frac{\varepsilon}{\varepsilon - 1}\chi M.$$

This equilibrium relationship means that there is an exactly zero effect of $P_{0,t}$ on the right-hand side: there is no complementarity in price-setting in this model when the nominal money supply is constant.

2.6 Summarizing the economy by p_0 and m

Under discretionary policy, the monetary authority will not choose to keep the nominal money supply constant. Therefore, the optimal pricing condition (11) will not simplify to a static equation. In general, however, equilibrium will be a function of just two variables: a measure of the price set by adjusting firms and a measure of monetary policy. We construct these variables by normalizing nominal prices and money by the single nominal state variable in this economy, the price set by firms in the previous period ($P_{1,t} = P_{0,t-1}$). Define the normalized money supply as

$$m_t = M_t/P_{1,t}, \tag{13}$$

and the normalized price set by adjusting firms in the current period as

$$p_{0,t} = P_{0,t}/P_{1,t}. \tag{14}$$

We can then express all variables of interest as functions of these two normalized variables. From (4), the normalized price level is a function of only $p_{0,t}$:

$$P_t/P_{1,t} = g(p_{0,t}),$$

where

(15) $$g(p_{0,t}) \equiv [\frac{1}{2}p_{0,t}^{1-\varepsilon} + \frac{1}{2}]^{\frac{1}{1-\varepsilon}}.$$

Aggregate demand is a function of both $p_{0,t}$ and m_t :

$$c_t = c(p_{0,t}, m_t) \equiv m_t/g(p_{0,t}).$$

This follows from the money demand equation:

$$c_t = \frac{M_t}{P_t} = \frac{M_t}{P_{1,t}}\frac{P_{1,t}}{P_t} = \frac{m_t}{g(p_{0,t})}.$$

Further, since $n_t = [\frac{1}{2}n_{0,t}+\frac{1}{2}n_{1,t}] = [\frac{1}{2}c_{0,t}+\frac{1}{2}c_{1,t}]$, we can use the individual demands together to show that total labor input is also pinned down by $p_{0,t}$ and m_t :

$$n_t = n(p_{0,t}, m_t) \equiv \frac{1}{2} \cdot c(p_{0,t}, m_t) \cdot [g\left(p_{0,t}\right)]^{\varepsilon} \cdot \left(p_{0,t}^{-\varepsilon} + 1\right).$$

Leisure is the difference between the time endowment and labor input. Marginal cost is

$$\psi_t = w_t = \frac{\partial u(c_t, l_t)/\partial l_t}{\partial u(c_t, l_t)/\partial c_t} = \chi c_t = \psi(m_t, p_{0,t}).$$

Another variable of interest is the gross inflation rate, P_{t+1}/P_t. It is determined by current and future p_0:

(16) $$\frac{P_{t+1}}{P_t} = \pi(p_{0,t}, p_{0,t+1}) \equiv \frac{g(p_{0,t+1})}{g(p_{0,t})}p_{0,t}.$$

This follows directly from writing the inflation rate as a a ratio of normalized variables:

$$\frac{P_{t+1}}{P_t} = \frac{P_{t+1}/P_{0,t}}{P_t/P_{1,t}} \cdot \frac{P_{0,t}}{P_{1,t}} = \frac{g(p_{0,t+1})}{g(p_{0,t})}p_{0,t}.$$

In a steady state, there is thus a simple relationship between inflation and the relative price, $\pi = p_0$.

2.7 Two distortions and monetary policy

The monetary authority in this model faces two distortions that are present in the private economy and can be influenced by monetary policy. First, there is a *markup distortion* that represents the wedge between price and marginal cost: it has consequences similar to those of a tax on labor income. The markup is just the reciprocal of real marginal cost,

$$\mu_t = \frac{1}{\psi_t} = \frac{1}{w_t} = \frac{\partial u(c_t, l_t)/\partial c_t}{\partial u(c_t, l_t)/\partial l_t} = \frac{1}{\chi c_t}.$$

From the derivations above, the markup depends on $p_{0,t}$ and m_t: $\mu_t = g(p_{0,t})/(\chi m_t)$. Second, there is a *relative price distortion* that represents a wedge between inputs and outputs:

$$n_t/c_t = \delta(p_{0,t}) \equiv \frac{1}{2} \cdot [g(p_{0,t})]^{\varepsilon} \cdot \left(p_{0,t}^{-\varepsilon} + 1\right).$$

The relative price distortion depends solely on $p_{0,t}$. It takes on a value of unity at $p_{0,t} = 1$ (this would be the case in a zero inflation steady state) and is higher for other values of $p_{0,t}$. The trade-off that the monetary authority typically faces between these two distortions is that choosing a higher money supply decreases the markup (good) and raises the relative price distortion (bad).

For the analysis of discretionary monetary policy, there is a strong implication of the summary role of p_0 and m. The monetary authority can always choose m freely and p_0 is determined by the private sector, conditional on m and on expectations of the future.[2] Thus the level of the predetermined price ($P_{1,t}$) does not restrict the outcomes a discretionary policymaker can achieve.[3]

[2] The monetary authority can always choose m freely inasmuch as any choice of M_t can be replicated by choosing $m_t = M_t/P_{1,t}$. However, a policy of keeping M_t constant is not the same as a policy of keeping m_t constant.

[3] If the future monetary authority made m depend on nominal levels, it might be optimal for the current monetary authority to do the same. Our focus on Markov equilibria means that we do not consider equilibria with this property.

3 Equilibrium with homogeneous policy

We begin by studying the nature of equilibrium price-setting ($p_{0,t}$) under perfect foresight. We assume that the monetary authority adopts a policy rule of the form

$$M_t = m_t P_{0,t-1}, \tag{17}$$

where m_t is viewed as the policy variable. That is, the money supply is proportional to the prices that adjusting firms set one period ago with a constant of proportionality m_t. We call this a *homogenous monetary policy rule.* This form of monetary accommodation of past nominal variables is characteristic of optimal monetary policy under discretion, for the following reason. The monetary authority is concerned about the real variables that enter in private agents' utility. It takes past prices as given and there is no mechanism by which the level of nominal predetermined prices necessarily constrains the behavior of a discretionary policymaker.[4] Thus, if we viewed M instead of m as the policy instrument, we would find that the optimizing monetary authority adjusted M_t proportionally to $P_{1,t}$, just as is specified in (17). It will economize slightly on notation and computation to view m_t as the policy instrument and there is no loss of generality. In a discretionary equilibrium, m_t will be chosen to maximize welfare; in this section m_t is arbitrary.[5]

A homogenous money supply rule means that the future money supply depends on the price set by adjusting firms today,

$$M_{t+1} = m_{t+1} P_{0,t}.$$

[4]The word "necessarily" appears because one could construct non-Markov equilibria in which all agents agreed that $P_{1,t}$ did constrain the monetary authority. See previous footnote. We do not study such equilibria.

[5]By contrast, under commitment, the monetary authority commits *not* to respond to $P_{1,t}$, and the choice is over sequences of M_t. King and Wolman (1999) study optimal policy with commitment in the model used here.

Consequently, under homogeneous policy and using the preferences introduced above, it follows from the efficient price-setting condition (11) that the nominal price set by adjusting firms ($P_{0,t}$) satisfies

$$P_{0,t} = \frac{\varepsilon\chi}{\varepsilon - 1}\left(\left(1 - \theta_{t,t+1}\right) m_t P_{1,t} + \theta_{t,t+1} m_{t+1} P_{0,t}\right) \tag{18}$$

in equilibrium. The derivation of (18) from (11) involves (i) using the fact that nominal marginal cost is $\Psi_t = P_t \chi c_t$ given the specific utility assumption; (ii) using the money demand relationship ($M_t = P_t c_t$); and (iii) imposing the homogenous form of the monetary policy rule ($M_t = m_t P_{1,t}$). From (18), the normalized price set by adjusting firms ($p_{0,t}$) satisfies

$$\begin{aligned} p_{0,t} &= \left(\frac{\varepsilon\chi}{\varepsilon - 1}\right)\left(\left(1 - \theta_{t,t+1}\right) m_t + \theta_{t,t+1} m_{t+1} p_{0,t}\right) \\ &\equiv r(p_{0,t}, m_t, p_{0,t+1}, m_{t+1}). \end{aligned} \tag{19}$$

The weight on future nominal marginal cost, which was defined in (12), can be written in terms of current and future normalized prices as

$$\theta(p_{0,t}, p_{0,t+1}) = \frac{\beta\pi(p_{0,t}, p_{0,t+1})^{\varepsilon-1}}{1 + \beta\pi(p_{0,t}, p_{0,t+1})^{\varepsilon-1}}, \tag{20}$$

where we are now explicit about how $\theta_{t,t+1}$ depends on the present and the future. Equation (19) is a nonlinear difference equation in p_0 and m that must be satisfied in a perfect foresight equilibrium with homogeneous policy.

We view $p_{0,t}$ on the left-hand side of (19) as describing what an individual firm finds optimal given the actions of other price-setters and the monetary authority. On the right hand side, $p_{0,t}$ then represents all other adjusting firms' pricing behavior. The function on the right hand side represents the implications of those firms' behavior for the marginal revenues and costs of an individual firm. In other words, $r(.)$ is a *best-response function* for the individual firm. We restrict attention to symmetric equilibria, so that prices chosen by all adjusting firms are identical. We define complementarity in terms of a positive partial derivative of the response function with respect to its first argument. That is: with perfect foresight, there is complementarity if $\partial r(p_{0,t}, m_t, p_{0,t+1}, m_{t+1})/\partial p_{0,t} > 0$.

In section 3.2 below, we use the perfect-foresight best-response function (19) to describe point-in-time equilibria; this involves characterizing the fixed points for $p_{0,t}$, taking as given m_t, m_{t+1}, and $p_{0,t+1}$. This will then serve as an input to our analysis of discretionary equilibria. There, (19) will summarize private sector equilibrium for any action that the monetary authority contemplates, under perfect foresight.[6] With uncertainty, (19) will not hold exactly, but the mechanisms discussed here will still be relevant.

3.1 Complementarity under homogeneous monetary policy

There are two mechanisms for complementarity in (19) that will be operative in our analysis of point-in-time equilibria. First, holding fixed the weights, the normalized price $p_{0,t}$ has a positive effect on the right-hand side in (19): it enters linearly with a coefficient of $(\varepsilon\chi/(\varepsilon-1))\,\theta_{t,t+1}m_{t+1}$, which is positive because firms are forward-looking and the monetary authority raises nominal M_{t+1} proportionately with $p_{0,t}$. Since nominal marginal cost is proportional to the money stock, a higher future money stock provides an incentive for an individual firm to set a higher price in the current period. Hence the specification of monetary policy has introduced some complementarity into an economy in which it was previously absent.

Second, the weights in these expressions vary with the current normalized price $p_{0,t}$. This additional channel plays an important role in our analysis. A reference value for the weight

[6] If we impose $m_t = m_{t+1}$ then (19) describes the dynamics of $p_{0,t}$ for constant homogeneous policy. Such analysis might reveal interesting dynamics, and it does reveal the existence of two steady states. However it is not an input into our analysis of discretionary equilibrium, because under discretion policy is chosen sequentially.

$\theta_{t,t+1}$ is one-half, since (12) implies that the weight is $\beta/(1+\beta)$ if $P_t = P_{t+1}$ and since β is close to one. An upper bound on this weight is one: this is a situation where firms place full weight on the future. Increases in the weight raise the extent of the effect discussed above, i.e., they raise the coefficient $(\varepsilon\chi/(\varepsilon-1))\,\theta_{t,t+1}m_{t+1}$. The second mechanism is then that an increase in $P_{0,t}$ (or its normalized counterpart $p_{0,t}$) raises the weight on the future. This occurs because a firm's profits are not symmetric around its optimal price. As the firm's relative price rises, its profits decline gradually, asymptotically reaching zero as the price goes to infinity. By contrast, as the price falls, the firms profits decline sharply toward zero and may even become highly negative if the firm is not allowed to shut down its operations.[7] Thus, when $P_{0,t}$ increases for all other firms, future monetary accommodation – and the associated higher *nominal* price set by firms in the future – automatically lowers the firm's future relative price, raising the marginal revenue share associated with the future period. The costliness of a low relative price leads the firm to put increased weight on future marginal cost.

3.2 Point-in-time equilibria

Solving the monetary authority's problem under discretion means computing the point-in-time equilibria that correspond to all possible current policy actions and then picking the best action. Before studying this topic in detail in section 4, we begin by characterizing point-in-time equilibria for an arbitrary policy action in the current period. Point-in-time equilibrium refers to the values of $p_{0,t}$ that solve (19) for given current and future monetary actions and a given future price $p_{0,t+1}$. The mechanisms described above lead to the potential for multiple point-in-time equilibria.

[7] At this point in the analysis, we do not explicitly take into account the shut-down possibility. But, when we calculate discretionary equilibria, we do verify that the equilibria are robust to allowing firms to shut down.

As an input into our analysis it will be useful to define an inflationary monetary policy. To that end, note that in a zero inflation steady state, $p_0 = 1$. Such a steady state exists when the normalized quantity of money is $m^* \equiv (\frac{\varepsilon}{\varepsilon-1}\chi)^{-1}$. Furthermore, if $m = m^*$, zero inflation is the unique steady state; that is, $p_0 = 1$ is the unique solution to (19) that satisfies $p_{0,t} = p_{0,t+1}$when $m_t = m_{t+1} = m^*$. We refer to any $m > m^*$ as an *inflationary monetary policy*, because if inflation is positive in a steady state, then $m > m^*$, as we now show. From (19), given that $\pi = p_0$ in steady state, we have that in steady state,

$$m = \frac{1}{(\frac{\varepsilon}{\varepsilon-1}\chi)} \frac{\pi}{[1-\theta+\theta\pi]} = \frac{1}{[\theta + (1-\theta)(\frac{1}{\pi})]} m^*.$$

Thus, $\pi > 1$ if and only if $m > m^*$.

We assume now that the future money supply is given by $m_{t+1} > m^*$ (i.e. steady-state equilibria for the assumed value of m_{t+1} are inflationary) and that the normalized price set by firms in the next period is given, at $p_{0,t+1} > 0$. Under these assumptions, there are either two equilibria in the current period or equilibrium does not exist. In a knife edge case equilibrium is unique.

Proposition 1 *Given $p_{0,t+1} > 0$ and $m_{t+1} > m^*$,*
(a) if $m_{t+1} < m^* \cdot \left(1 + \frac{2}{\beta}\left(g\left(p_{0,t+1}\right)\right)^{1-\varepsilon}\right)$, then there exists $\breve{m} > 0$ such that for $m_t < \breve{m}$ there are two equilibria in period t, for $m_t > \breve{m}$ no equilibrium exists in period t, and for $m_t = \breve{m}$ there is a unique equilibrium in period t;
(b) if $m_{t+1} \geq m^* \cdot \left(1 + \frac{2}{\beta}\left(g\left(p_{0,t+1}\right)\right)^{1-\varepsilon}\right)$, then no equilibrium exists in period t for any m_t.

Proof. see appendix. ■

Point-in-time equilibria are fixed points of the best-response function for current period price-setters, which we write without time subscripts, using superscript prime to denote next period:

$$p_0 = r\left(p_0; m, p_0', m'\right) = \frac{1}{m^*}\left[\left(1-\theta\left(p_0, p_0'\right)\right)m + \theta\left(p_0, p_0'\right)m' p_0\right] \tag{21}$$

No expectation operator appears because we are assuming, for the purposes of this section, that there is no uncertainty about future m and – more importantly – future p_0.

The two mechanisms described previously work to produce multiple point-in-time equilibria for arbitrary homogeneous monetary policy. The first is that monetary policy is accommodative: if all other firms set higher prices today, the future nominal money stock will be higher in proportion, as will future marginal cost. This effect of current pricing on future nominal money is given by $m'p_0$ in (21). The second mechanism is that if all other firms raise prices today, then the future inflation rate will rise (it is easy to show that $\partial\pi(p_0, p_0')/\partial p_0 > 0$). An individual firm today will then place higher weight on future nominal marginal costs, because high future inflation shifts the firm's marginal revenue toward the future period.

Figure 1 illustrates the multiplicity of point-in-time equilibria for $m = m' > m^*$, for two different beliefs about future p_0. The dashed line is the 45^0 line that identifies fixed points – which are the point-in-time equilibria.[8] The two points marked with asterisks ('*') on the 45^o line, $\underline{p}_0$ and $\bar{p}_0$ are both steady state equilibria, because the policy action does not change and the equilibrium price does not change from the current to the future period. The solid line is the best-response function when agents expect $\underline{p}_0$ in the future, with certainty. The low steady state ($\underline{p}_0$) is a point-in-time equilibrium when agents expect $\underline{p}_0$ in the future, but the high steady state is not, because in that steady state agents expect $\overline{p}_0$ rather than $\underline{p}_0$ in the future. In fact, the second point-in-time equilibrium must be at a higher level than $\overline{p}_0$ because a larger increase in the weight on the future is required for the second equilibrium to occur. The dotted line shows the best-response function when agents expect $\overline{p}_0$ in the future, with certainty. In this case the higher of the two steady-state equilibria is a point-

[8]The proof of Proposition 1 characterizes fixed points of the best response function, but it does not establish the shape of the best response function as displayed in Figure 1. It is straightforward to establish that shape by combining the proposition with the facts that (i) $r(0; m, p_0', m') > 0$ and (ii) $\partial r/\partial p_0 > 0$.

in-time equilibrium, but the the low-inflation point-in-time equilibrium is now higher than the low-inflation steady state.

From (21), note that for given m', lower m drives down the lower price equilibrium and drives up the higher price equilibrium. Lower current m shifts the best-response function down, with lower current marginal cost reducing the firm's optimal price for any price set by other firms. The point-in-time equilibrium with a lower price falls, but because the basic properties of the response function are unchanged, there is still a second equilibrium, now at a higher level of p_0; at this high level of p_0, high future marginal cost offsets the lower current marginal cost. Current monetary policy actions thus affect the two equilibria in very different ways.

Figure 1 together with (21) illustrate that beliefs about both current and future equilibrium selection can affect the opportunities available to a discretionary monetary authority. Raising the current money supply shifts out the best-response function for firms, resulting in a lower high-p_0 equilibrium and a higher low-p_0 equilibrium. The likelihood of each equilibrium in the present will thus alter the trade-off facing the monetary authority. Beliefs about *future* equilibrium selection shift the current period best-response function for a given current money supply and thus also alter the trade-off facing the current monetary authority.

4 Discretion under perfect foresight

In a perfect foresight discretionary equilibrium, the current monetary authority sets the money stock to maximize the representative private agent's welfare, subject to

1. The behavior of the future monetary authority (m').

2. The behavior of firms in the future (p_0').

3. Optimal pricing by firms in the current period (p_0). The monetary authority must

have beliefs about the selection rule used to determine p_0 when a contemplated value of m leads to multiple equilibrium values of p_0.

Two conditions define a stationary perfect foresight equilibrium with discretion: (i) the current and future monetary authority each choose the same action; and (ii) the selection rule specifies parametrically that only one equilibrium will prevail in every period. It is common knowledge which equilibrium will prevail.

As we noted above, it is the essence of discretion in monetary policy that certain predetermined nominal variables are taken as given by the monetary authority. Here, the current money supply is set proportionally to the previously set price, $P_{1,t}$. This leads us to view m as the monetary authority's choice variable.[9] Our analysis of equilibrium under arbitrary choice of m revealed that in general there were either two point-in-time equilibria or no point-in-time equilibria, as long as future policy was expected to be inflationary. This leads us to expect multiple discretionary equilibria.

We used an individual firm's best-response pricing function to show that there would be multiple point-in-time equilibria under arbitrary homogeneous policy and thus in a discretionary equilibrium. This approach embodied the view that the multiplicity results from complementarity among firms, induced by the nature of policy. However, there is another perspective on the multiplicity here that involves policy complementarity. Instead of emphasizing that the price set by an individual firm depends on the price set by all other firms, this alternative approach emphasizes that the future nominal money stock depends on current period expectations of the future nominal money stock. The future nominal money stock is proportional to the current nominal price set by adjusting firms and that price incorporates expectations of future policy. Just as we found two fixed points to the pricing best response

[9] In general, the actions of a discretionary monetary authority will have a homogeneneity property in some index of predetermined nominal variables. Here that homogeneity allows us to eliminate the single predetermined nominal price from the policy problem. We can then proceed *as if* there are no state variables.

function, we would also find two fixed points to a policy response function, which plotted M_{t+1} vs. $E_t M_{t+1}$, holding fixed m_t, m_{t+1} and $p_{0,t+1}$. We chose to present our results from the perspective of complementarity among firms, but the reader should keep in mind that those results have an equally valid interpretation as stemming from policy complementarity of the sort just described. In some contexts, there could be conceptual or computational advantages to taking this alternative perspective.

4.1 Constructing Discretionary Equilibria

We look for a stationary, discretionary equilibrium, which is a value of m that maximizes $u(c, l)$ subject to the constraints above when $m' = m$. We have used two computational approaches to find this fixed point. A comparison of the two approaches is revealing about the nature of the multiple equilibria we encounter.

The first computational method involves iterating on steady states. We assume that all future monetary authorities follow some fixed rule m'. Next, we determine the steady state that prevails including the value of p_0'. Then, we confront the current monetary policy authority with these beliefs and ask her to optimize, given the constraints including the selection rule. If she chooses an m such that $|m - m'|$ is sufficiently small, then we have an approximate fixed point. If not, then we adjust the future monetary policy rule in the direction of her choice and go through the process again until we have achieved an approximate fixed point. This approach conceptually matches our discussion throughout the text, but leaves open an important economic question: are the equilibria that we construct critically dependent on the infinite horizon nature of the problem?

The second computational method involves backward induction on finite horizon economies. We begin with a last period, in which firms are not forward-looking in their price setting and deduce that there is a single equilibrium, including an optimal action for the monetary authority $m_T > m^*$ and a unique equilibrium relative price $p_{0,T}$. Then, we step back one period, taking as given the future monetary action and the future relative price. We find that there are two private sector equilibria. In fact, this is inevitable, because the first step backwards creates a version of our point-in-time analysis above. Consequently, this approach establishes that the phenomena are associated with forward-looking pricing and homogenous monetary policy, rather than with an infinite horizon. To construct stationary nonstochastic equilibria using this approach, we can iterate backwards from the last period, computing the optimal policy, $\{m_T, m_{T-1}, ...\}$ and stop the process when $|m_{t+1} - m_t|$ is small, taking $m_t = m$ as an approximate fixed point.

In either computational approach, our work adopts the perspective that the relevant sequential dynamic equilibrium is one that is Markovian, as in Krusell and Rios-Rull [1999]. In general, this equilibrium concept requires the actions of the policymaker to depend on a set of fundamental state variables that have intrinsic relevance to the equilibrium. In our setting, the only state variable ($P_{1,t}$) has been shown to have no intrinsic relevance to the real equilibrium. The search for a nonstochastic Markov equilibrium corresponds then to determining constant levels of public and private actions. This process reveals two nonstochastic Markov equilibria. We then consider a stochastic discretionary equilibrium in which each period's equilibrium outcome is determined by a sunspot that shifts private sector beliefs. With this extension, we continue to assume that the monetary authority makes its actions a function of the state variables that are relevant to the private sector. We focus on Markov equilibria because these impose the most structure on the problem (making clear that our multiplicity arises from a single source) and provide the most tractable solution. Furthermore, the Markov equilibria of the model have natural analogues in a finite-horizon version of the model, making it clear that our results do not depend on whether the model is literally an infinite horizon one or simply the convenient stationary limit of a sequence of finite horizon models.

The numerical examples that we study next have the following common elements. The demand elasticity (ε) is 10, implying a gross markup of 1.11 in a zero inflation steady state. The preference parameter (χ) is 0.9 and, for convenience, we set the time endowment to 5. Taken together with the markup, this implies that agents will work one fifth of their time ($n = 1$) in a zero inflation steady state. With zero inflation, $c = n = 1$ since there are no relative price distortions and thus $m^* = 1$. A first-best outcome would dictate that $u(c, l)$ be maximized subject to $c = (1 - l)$. For the specified preferences, this leads to a first order condition $\frac{1}{n} = \chi$ or an efficient level of work (n) of 1.11. So, cutting the gross markup to 1.0 leads to an 11.1 percent increase in work and output.

4.2 Optimistic Equilibrium

If the discretionary monetary authority knows that the low p_0 equilibrium will prevail, then its problem is to maximize

$$u(c, l) + v(\underline{p_0}'; m')$$

where $v(\underline{p_0}'; m')$ denotes the future utility that corresponds to a steady state with m' and selection of the low-p_0 equilibrium with probability 1.0. The maximization is subject to

$$\begin{aligned} c &= c(m, \underline{p_0}) \\ l &= l(m, \underline{p_0}) \\ \underline{p_0} &= r(\underline{p_0}, \underline{p_0}', m, m'), \end{aligned}$$

where $r\,()$ denotes the response function on the right hand side of (21) and the presence of $\underline{p_0}$ instead of p_0 is meant to imply that we place probability one on the low-p_0 fixed point of the response function. The monetary authority understands that future utility and current price determination is influenced by the actions of the future monetary authority, but has no way of influencing its behavior or the future price that will prevail. So, the monetary authority maximizes current period utility.

Exploiting initial conditions. Figure 2 provides some insight into the nature of the monetary authority's problem if it knows that future monetary policy will be noninflationary ($m' = m^*$, $p_0' = 1$). The figure displays welfare, the two distortions and the current normalized price set by adjusting firms for a range of monetary actions. Note that in Figure 2 there is a unique point-in-time equilibrium for every monetary action. This does not contradict Proposition 1, because future monetary policy is noninflationary in Figure 2. In fact, one can see from (21) that if $m' = m^*$, then the unique current-period equilibrium is $p_0 = m/m^*$. This linear increasing relationship between p_0 and m is displayed in panel D.

The current monetary authority optimally adopts an inflationary monetary policy when $m' = m^*$ (choosing $m > m^* = 1$) because it can reduce the markup and stimulate consumption toward the first-best level. It does not drive the gross markup all the way to one because an increase in m produces relative price distortions. While the relative price distortions are negligible near the noninflationary steady state, they increase in a convex manner as monetary policy stimulates the economy. Figure 2 illustrates the sense in which New Keynesian models capture the incentive for stimulating the economy at zero inflation, as described in Kydland and Prescott [1977] and Barro and Gordon [1983]. The optimal monetary action in this figure does not represent a discretionary equilibrium, because future policy will not deliver zero inflation in equilibrium.

An inflation bias equilibrium. Figure 3 illustrates the policy problem for a monetary authority under optimistic beliefs, in equilibrium. Panel A shows the policymaker's objective function, which can be thought of as an indirect utility function: the relevant portion for the current discussion is the solid line, which reaches a maximum at the value of m/m^* =1.01.[10]

[10] Even in the discretionary equilibrium with optimism, there are two point-in-time equilibria each period. At any given level of m, panel D shows that there are high and low equilibrium values of p_0. One sees here the effect described in section 3.2: as m increases, the two equilibrium values of p_0 converge, and for high enough m there is no equilibrium.

This implies a stationary relative price (p_0) of 1.022, which is determined along the lines of Figure 2 with agents expecting $p_0' = p_0$ and $m' = m$. Given that there is a steady state, $\pi = p_0$ and this relative price thus implies an inflation rate of 2.2 percent per quarter. At this inflation rate, the monetary authority faces sufficiently increasing marginal relative price distortions that it chooses not to further increase m in an effort to further reduce the markup. Notably, the stationary markup departs little from its value at zero inflation. Stationary consumption is 99.96 percent of its zero inflation value, so that the markup has changed negligibly (recall that the markup and consumption are inversely related by $\mu_t = (c_t\chi)^{-1}$ with the preference specification used here). Figure 3 illustrates the sense in which this model delivers a discretionary equilibrium with the inflation bias stressed by Kydland and Prescott [1977] and Barro and Gordon [1983].

4.3 Pessimistic Equilibrium

We next suppose that the monetary authority instead knows that the high p_0 equilibrium will always prevail. Its incentives are sharply different from the optimistic case. Looking at Figure 3, we can see these incentives in the *dashed* lines, which describe point-in-time equilibrium when the private sector and the monetary authority assume that the future is described by $\underline{m}, \underline{p}_0$ while the present is described by $\bar{p}_0$. The monetary authority has a clear incentive to raise $m > \overline{m}$ since this lowers the markup *and* relative price distortions, with utility being maximized when m is sufficiently high that the point-in-time best response function of Figure 1 is tangent to the 45-degree line. Here the monetary authority "takes policy to the limit" of the set of equilibria that are imposed as its constraints. Because Figure 3 assumes optimism (that is, the low-p_0 point-in-time equilibrium occurs with probability one in the future), there are some inconsistencies in using Figure 3 to discuss a pessimistic equilibrium. Notably, the monetary authority can lower the markup to less than one, in which case some of the firms in the economy are making losses. But the picture tells the right story: nearer the discretionary equilibrium that is described by a level $\overline{m}$, the monetary authority still has the same incentives to raise m, but it does so without producing the curious behavior of the markup shown here.

In fact, it is not necessary to make a complicated set of fixed point computations in this case. A tangency equilibrium is one in which $\partial r(p_{0,t}, m_t, p_{0,t+1}, m_{t+1})/\partial p_{0,t} = 1$. Therefore, we can simply solve the stationary version of the equation,

$$p_{0,t}\frac{\partial r(p_{0,t}, m_t, p_{0,t+1}, m_{t+1})}{\partial p_{0,t}} = r(p_{0,t}, m_t, p_{0,t+1}, m_{t+1}),$$

to calculate the equilibrium value of p_0 (this is one equation in one unknown p_0 because the $m = m'$ drops out). We can then determine the relevant m from the equation $p_0 = r(p_0, p_0, m, m)$.

In our numerical example, there is a discretionary equilibrium with $p_0 = 1.17$, so that there is a 17 percent quarterly inflation rate in the pessimistic equilibrium with optimal discretionary policy. The associated value of m/m^* is 1.0295. This value is larger than the one used to construct Figure 3, as it should be: a higher level of m is necessary to produce a tangency equilibrium in the pessimistic case.

Taken together, the optimistic equilibrium result from section 4.2 and the pessimistic equilibrium that we just constructed, we see that there are two steady-state equilibria with discretionary optimal monetary policy. The inflation rates are quite different across the two equilibria: about 2 percent (per quarter) in one case and about 17 percent in the other. The high inflation steady state is characterized by both a markup and a relative price distortion that are roughly two percent higher than in the low inflation steady state. These factors combine to make consumption lower by about the same amount in the high inflation steady state.

5 Stochastic equilibria

The generic existence of two point-in-time equilibria and two steady-state equilibria for arbitrary homogeneous policy suggests the existence of discretionary equilibria that involve stochastic fluctuations. We now provide an example of such an equilibrium. We assume that there is an i.i.d. sunspot realized each period which selects between the two private sector equilibria: in each period, the low-p_0 outcome occurs with probability 0.6, the high-p_0 outcome occurs with probability of 0.4 and this is common knowledge.[11]

In order for its maximization problem to be well-defined, the monetary authority must have beliefs about the current and future distribution over private-sector equilibria. Above, these beliefs were degenerate. Now that they are nondegenerate, the problem is slightly more complicated. Letting α be the probability of the low-p_0 outcome (where we denote the low- and high-p_0 outcomes as p_0^L and p_0^H to distinguish them from their steady-state counterparts), the monetary authority maximizes

$$\{\alpha u(c(m,p_0^L),l(m,p_0^L))+(1-\alpha)u(c(m,p_0^H),l(m,p_0^H))\}+\beta v'$$

where v' denotes the future expected utility, which again cannot be influenced by the current monetary authority. It is important to stress that the low and high p_0 values are influenced by the sunspot probabilities, since they satisfy the equations

$$(22)\quad p_0=\frac{1}{m^*}\left[\left(\frac{1}{1+\beta E\pi\left(p_0,p_0'\right)^{\varepsilon-1}}\right)m+\left(\frac{\beta p_0}{1+\beta E\pi\left(p_0,p_0'\right)^{\varepsilon-1}}\right)E\left\{\pi\left(p_0,p_0'\right)^{\varepsilon-1}m'\right\}\right],$$

where expectations are taken over the distribution of the sunspot variable. For example,

$$E\pi\left(p_0,p_0'\right)^{\varepsilon-1}=\alpha\pi\left(p_0,p_0^L\right)^{\varepsilon-1}+(1-\alpha)\,\pi\left(p_0,p_0^H\right)^{\varepsilon-1}.$$

[11] Our model does not pin down the distribution of the sunspot variable. However, some restrictions on that distribution are imposed by the requirement that every firm's profits be nonnegative in each period. For example, if α is 0.75 rather than 0.6, this condition is violated in the low-p_0 state, and no discretionary equilibrium exists. As in Ennis and Keister [forthcoming], it would be interesting to study whether adaptive learning schemes would further restrict the distribution of the sunspot variable.

Because the sunspot is i.i.d., this expression holds for both the low and high current value of p_0. Note that uncertainty prevents us from writing (22) as the simple weighted average that we used with perfect foresight.

5.1 Constructing Discretionary Equilibria

We can again apply the two computational approaches described in the previous section to construct Nash equilibria. In implementing these, we assume that the monetary authority and the private sector share the same probability beliefs.

5.2 Optimal discretionary policy

The relevant trade-offs for the discretionary monetary authority are illustrated in Figure 4. In panel A, there is a heavy dotted line between the objective function for the low-p_0 private-sector equilibrium (the solid line) and the objective function for the high-p_0 private sector equilibrium (the dashed line): this is the monetary authority's expected utility objective, which is a weighted average of the two other objectives. The monetary authority chooses an optimal action that is about 1.0202, which is more stimulative than the earlier equilibrium action (1.01, shown in Figure 3) that was appropriate under extreme optimism ($\alpha = 1$). But it is smaller than the equilibrium action appropriate under extreme pessimism ($\alpha = 0$). Figure 4 also highlights that the specific values taken on by p_0 in the optimistic and pessimistic equilibrium are endogenously determined, by current monetary policy and the sunspot probabilities.[12]

[12] By contrast, in the work by Albanesi, Chari and Christiano [2003] discussed below, the values of endogenous variables are not affected by the probability structure of extrinsic uncertainty.

5.3 Effects of sunspots

Consider now the effects of a sunspot on equilibrium quantities. We take as the reference point the levels in the low-p_0 private-sector equilibrium, which involve a markup of about 1.11 (close to the zero inflation markup) and a normalized price that is close to one. If the economy suddenly shifts to the high-p_0 private sector equilibrium as a result of the sunspot, then firms become much more aggressive in their adjustments. With the nominal money stock fixed ($M_t = mP_{1,t-1}$), there is a decline in real aggregate demand since the price level rises. Consumption and work effort accordingly fall. Alternatively, the average markup rises dramatically, increasing distortions in the economy, to bring about this set of results. Quantitatively, in Figure 4, the rise in the markup is from about 1.12 to about 1.17, so that there is roughly a 4.5 percent increase in the markup. Given that markups and consumption are (inversely) related proportionately, there is a 4.5 percent decline in consumption.

Although the sunspot process has only two states, leading to two sets of point-in-time equilibrium allocations, there are four possible realizations of the inflation rate. From equation (16), the current inflation rate depends on both the current and previous values of p_0 and thus two states for p_0 leads to four realizations of inflation. When the state persists for more than one period, the flavor of our steady state results above carries over, in that inflation is high when p_0 is high. However, transitions from one sunspot state to the other lead to an interesting phenomenon: the transition to the low-p_0 state is accompanied by a higher inflation rate than the transition to a high-p_0 state. Because real allocations are governed entirely by the current state, there is no simple Phillips-curve relationship in a discretionary equilibrium with sunspots.

6 Links to existing monetary policy literature

The study of monetary policy under discretion began with the seminal papers by Kydland and Prescott [1977] and Barro and Gordon [1983], which we will refer to as KPBG, and has included hundreds of subsequent papers. We thus must be highly selective in our discussion of the existing literature. We confine our discussion to three topics: (i) links to the original KPBG analysis; (ii) links to recent work that provides explicit micro foundations for the KPGB analysis; and (iii) links to two recent studies that also find multiplicity under discretion.

6.1 The KPBG analysis

The key prediction of the KPBG model was that there should be inflation bias, specifically that there would be positive inflation under discretion when zero inflation was optimal under commitment. To generate this result, as stressed by Barro and Gordon [1983, p. 593], it is necessary that output is inefficiently low, but can be raised by policies that also produce unexpected inflation. There are costs of actual inflation, so that a discretionary equilibrium exhibits an inflation bias. The specific model that captures these ideas involves a quadratic monetary authority objective and an economic model consisting of linear behavioral equations. There is thus a unique discretionary equilibrium in the standard model (absent reputational effects or trigger strategies).

Our model suggests that there can be more than one constant rate of inflation that is a discretionary equilibrium, so that there can be equilibria with different levels of inflation bias. In addition, as stressed above, it suggests that discretionary policy opens the door to real fluctuations not linked to fundamentals.

6.2 New Keynesian Models

An important recent literature works out how the standard KPBG model can be derived from a fully articulated New Keynesian framework. The key ingredients of the models in this literature are that output is inefficiently low due to monopoly distortions; that the monetary authority has temporary leverage over the real economy because of staggered price setting; and that the costs of actual inflation are welfare losses associated with relative price distortions. Based on these micro foundations, the standard approach in the modern literature on discretionary equilibria – for example, in the well-known work of Clarida, Gali and Gertler [1999] – is to employ a quadratic policy objective and a related linear economic model which contains forward-looking inflation. However, the objective and model are now viewed as a quadratic approximation to the utility function and a linear approximation of the forward-looking economic model, with both approximations around zero inflation as developed in detail by Woodford [2003, chapters 5 and 6].

By contrast, our analysis takes a basic fully articulated New Keynesian model, without linearizing and shows that there are multiple equilibria.[13] Like the rest of the modern literature, our model features costs of stimulative policies – which bring about actual inflation – stemming from relative price distortions across goods. It also features benefits from unexpected stimulative policies, which lower monopoly markups and raise output toward the first best level. Our model is explicitly dynamic, with firms forecasting future inflation when setting nominal prices for two periods.

[13] Much of the New Keynesian literature uses the Calvo assumption of a constant probability of price adjustment. The Calvo assumption implies that a positive fraction of firms charge a price set arbitrarily far in the past. For many purposes this formulation has the advantage of tractability. For our purposes however the Calvo assumption is more complicated than two period staggered pricing, because it would add a real state variable to the monetary authority's problem. As we discuss below, adding a real state variable leaves intact the fundamental mechanism generating multiplicity.

It should be clear from Figure 1 that nonlinearity is central to the multiplicity of point-in-time equilibria we describe: in order to have multiple fixed points to the best response function, it must be nonlinear. We have stressed above that multiple equilibria occur because of complementarity among price-setting firms that is induced by the response of future policy to current prices, taking as given the nonlinear nature of the best response function.

A fairly standard approach to studying dynamic models is to use the nonlinear equations of the model to solve for a steady state and then to use linear approximation to study local dynamics. Applied to our model, this strategy would isolate two steady-state points and then one could study the local dynamics near each. We conjecture that many previous analyses of discretionary equilibrium in New Keynesian models have not uncovered multiple steady states partly because they employed approximations around zero inflation.

Other analyses of discretionary policy in New Keynesian models likely have missed multiplicity of equilibria for a more subtle analytical reason: they have used a "primal approach." That is, they have specified a planner who can choose allocations (thus, prices), subject to those allocations being consistent with private-sector equilibrium. By contrast, the approach taken in this paper is to specify a policymaker who chooses an instrument and must accept whatever equilibria correspond to the instrument setting.

If the policymaker can commit to future actions, the distinction between planning problem and policy problem is immaterial in New Keynesian models. However, absent commitment, the distinction becomes important: the planner's formulation rules out the steady state with lower welfare. To see this, consider a planner in the current period who knows that the future will be characterized by the steady state with lower welfare (higher p_0). It is optimal for the planner to pick allocations that correspond to a low value of p_0 in the current period and thus the low-welfare steady state is not an equilibrium to the planning problem.[14]

[14] Wolman [2001] illustrates the exact discretionary solution to the planner's problem in this model, and Dotsey and Hornstein [2003] solve the discretionary planner's problem of this model using an LQ

By contrast, a *policymaker*– who can only choose m – must respect private agents' beliefs. If agents are pessimistic today and in the future, then the current policymaker chooses an m such that the low-welfare steady state outcome is realized today.

6.3 Recent work on multiplicity under discretion

Our paper is related to recent work by Albanesi, Chari and Christiano [2003] and Dedola [2002], both of which find multiple equilibria under discretionary monetary policy. These analyses share one key element with our analysis, in that a portion of monopolistically competitive firms must set prices before the monetary authority's action in each period, but they differ in the specific nature of this friction. They also share one common element with each other, which is distinct from our analysis, in that the multiplicity depends in a central manner on the interaction of money demand and price-setting.

The modelling framework of Albanesi, Chari and Christiano (henceforth, ACC) is essentially static on the pricing dimension: at the start of each period, a portion of firms set prices before the monetary policy authority's action and a portion set them after it. The stimulative policies that produce inflation in their model also raise nominal interest rates and lead to money demand distortions, either by driving a relative price wedge between the cost of buying goods on cash and credit or by increasing transactions time. The monetary authority thus faces a trade-off between the benefits of driving down the markup and these costs. In our model, instead of the costs of realized inflation being related to money demand, they involve price distortions across goods whose prices were set in different periods.

While ACC find multiple equilibria under discretionary policy, the multiplicity is of a different sort than the multiplicity in our model. We find multiple private sector equilibria, given the policy action. In ACC, equilibrium is unique given the policy action: there are multiple equilibrium policy actions and each one corresponds to different private sec-

approximation. In neither case does multiplicity arise.

tor actions. If there are sunspots which switch the economy between equilibria, there are also important differences in the consequences that are suggested by our model from those suggested by ACC. In our setting, if a high inflation equilibrium occurs when agents attach low probability to such an event, then there will be a decline in output because aggregate demand will fall and the average markup will increase. By contrast, in ACC, a switch from low inflation to high inflation will have little effect on the average markup or output, with the main difference being the extent of money demand distortions.

In our model, beliefs about future outcomes affect the nature of the current policy problem because firms setting their price in the current period care about both current and future monetary policy. By contrast, in ACC, there is no feedback between the likelihood that economic agents attach to future equilibria and the levels of inflation and output at a point in time. Accordingly beliefs about the future are of no bearing for current events.

Dedola [2002] studies discretionary policy in a Rotemberg-style model of pricing and finds multiple equilibria as well. Dedola models money demand using a cash-in-advance constraint and the multiple equilibria are related to the money demand specification (as in ACC). For some parameter choices, Dedola reports multiple steady-state discretionary equilibria consistent with the nonlinear equations of his model. He also reports nonuniqueness of linear approximation dynamics near one of those steady-states. Taken together with this paper, Dedola's results suggest the importance of careful steady state analysis when applying linear approximation methods to the study of discretionary equilibrium.

7 Discretion and multiplicity more generally

Multiple equilibria arise under discretion in our model because of policy-induced complementarity among private agents. The complementarity involves interaction between forward-looking firms and a future policymaker who will respond to the state variable determined by those firms. We will argue here that similar types of complementarity are present more generally when policy is formulated without commitment. For generalizations of our staggered pricing model we know this to be true and thus we speculate on some empirical implications of monetary discretion. We then describe some other contexts in which discretion can lead to multiple equilibria because of policy-induced complementarity among private agents.

7.1 Greater Price Stickiness

Multiplicity of equilibrium under discretion is not an artifact of two-period staggered pricing. The key model element generating multiplicity is the existence of a nominal state variable (here, the nominal price set by firms that adjusted their price in the previous period). With prices set for more than two periods, such a nominal state variable would still exist, but it would be an index of those nominal prices charged in the current period but chosen in previous periods. Furthermore, there would be real state variables, namely ratios of the current period nominal prices chosen by firms in previous periods. Solving for an equilibrium under discretion is more complicated when there is a real state that constrains the monetary authority. In Khan, King and Wolman [2001], we show that multiple equilibria also arise with three-period staggered pricing. However, that analysis is conducted using backward induction on a finite horizon model and we encounter some headaches even with a two-period horizon. There are discontinuities in the monetary authority's policy functions, which makes it computationally difficult – though not impossible – to extend the horizon beyond two periods. For this reason we chose to focus here on the model without a real state variable, where we are able to characterize equilibrium with an infinite horizon. Since much of the sticky price macroeconomic literature has focused on the Calvo stochastic adjustment model, it would be of interest to explore the interaction of discretion and forward-looking price-setting in that context. The Calvo model would also lead to a real state, as well as a nominal one, so that similar complications to those discussed above would likely arise.

7.2 Empirical implications

There are tantalizing empirical implications of the kind of model we have discussed here. First, a model with multiple steady state rates of inflation can potentially explain why monetary policymakers can be caught at a high rate of inflation, in what Chari, Christiano and Eichenbaum [1998] call an "expectations trap." For this reason, models along the lines considered here could potentially explain wide variation in inflation rates across countries or time periods displaying similar structural features.

Second, the effect of sunspots on economic activity that we discussed in section 5.3 above is a situation of "unexpected stagflation" arising because of shifting beliefs. Goodfriend [1993] describes post-war U.S. recessions as arising from "inflation scares," situations in which markets suddenly come to expect higher inflation and a contraction in aggregate demand occurs. The effect that we describe above seems to capture some aspects of this perspective, but it does not involve the increases in long-term expectations of inflation reflected in market interest rates. To consider such effects, which may be important for understanding the interaction of the U.S. central bank with the real economy during the post-war period, one could introduce persistence into the sunspot process determining equilibrium selection.

7.3 Discretionary policy in other contexts

Our emphasis in this paper has been on the link between lack of commitment for monetary policy and multiple equilibria. However, the nature of the mechanism by which lack of commitment leads to multiple equilibria suggests that the phenomenon is more general. Whenever private agents' forecasts of future policy affect an endogenous state variable to which future policy responds, there is the potential for policy-induced complementarity among private agents actions. Just as in our model, even without “structural” complementarity among private agents, discretionary policy can create complementarity and lead to multiple equilibria.

A slight modification of Kydland and Prescott’s [1977] flood control example fits into this framework. Suppose private agents choose among two locations, one of which experiences flooding with positive probability. Agents have idiosyncratic preferences over the two locations. After agents choose locations, a government chooses whether to impose taxes and undertake costly flood control. Plausibly, flood control involves an important element of fixed costs: one must build a dam of a minimum size to control floods. For this reason, it seems plausible as well that an optimizing government would be willing to let a small number of inhabitants be flooded, but will undertake flood control if enough agents move to the flood plain. For this reason, there would be multiple equilibria here of the sort that arise in our model. When a single agent believes that no others will move to the flood plain, she knows that the government will not protect her and, therefore, chooses not to locate in the flood plain. When the agent believes that many others will move to the flood plain, she knows that the government will protect her and, therefore, she chooses to live in the flood plain. Thus, there is complementarity in agents’ location decisions. That complementarity is not intrinsic, but is induced by the fact that location decisions determine a state (population in the floodplain) to which the future policymaker responds in a discretionary manner. By contrast, if the policymaker could commit in advance to its action, there would be a unique equilibrium.

Another example of lack of commitment leading to multiple equilibria comes from Glomm and Ravikumar’s [1995] model of public expenditure on education. In their OLG model, young agents choose how much time to devote to learning, given their expectation of the income tax rate in the next period. Individual young agents’ decisions in the current period determine next period’s individual and aggregate stock of human capital. In turn, next period's government chooses the optimal tax rate as a function of the aggregate stock of human capital, in order to fund valued public education. Thus, the future tax rate effectively responds to expectations of the future tax rate. Or, equivalently, an individual agent's current decision about human capital accumulation depends on aggregate decisions because the aggregate determines the future state (capital) to which policy will respond. For certain parameterizations the policy response induces sufficient complementarity among private agents' decisions that there are multiple equilibria. Again, with commitment the multiplicity disappears.

8 Summary and conclusions

We have described equilibria under discretionary monetary policy in a basic New-Keynesian model with two-period staggered price setting. The trade-off that our monetary authority faces is a familiar one. Output is inefficiently low because firms have monopoly power. This creates an incentive for the monetary authority to provide unexpected stimulus, exploiting the pre-set prices and raising output. However, when it exploits preset prices, the monetary authority also raises the dispersion of prices, leading to an inefficient allocation of resources. In equilibrium, the monetary authority is balancing the marginal contribution of these two effects.

While the monetary policy trade-off is familiar, the nature of equilibrium is not. Discretionary monetary policy leads to multiple equilibria. The multiplicity occurs because of complementarity in pricing behavior that is induced by the monetary authority's natural tendency to treat the level of pre-set nominal prices as a bygone. Under discretion, the monetary authority moves the nominal money supply proportionately with the nominal level of pre-set prices. This feature of monetary policy means that higher prices set by firms in the current period will lead to a higher money supply (and even higher prices) in the subsequent period. Understanding this mechanism, an individual firm adjusting its price in the current period finds it optimal to raise its price in response to higher prices set by other firms. There is complementarity in pricing and it leads to multiple equilibria.

When we consider discretionary equilibria that are driven by a sunspot variable, the equilibria involve random fluctuations between different real outcomes.[15] If all firms choose to raise prices by a large amount because they (rationally) believe that others are raising prices, the result is a reduction in real aggregate demand and a decline in output relative to the level that would prevail if smaller price adjustments took place. Economic volatility then, as well as high inflation, may be a cost of discretion in monetary policy.

The mechanism leading to complementarity and multiple equilibria here transcends our example of monetary policy in a staggered pricing model. Other environments which share two features have the potential to generate similar results. First, private agents must be forward-looking and their actions must be influenced by their expectations about future policy. Second, private agents' actions must determine a state variable to which future policy responds. These features seem quite widespread, suggesting that lack of commitment may be an important cause of economic instability.

[15] The distribution of the sunspot variable shifts the equilibrium, and while we do not pin down that distribution, it is an integral part of the definition of equilibrium. Thus far, we have only considered i.i.d. sunspot variations, so as to produce the simplest possible explanation of the source and nature of multiple equilibria. In future work, we plan to extend the analysis to the implications of persistent sunspots. This extension would allow us to take the model more seriously as a potential explanation for some of the volatility observed in actual macroeconomic time series.

A Appendix: Proof of Proposition 1

Proposition 1. Given $p_{0,t+1} > 0$ and $m_{t+1} > m^*$,

(a) if $m_{t+1} < m^* \cdot \left(1 + \frac{2}{\beta}\left(g\left(p_{0,t+1}\right)\right)^{1-\varepsilon}\right)$, then there exists $\breve{m} > 0$ such that for $m_t < \breve{m}$ there are two equilibria in period t and for $m_t > \breve{m}$ no equilibrium exists in period t;

(b) if $m_{t+1} \geq m^* \cdot \left(1 + \frac{2}{\beta}\left(g\left(p_{0,t+1}\right)\right)^{1-\varepsilon}\right)$, then no equilibrium exists in period t for any m_t.

Proof. We eliminate time subscripts and use superscript prime to denote $t+1$ variables. From (19), point-in-time equilibrium values of p_0 are solutions to

$$m^* - \frac{m}{p_0} = \theta\left(p_0, p_0'\right) \cdot \left(m' - \frac{m}{p_0}\right). \tag{23}$$

This follows from multiplying both sides of (19) by m^*/p_0. Subtracting $\theta\left(p_0, p_0'\right) \cdot \left(m^* - \frac{m}{p_0}\right)$ from both sides of (23), we have

$$\left(1 - \theta\left(p_0, p_0'\right)\right) \cdot \left(m^* - \frac{m}{p_0}\right) = \theta\left(p_0, p_0'\right) \cdot \left(m' - m^*\right), \tag{24}$$

and thus

$$\left(m^* - \frac{m}{p_0}\right) = \frac{\theta\left(p_0, p_0'\right)}{\left(1 - \theta\left(p_0, p_0'\right)\right)} \cdot \left(m' - m^*\right). \tag{25}$$

Using the definitions of $\theta()$, $\pi()$ and $g()$ in (20), (16) and (15), it follows that

$$\left(m^* - \frac{m}{p_0}\right) = \beta\left(g\left(p_0'\right)\right)^{\varepsilon-1}\left(\frac{1}{2} + \frac{1}{2}p_0^{\varepsilon-1}\right)\left(m' - m^*\right). \tag{26}$$

Finally, multiplying (26) by p_0/m^* yields the expression whose properties will allow us to complete the proof:

$$p_0 = \frac{m}{m^*} + \beta\left(g\left(p_0'\right)\right)^{\varepsilon-1}\left(\frac{m'}{m^*} - 1\right)\left(\frac{1}{2}\right)\left(p_0 + p_0^{\varepsilon}\right), \tag{27}$$

for fixed $m' > m^*$ and fixed $p_0' > 0$.

In (23) through (27) we have simply manipulated condition (19), which implicitly defines the equilibrium values of p_0. Thus, characterizing the positive fixed points of the right hand side of (27) will allow us to characterize the equilibrium values of p_0. Denote the right hand side of (27) by $h(p_0)$. That is,

$$h(p_0) \equiv \frac{m}{m^*} + \beta \left(g\left(p_0'\right)\right)^{\varepsilon-1} \left(\frac{m'}{m^*} - 1\right) \left(\frac{1}{2}\right) \left(p_0 + p_0^{\varepsilon}\right). \tag{28}$$

(i) Since $\varepsilon > 1$ and $m' > m^*$, $h(p_0)$ is strictly increasing and strictly convex.

(ii) $h(0) = m/m^* > 0$.

(iii) From (i) and (ii), there are typically either two or zero positive fixed points of $h()$. In a knife edge case where $h()$ is tangent to the 45-degree line there is a unique positive fixed point. A unique positive fixed point accompanied by a negative fixed point is ruled out by (i) and (ii).

(iv) Define p_0^s to be the value of p_0 for which $h'(p_0) = 1$. That is,

$$1 = \beta \left(g\left(p_0'\right)\right)^{\varepsilon-1} \left(\frac{m'}{m^*} - 1\right) \left(\frac{1}{2}\right) \left(1 + \varepsilon \left(p_0^s\right)^{\varepsilon-1}\right). \tag{29}$$

We can manipulate (29) to express p_0^s explicitly as

$$p_0^s = \left\{ \frac{1}{\varepsilon} \left[\frac{1}{\beta \left(g\left(p_0'\right)\right)^{\varepsilon-1} \left(\frac{m'}{m^*} - 1\right)\left(\frac{1}{2}\right)} - 1 \right] \right\}^{1/(\varepsilon-1)}. \tag{30}$$

Note that p_0^s is independent of m.

(v) If $\beta \left(g\left(p_0'\right)\right)^{\varepsilon-1} \left(\frac{m'}{m^*} - 1\right)\left(\frac{1}{2}\right) \geq 1$, then $h'(0) \geq 1$. In this case $h(p_0) > p_0$ for all $p_0 > 0$, completing the proof of part (b).

(vi) Now assume $\beta \left(g\left(p_0'\right)\right)^{\varepsilon-1} \left(\frac{m'}{m^*} - 1\right)\left(\frac{1}{2}\right) < 1$. Let $\breve{m}$ denote the value of m such that $h(p_0^s; \breve{m}) = p_0^s$. That is, when $m = \breve{m}$, there is a tangency between $h()$ and the 45-degree line and thus a unique equilibrium value of $p_0 = p_0^s$. To complete the proof of part (a), we need to show that $\breve{m}$ exists and is unique. Using (29) and (28) together with the definition of $\breve{m}$, we have

$$p_0^s = \frac{\breve{m}}{m^*} + \frac{p_0^s + \left(p_0^s\right)^{\varepsilon}}{1 + \varepsilon \left(p_0^s\right)^{\varepsilon-1}}.$$

After multiplying both sides by $1 + \varepsilon \left(p_0^s\right)^{\varepsilon-1}$, we can solve explicitly for $\breve{m}$ as

$$\breve{m} = m^* \cdot \frac{(\varepsilon - 1)\left(p_0^s\right)^{\varepsilon}}{1 + \varepsilon \left(p_0^s\right)^{\varepsilon-1}};$$

$\breve{m}$ exists and is unique. When $m = \breve{m}$ there is a unique equilibrium (positive fixed point of $h()$), with $h()$ tangent to the 45-degree line at $p_0 = p_0^s$. Because $h()$ is strictly increasing and strictly convex, $h(p_0^s) < p_0^s$ when $m < \breve{m}$, implying two equilibria and $h(p_0^s) > p_0^s$ when $m > \breve{m}$, implying $h(p_0) > p_0$ for all $p_0 > 0$ and thus nonexistence of equilibrium. This completes the proof of part (a). ■

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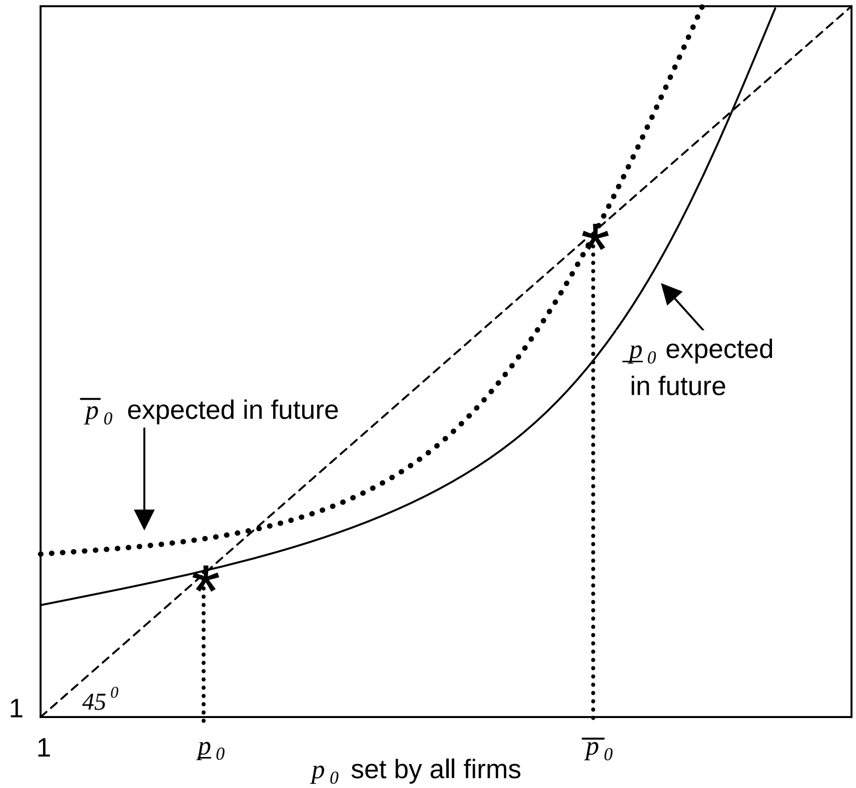

Figure I. Point-in-Time Best-Response Functions
for $m = m^{/} > m^*$

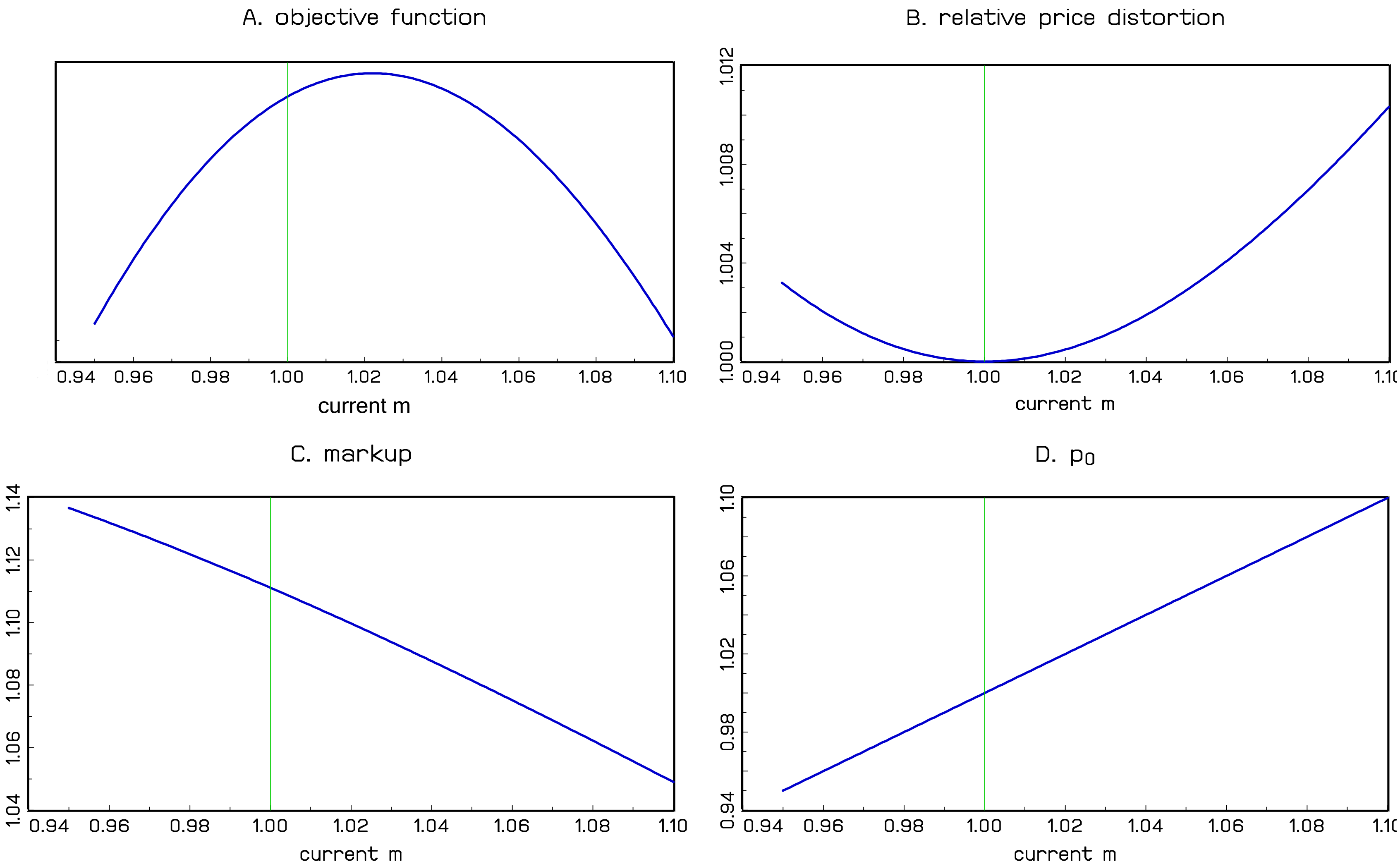

Figure II. The Temptation to Stimulate a Zero-Inflation Economy
(m'=1.0)

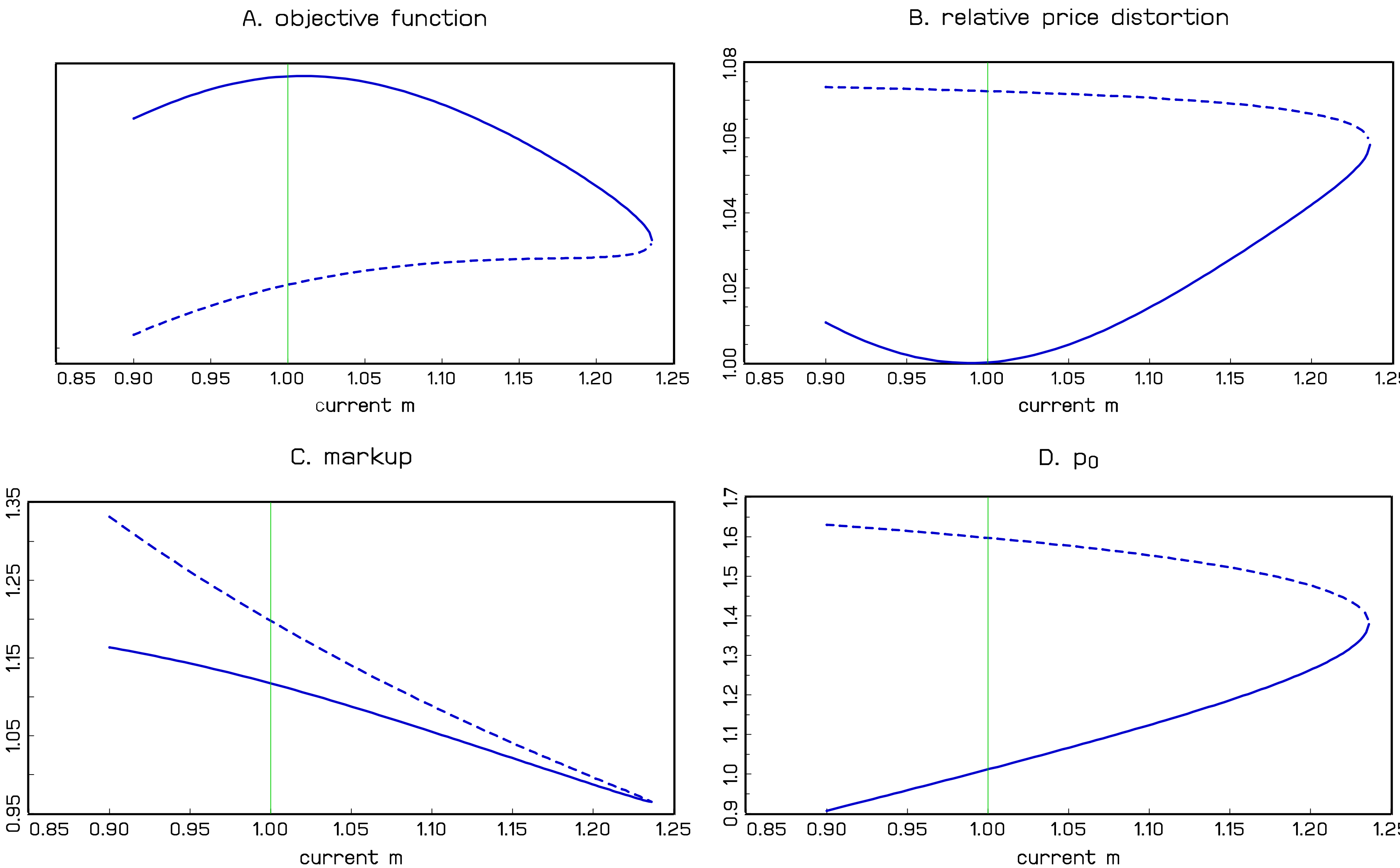

Figure III. Discretionary Equilibrium with Optimism (p_0 expectations)
Equilibrium *m=1.010*

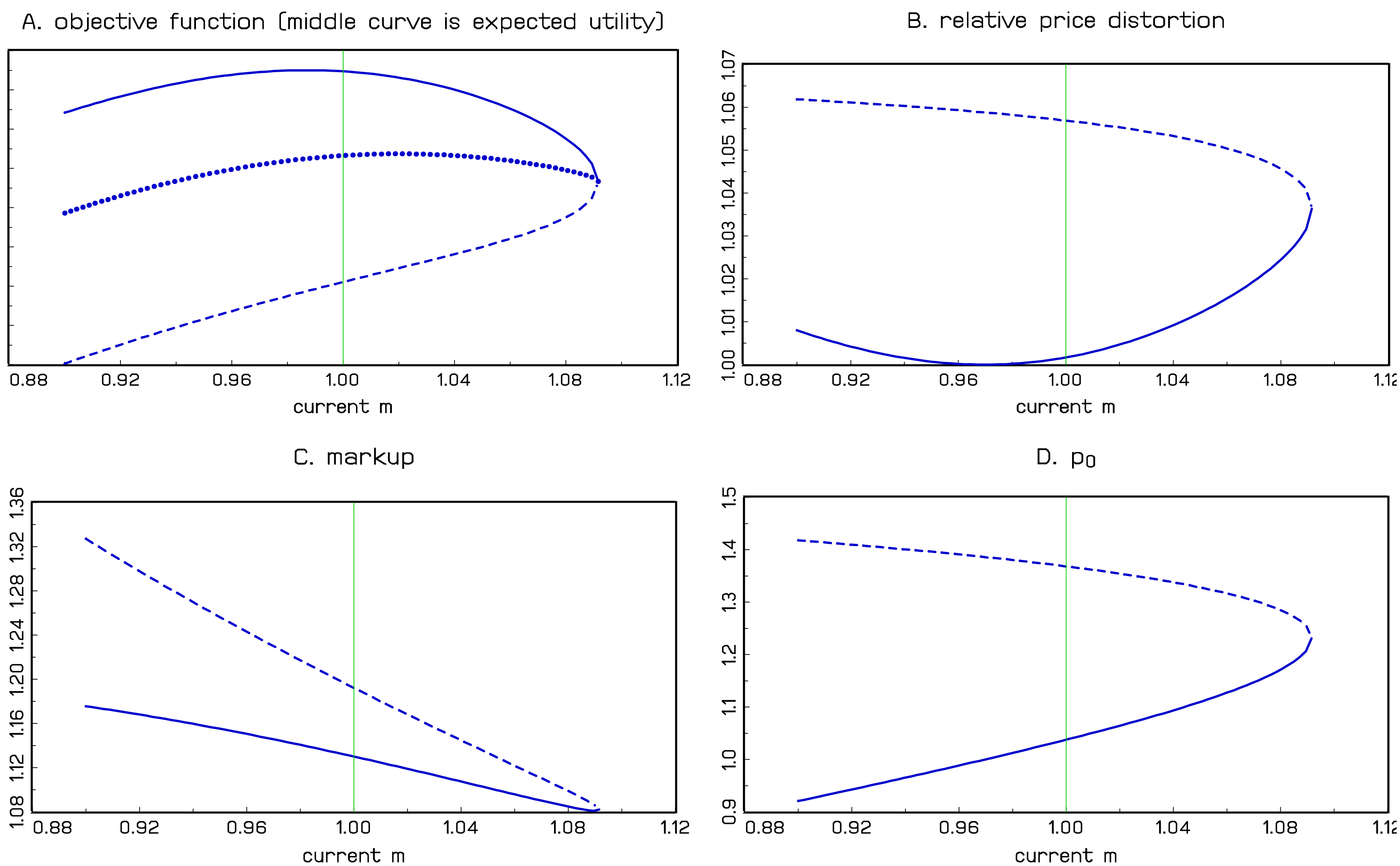

Figure IV. Discretionary Equilibrium with Prob(p_0 is low)=*0.6*
Equilibrium *m=1.020156*