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Abstract

Does capital-embodied technological change play an important role in shaping labor market outcomes? To address this question, we develop a model with vintage capital and search-matching frictions where irreversible investment in new vintages of capital creates heterogeneity in productivity among firms, matched as well as vacant. We demonstrate that capital-embodied technological change reduces labor demand and raises equilibrium unemployment and unemployment durations. In addition, the presence of labor market regulation—we analyze unemployment benefits, payroll and income taxes, and firing costs—exacerbates these effects. Thus, the model is qualitatively consistent with some key features of the European labor market experience, relative to that of the United States: it features a sharper rise in unemployment and a sharper fall in the vacancy rate and the labor share. A calibrated version of our model suggests that this technology-policy interaction could explain a sizeable fraction of the observed differences between the United States and Europe.

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1 Introduction

Over the past three decades, the nature of growth and productivity advancement has gone through a profound transformation: technological innovations *embodied* in new and more productive capital goods—especially in information and communication equipment and software—have represented the major source of output growth in the United States and Europe (Jorgenson, 2001; Colecchia and Schreyer, 2002). Productivity growth embodied in new vintages of capital has accelerated significantly over the past 30 years, from 2 percent per year in the 1960s to 4.5 percent in the 1990s (Gordon, 1990; Cummins and Violante, 2002). As a result, the emphasis in growth accounting exercises has moved from factor-neutral productivity to "investment-specific productivity" (Greenwood, Hercowitz and Krusell, 1997).

How does technological change, especially in this form, influence labor-market outcomes? An extensive literature argues that this episode of capital-embodied technical change has a "skill-biased" character, whereby the productivity of high-skilled workers has risen in relative terms with the arrival of new technologies (e.g., Katz and Murphy, 1992; Krusell et al., 2000). In this paper, we examine the hypothesis that new technology may have consequences that go far beyond changes in relative productivities of workers: in the context of a *labor market with frictions*, a capital-embodied technology acceleration may reduce firms' incentives to create new jobs, increase unemployment, and reduce the labor share. We particularly look at how labor-market policy interacts with technological change in shaping labor-market outcomes.

We formulate a Diamond-Mortensen-Pissarides-style matching model—now a standard way of formalizing labor-market frictions—with an aggregate matching function and wages set by Nash bargaining. We assume that workers are identical, but that there is firm heterogeneity induced by technological change embodied in new vintages of capital. We introduce three policy variables that feature prominently in actual labor markets: welfare benefits, taxes, and employment protection. The model features a stark *technology-policy interaction*: the impact of an acceleration in capital-embodied productivity on unemployment depends on the size of the labor-market policy variables. More rigid institutions (more generous benefits, higher taxes and larger firing costs) exacerbate the long-run creative-destruction effect of a capital-embodied productivity shock on unemployment, particularly along the unemployment duration margin. That is, an acceleration in capital-embodied productivity growth reduces *labor demand* more in economies with rigid institutions. Interestingly, the marginal effect of each institution is increasing in the magnitude of the others: the policy bundle we examine has a much stronger impact than the sum of the impacts of the three policies taken individually.

We study if the common acceleration of capital-embodied technological change, together

with differences in labor market institutions, can account for the different labor market outcomes in Europe and the United States since the 1960s. Krugman (1994) was the first to propose the technology-institutions perspective for studying the differential rise in unemployment between the United States and Europe. Whereas the U.S. unemployment rate did not change much, it was about 5 percent in the 1960s and about 6 percent in the 1990s, Europe's unemployment rate increased substantially, from 2 percent in the 1960s to 10 percent in the 1990s (see Figure 1A). But labor-market outcomes in Europe and the United States have also differed systematically in dimensions other than the unemployment rate. The worse unemployment performance of Europe was accompanied by a relative decline of both the labor share and the vacancy-employment rate (see Figures 1B and 1C). In the United States, the labor share shows only a modest decline since 1960 (around 3 percentage points), while in Europe its decline is twofold.[1] The vacancy-employment ratio remained roughly constant in the United States, while it declined by 40 percent in Europe. These differences in the behavior of job creation rates and labor-income shares suggest that the economic forces at work in the European and U.S. labor markets operate, to a large extent, through the labor demand channel.

The mechanism we propose is qualitatively consistent with these facts. Moreover, in a quantitative exploration, we show that a calibrated version of our model can account for a sizeable fraction of the rise in European unemployment, and the surge in unemployment is entirely due to longer durations, as seen in the data. In addition, the model generates a decline in the labor share and in the vacancy rates of a magnitude similar to the data.

Our analysis focuses on changes in labor demand and complements the "labor supply" view of European unemployment. The supply view conjectures that the technological shock accelerated the depreciation of human capital, or increased the importance of "mismatch," and in response to these shocks European workers chose unemployment over employment due to the generosity of the welfare state (as, for example, in Ljungqvist and Sargent, 1998; see also Marimon and Zilibotti, 1999).

Research on capital-embodied productivity growth in the context of frictional labor markets was pioneered by Aghion and Howitt (1994) and Mortensen and Pissarides (1998). Their setup has become the "industry standard" in all subsequent work.[2] However, this standard search-vintage model displays one stark, and clearly unrealistic, feature that so far has gone unnoticed: even though firms in operation (i.e., those matched with a worker) are technologically heterogeneous, job vacancies are all alike—all vacant firms have the best technology in

[1] See Appendix A.1 for details on the data construction for Figure 1.

[2] The literature is vast. Examples include Caballero and Hammour (1998), Cohen and Saint-Paul (1994), den Haan, Ramey and Watson (2000), Lopez-Salido and Michelacci (2004), Pissarides and Vallanti (2004), and Postel-Vinay (2002).

place. The degenerate vintage distribution of vacant firms is the result of assumptions made in the name of tractability. Mortensen and Pissarides (1998) assume that vacant firms purchase capital only after meeting the worker, thus eliminating the possibility that newly created firms would remain, unmatched, in the labor market as their capital depreciates. Aghion and Howitt (1994) assume that firms make their investment decision before searching for a suitable worker, but they also assume that all worker-firm meetings occur after a deterministically fixed interval; thus, when vacancies meet workers, they are all homogeneous. More recently, Lopez-Salido and Michelacci (2004), and Pissarides and Vallanti (2004) assume that vacant firms can freely adopt the best available technology, i.e., producing with the leading-edge machine does not require a costly purchase of capital, but merely a flow cost of posting a vacancy to search in the labor market.[3]

Our framework builds on the standard search-vintage model, but it is significantly richer. In our setup, new firms purchase new capital upon entry and then proceed to search for workers in the frictional labor market. Because the investment cost is *sunk* at the search stage, not only matched but also vacant firms are fundamentally heterogeneous: they may be firms with new capital that just entered and that did not yet find workers, or they may be older, previously matched firms with old capital that were hit by (exogenous) separation shocks.

Moreover, the standard search model with vintage capital features a second unappealing theoretical property: the existence of a non-degenerate vintage distribution of matched capital in equilibrium is not "genuine": it is solely the result of the matching friction, i.e., of the fact that finding a worker to fill the vacancy is costly and takes time. As frictions disappear, the vintage structure collapses to a degenerate distribution where all operating firms use the latest technology. Our model, instead, is a natural extension of the competitive irreversible vintage capital model (Jovanovic, 1998) to an economic environment with matching frictions. Thus, the vintage structure survives even when frictions vanish: as we raise the efficiency of the matching process—a parameter in the matching function—wages at different firms converge toward the competitive wage, the lifespan of capital decreases toward its competitive non-zero value, and the economy reaches full employment.

Both the nontrivial distribution of vacancies across vintages and the fact that, in equilibrium, vacant firms do not have constant (i.e., zero) value make the analysis more challenging.

[3]To avoid vacancy heterogeneity both Aghion and Howitt (1994) and Mortensen and Pissarides (1998) must also deal with separations that occur before old machines are scrapped when the match surplus is zero. Mortensen and Pissarides (1998) allow for exogenous separations, but the machine is destroyed during separation. Aghion and Howitt (1994) do not have exogenous separations, and they only consider equilibria where the useful economic life time of a machine is shorter than the time it takes a worker to find a new machine. The somewhat more extreme assumption in Lopez-Salido and Michelacci (2004) and Pissarides and Vallanti (2004) allows them to incorporate workers' quits into the analysis.

Nevertheless, the model remains tractable, and we present closed-form solutions for the vacancy and employment distributions. We identify parametric conditions for the existence and uniqueness of a stationary equilibrium, and we show that the equilibrium of the economy can still be represented in a two-dimensional diagram. That is, as in standard matching-model analysis, we identify a "job creation curve" and a "job destruction curve" in the space defined by the scrapping age of capital and labor-market tightness. We also study analytically how a higher rate of capital-embodied technological change influences the steady-state labor-market equilibrium. Tractability is maintained when a set of common labor-market policies—unemployment benefits, payroll and income taxes, and firing costs—are considered. We examine both how each of these policies influences labor market outcomes by themselves and how the presence of policy—especially the combination of different policies—interacts with technological change in influencing the labor market.

Our view on how new technology enters the economy coincides witha that of Aghion and Howitt (1994): technical change is implemented through "creative-destruction." Since existing matches cannot adopt the newest vintage, matches with obsolete capital have to be dissolved, and workers have to transit through an intermittent unemployment stage. Thus, a faster rate of embodied technical change requires more frequent reallocation of labor, that is, it increases the unemployment rate. Mortensen and Pissarides (1998) propose an alternative version of the search-vintage model where existing matches are allowed to upgrade their machine without destroying the match.[4] Even though technical change remains embodied in capital in the economy with upgrading, technical change now has more the appearance of "disembodied" technical change. This feature is most obvious if one considers the limiting case, when the cost of upgrading becomes arbitrarily small. A faster rate of embodied technical change then implies higher future productivity of machines, and it raises the capital value of new vintages. In equilibrium, faster technical change then raises the entry rate of new vintages and reduces the unemployment rate. Qualitatively, the distinction between "creative destruction" and "upgrading" is important because it *can* lead to opposite comparative statics of technological change on equilibrium unemployment. In Hornstein, Krusell and Violante (2005a) we have performed a quantitative, numerically executed, robustness exercise with regard to this assumption within the context of the baseline version of the model in this paper, i.e., the model without policy. The findings there are reassuring: when the model is calibrated to the U.S. labor market, the long-run comparative static results for the key labor-market variables are almost identical with and without upgrading.

[4]Lopez-Salido and Michelacci (2004) and Pissarides and Vallanti (2004) have also studied versions of the search-vintage model with upgrading.

The remainder of the paper is organized as follows. In Section 2 we outline the economic environment and solve the model. In Section 3 we prove the existence and uniqueness of equilibrium. In Section 4 we introduce the policy variables and study the technology-policy interaction present in our model. First, we characterize the qualitative comparative statics with respect to technology and policy parameters; second, we calibrate the model and explore the quantitative importance of our economic mechanism in explaining the U.S.-Europe differential. Section 5 concludes the paper.

2 The Economy

Demographics and preferences– Time is continuous. The economy is populated by a measure one of ex-ante equal, infinitely lived workers. The workers are risk-neutral and discount the future at rate r. Unemployed workers derive flow utility $\ell(t) > 0$ from leisure/home-production. Employed workers supply one unit of labor inelastically.

Technology– Production requires pairing one machine and one worker and yields a homogeneous output good. There are both capital-embodied and disembodied productivity improvements. The level of disembodied technology $z(t)$ grows at rate ψ, while the amount of efficiency units embodied in new machines grows at rate γ. At time t, a production unit of age a has $k(t,a)$ efficiency units of capital and, once paired with a worker, produces output

$$y(t,a) = z(t)\,k(t,a)^{\omega} = z_0 e^{\psi t}\left[k_0 e^{\gamma(t-a)} e^{-\delta a}\right]^{\omega}, \tag{1}$$

where $\omega > 0$. In what follows we set, without loss of generality, $z_0 = k_0 = 1$.

At any t, there is an infinite supply of potential entrant firms. Entry requires the purchase of a machine embodying the technology of vintage t at the cost Ie^{gt}. Once capital is installed, it becomes fully firm-specific, i.e., the investment is irreversible, and its physical depreciation rate is δ. At this point the firm is ready to match with a worker and begin production.

Rendering the growth model stationary– We focus on the steady state of the normalized economy; this corresponds to a balanced growth path where the actual economy grows at rate $g \equiv \psi + \omega\gamma$. To make the model stationary, we normalize all variables dividing by the term e^{gt} (see Appendix B). The normalized cost of a new production unit is I, and the normalized output of a production unit of age a is $e^{-\phi a}$, where $\phi \equiv \omega(\gamma + \delta)$; thus, output is defined relative to the newest production unit. The parameter ϕ represents the effective depreciation rate of capital obtained as the sum of physical depreciation δ and technological obsolescence γ. Finally, for the labor market to be "viable," we require that the normalized flow value of leisure ℓ be strictly less than output produced on the frontier machine:

A0 [viability of labor market]: $\ell < 1$

Competitive benchmark– In a frictionless labor market, there is no unemployment and, in steady state, there is a unique constant market-clearing wage w. Consider a price-taker firm that sets up a new vintage machine.[5] The firm optimally chooses the exit age $\bar{a}$ that maximizes the present value of lifetime profits

$$\Pi(w) = \max_{\bar{a}} \int_0^{\bar{a}} e^{-(r-g)a} \left(e^{-\phi a} - w\right) da,$$

where $\Pi(w)$ is the profit function. Profit maximization leads to the condition $w = e^{-\phi \bar{a}}$, stating that the rental price of labor services equals the productivity of the oldest machine, which is also the marginal productivity of labor. Free entry of firms implies $I = \Pi(w)$. This condition determines the exit age $\bar{a}$, and pins down the equilibrium wage. Using the profit-maximization condition $w = e^{-\phi \bar{a}}$, the free entry condition becomes

$$I = \int_0^{\bar{a}} e^{-\bar{r}a} \left[1 - e^{-\phi(\bar{a}-a)}\right] da. \tag{2}$$

For notational ease, we have defined $\bar{r} \equiv r - g + \phi = r - \psi + \omega\delta$, which must satisfy

A1 [recoverability of the investment cost]: $\bar{r}I < 1$.

This condition is natural: unless one can recover the initial capital investment I at zero wages over an infinite lifetime, with $\int_0^\infty e^{-\bar{r}a} da = 1/\bar{r}$ being the net profit from such an operation, it is not profitable to ever start any firm. We are now ready to state the following:

Proposition 1 [frictionless equilibrium]: *(a) There exists a unique stationary competitive equilibrium with production, if and only if A0-A1 hold, and the* exit age satisfies $\bar{a}^{CE} \leq \bar{a}^{\max} = -\ln(l)/\phi$. *(b) The stationary equilibrium is socially efficient.*

The Proof is in Appendix C. The logic of the proof of *(a)* is simple: since flow profits are monotonically declining in age relative to the new production unit—because of depreciation and obsolescence—while the labor cost is the same for all vintages, there is a unique exit age. Furthermore, the wage at the exit age has to exceed the value of leisure, that is the exit age cannot be too big. As usual, proving efficiency of the equilibrium stated in part *(b)* requires showing the equivalence between the marginal conditions for the planner and those for the firms, in the decentralized economy. We now turn to the frictional case.

[5] Profit-maximizing firms always choose the newest capital vintage. The key behind this argument is that the labor required to operate new machines is constant over time, which is why new technologies are better; in fact, technological change allows firms to pair their worker with more and more efficiency units of capital over time by using newer and newer equipment. A firm choosing to invest in old capital would, once in operation, generate lower output at the same wage cost. The lower initial installation cost of the old machine would compensate these losses only partially. This argument is easy to verify mathematically, so we omit its proof in the text.

Frictional labor market and vacancy heterogeneity– Now suppose that the labor market is frictional, in the sense of Pissarides (2000): an aggregate matching function governs job creation. Searching is costless: it only takes time. At the exogenous rate σ existing matches dissolve, and both worker and firm join the pool of searchers.[6]

In this environment vacant firms are heterogeneous: their capital is of different vintages. The source of heterogeneity is twofold. Newly created firms stay idle until they find a worker; exogenously separating firms will also become idle. What drives vacancy heterogeneity is that the capital expense of creating a new firm is sunk, and there is no additional cost to search for vacant firms. Thus, rather than emphasizing firms' costs of "posting vacancies," as in the standard matching model, we stress the up-front, irreversible costs of investing in equipment, as in the typical vintage capital model.

Random matching– The meeting process between workers and production units is random and takes place in one pool comprising all unemployed workers (i.e., there is no on-the-job search) and all vacant firms. Workers cannot direct their search since they do not observe firms' age. We assume that the number of matches is determined by a matching function $m(v,u)$ with constant returns to scale, where $v \equiv \int_0^\infty v(a)da$ is the total number of vacancies, $v(a)$ denotes the measure of vacant firms of age a, and u is the total number of unemployed workers. We also assume that $m(v,u)$ is strictly increasing in both arguments and satisfies some regularity conditions.[7]

A firm meets a worker at rate λ_f. The rate at which a worker meets a firm with capital of age a is $\lambda_w(a)$ and the unconditional rate at which she meets any firm is $\lambda_w \equiv \int_0^\infty \lambda_w(a)da$. Using $\theta \equiv v/u$ to denote labor market tightness, we have

$$\lambda_f = \frac{m(\theta,1)}{\theta}, \text{ and} \tag{3}$$

$$\lambda_w(a) = m(\theta,1)\frac{v(a)}{v}. \tag{4}$$

The expression for the meeting probability in (3) provides a one-to-one (strictly decreasing) mapping between λ_f and θ. In the following changes in λ_f stand in for changes in θ.

Surplus sharing– In the presence of frictions with transferable utility, the pair faces the bilateral monopoly problem of how to share the quasi-rents generated by the match. Let the values for workers matched with vintage a firms be $W(a)$ and $J(a)$, and let the values for idle

[6] We omitted separations from the description of the competitive equilibrium because, without frictions, it is immaterial whether the match dissolves exogenously or not as the worker can be replaced instantaneously by the firm at no cost.

[7] In particular, $m(0,u) = m(v,0) = 0$, $\lim_{u\to\infty} m_u(v,u) = \lim_{v\to\infty} m_v(v,u) = 0$, and $\lim_{u\to 0} m_u(v,u) = \lim_{v\to 0} m_v(v,u) = +\infty$.

workers and vintage a firms be U and $V(a)$. The total expected joint surplus of a match, from age a onward, is

$$S(a) \equiv J(a) + W(a) - V(a) - U. \tag{5}$$

The surplus is divided between the parties by the wage rule. As is standard in the literature, we choose the Nash bargaining solution for wages. Given risk-neutrality of firms and workers, the wage is determined such that, at every instant, a fraction β of the total surplus $S(a)$ of type a match goes to the worker and a fraction $(1-\beta)$ goes to the firm, implying

$$W(a) = U + \beta S(a) \text{ and } J(a) = V(a) + (1-\beta)S(a), \tag{6}$$

so that each party always obtains at least his or her threat point (or outside option).

Values– Let $w(a)$ be the wage paid by an age a firm. Under the Nash bargaining solution, every decision is jointly taken, and hence privately efficient. Therefore, flow values for market participants solve the following differential equation system:

$$\begin{aligned}
(r-g)V(a) &= \max\left\{\lambda_f\left[J(a)-V(a)\right] + V'(a), 0\right\}, & (7)\\
(r-g)J(a) &= \max\left\{\left[e^{-\phi a} - w(a)\right] - \sigma\left[J(a)-V(a)\right] + J'(a), (r-g)V(a)\right\}, & (8)\\
(r-g)U &= \ell + \int_0^\infty \lambda_w(a)\left[W(a)-U\right]da, \text{ and} & (9)\\
(r-g)W(a) &= \max\left\{w(a) - \sigma\left[W(a)-U\right] + W'(a), (r-g)U\right\}. & (10)
\end{aligned}$$

The derivatives of the value functions with respect to a will be negative and represent flow losses due to the aging of capital.

Two remarks are in order. First, the lower bound on the vacancy value in equation (7) reflects the irreversibility assumption: the resale price of installed capital is zero. Second, equation (8) illustrates an implicit "technological" restriction in the economy: in order to produce with new capital, a firm with an old machine must separate from its worker, install the new capital, and return on the labor market in search for a new hire. In other words, reallocation of labor from less to more productive capital requires an intervening unemployment spell.[8]

[8] Mortensen and Pissarides (1998), Lopez-Salido and Michelacci (2004), and Pissarides and Vallanti (2004), among others, allow existing matches to upgrade with some probability, or at some cost. In Hornstein, Krusell and Violante (2005a) we conclude that allowing costly upgrading during the match in calibrated versions of this class of models does not change the main *long-run* comparative statics, quantitatively. We explain this finding in light of the fact that, when the model is calibrated to the U.S. economy, vacancy durations are short—around one month according to JOLTS data—so that the matching frictions from the point of view of the firm are, on average, quantitatively minor. Hence, a retooling firm in an economy where upgrading during the match is not allowed faces only a negligible additional hiring cost compared to such a firm being in an economy with upgrading. This, clearly, does not mean that the short- or medium-run dynamics are unaffected: Lopez-Salido and Michelacci (2004) argue precisely that the short-run response of the economy with upgrading is very different from that of an economy with creative-destruction.

2.1 Solving the matching model

We now show that we can characterize the equilibrium of the matching model in terms of two variables: the exit age at which matched firms are scrapped, $\bar{a}$, and the rate at which vacant firms meet workers, λ_f. These two variables are jointly determined by two equilibrium conditions: a free-entry condition for vacancies and the joint match dissolution decision. Two useful steps of the characterization will be, first, establishing that the scrapping age for a vacancy, $\hat{a}$, equals the exit age for a matched firm, $\bar{a}$; second, showing that the equilibrium vintage distribution of vacant and matched machines has a closed-form expression.

2.1.1 The surplus function

In this class of models, all joint decisions are surplus maximizing. Thus, it is useful to start by stating the (flow version of the) surplus equation

$$(r-g)S(a) = \max\{e^{-\phi a} - \sigma S(a) - \lambda_f(1-\beta)S(a) - (r-g)U + S'(a), 0\}. \tag{11}$$

This asset-pricing-like equation is obtained by combining equations (5)-(8) and (10): the growth-adjusted return on the surplus on the left-hand side equals the flow gain on the right-hand side, where the flow gain is the maximum of zero—the resale value of capital—and the difference between inside and outside flow values. The inside value includes a production flow, $e^{-\phi a}$, a flow loss due to the probability of separation, $\sigma S(a)$, and changes in the values for the matched parties due to obsolescence, $J'(a) + W'(a)$. The outside option flows are the expected flow gain from finding a worker for a vacancy, $\lambda_f(1-\beta)S(a)$, the change in the value for the vacant firm, $V'(a)$, and the flow value of unemployment, $(r-g)U$.

The solution of the first-order linear differential equation (11) is the function

$$S(a) = \int_a^{\overline{a}} e^{-(r-g+\sigma+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - (r-g)U\right] d\tilde{a}, \tag{12}$$

where we have used the boundary condition, $S(\bar{a}) = 0$, associated with the fact that at age $\bar{a}$ the match is destroyed. For values of a below $\bar{a}$, the surplus is strictly positive and decreasing in age a for two reasons: first, the time-horizon over which the flow surplus accrues to the pair shortens with a; second, a job's output declines with age relative to the worker's outside option, $(r-g)U$, since vacancies embody newer and newer technologies over time.

Equation (12) contains a non-standard term: the firm's outside option of remaining vacant with its machine reduces the surplus by increasing the "effective" discount rate through the term $(1-\beta)\lambda_f$. Thus, the quasi-rents in the match decrease as the bargaining power of the idle firm and its meeting rate increase.

Since the surplus starts positive and falls over time, the worker-firm pair separates when the *flow surplus* from the match is exactly zero. This separation rule together with equation (12) implies that the exit age $\bar{a}$ satisfies

$$e^{-\phi \bar{a}} = (r-g)U. \tag{13}$$

The left-hand side of (13) is the net output of the oldest match in operation, whereas the right hand side is the flow-value of an idle worker: firms with old-enough capital shut down because workers have become too expensive, since the average productivity of vacancies and, therefore, the workers' outside option of searching grow over time. This equation resembles the profit-maximization condition in the frictionless economy, with the worker's flow outside option, $(r-g)U$, playing the role of the competitive wage rate.[9]

We can now use (13) to rewrite the surplus function (12) only in terms of the two endogenous variables $(\bar{a}, \lambda_f)$:

$$S(a; \bar{a}, \lambda_f) = \int_a^{\bar{a}} e^{-(r-g+\sigma+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\phi \tilde{a}} - e^{-\phi \bar{a}} \right] d\tilde{a}. \tag{14}$$

In this equation, and occasionally below, we use a notation that shows an explicit dependence of $\bar{a}$ and λ_f. From (14) it is immediate that $S(a; \bar{a}, \lambda_f)$ is strictly increasing in $\bar{a}$ and decreasing in λ_f. A longer life-span of capital increases the surplus at each age because it lowers the flow value of the worker's outside option, as evident from (13). A higher rate at which idle firms meet workers reduces the surplus because it increases the outside option of the firm.

2.1.2 The free-entry and the job-destruction conditions

The differential equation (7) together with the surplus-sharing rule (6) imply that the net-present-value of a vacant firm equals

$$V(a; \bar{a}, \lambda_f) = \lambda_f (1-\beta) \int_a^{\hat{a}} e^{-(r-g)(\tilde{a}-a)} S\left(\tilde{a}; \bar{a}, \lambda_f\right) d\tilde{a}. \tag{15}$$

Since vacant firms do not incur in any direct search cost and old capital has no resale value, they will exit the market at an age where surplus equals zero, from which it follows immediately that the exit age of vacant and matched machines are the same, $\hat{a} = \bar{a}$. Since in equilibrium there are no profits from entry, we must have that $V(0; \bar{a}, \lambda_f) = I$, or

$$I = \lambda_f (1-\beta) \int_0^{\bar{a}} e^{-(r-g)a} S(a; \bar{a}, \lambda_f) da. \tag{JC}$$

[9] In section 2.1.3 we show that the lowest wage paid in the economy $w(\overline{a})$ exactly equals the flow value of unemployment.

This free-entry (or *job creation*) condition requires that the cost of creating a new job, I, equals the value of a vacant firm at age zero which, in turn, is the expected present value of the profits it will generate: a share $1-\beta$ of the discounted surpluses produced by a match occurring at the instantaneous rate λ_f. This is our first equilibrium condition in the two unknowns $(\bar{a}, \lambda_f)$.

By substituting (6) and (9) into (13), we obtain the optimal separation (or *job destruction*) condition

$$e^{-\phi\bar{a}} = \ell + \beta \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) S(a; \bar{a}, \lambda_f) da. \tag{JD}$$

This is the second of our two equilibrium conditions in the two unknowns $(\bar{a}, \lambda_f)$. Note that the value of search on the right hand side of (JD) depends on the steady-state distribution of vacancies which, in turn, is a function, yet to be characterized, of the two endogenous variables.

2.1.3 Wage determination

By using the surplus-based definition (6) of the value of an employed worker $W(a)$ and the differential equation for the surplus (11) in equation (10), we obtain the wage function

$$w(a) = (r-g)U + \beta \left[e^{-\phi a} - (r-g)U - \lambda_f (1-\beta) S(a) \right]. \tag{16}$$

The Nash wage exceeds the flow value of unemployment by a fraction β of the quasi-rents. The last term $-\lambda_f (1-\beta) S(a)$ is age-specific and it captures the value, for an old firm, of becoming vacant; in standard models, this value is zero for every firm. Although both the inside value (of the worker-firm pair) and the outside value (of the firm) decrease in a, it is straightforward to verify that wages will be monotonically declining in a.

The wage equation also confirms that no transfer between the parties can extend the duration of the relationship beyond the separation age $\bar{a}$. Evaluating (16) at $\bar{a}$ and using the destruction condition (13) demonstrates that $e^{-\phi\bar{a}} = w(\bar{a}) = (r-g)U$. The flow profits at $\bar{a}$ are zero, so the firm is indifferent between continuing to operate and shutting down (the first equality) and, at the same time, the worker is indifferent between working and entering unemployment (the second equality).

2.1.4 The stationary distributions

To complete the characterization of the equilibrium, we need to derive explicit expressions for the matching probabilities in terms of the endogenous variables $(\bar{a}, \lambda_f)$. This requires solving for the equilibrium vintage distribution of vacant (and matched) capital.

Denote by $\mu(a)$ the measure of workers employed by firms of age a, and denote total employment by μ. The inflow of new firms into the economy is $v(0)$: new firms acquire a machine

of the latest vintage and proceed to the vacancy pool. Thereafter, these firms transit stochastically back and forth between the vacant and matched status (after being matched, a firm can become vacant at any age $a < \bar{a}$ at rate σ), and they exit at $a = \bar{a}$, whether vacant or matched. This means that $v(a) + \mu(a) = v(0)$, for all $a \in [0, \bar{a})$. The functions $v(a)$ and $\mu(a)$ jump down to zero discontinuously at $\bar{a}$. For $a \in [0, \bar{a})$, the evolution of $\mu(a)$ follows

$$\dot{\mu}(a) = -\sigma\mu(a) + \lambda_f v(a) = \lambda_f v(0) - (\sigma + \lambda_f)\mu(a). \tag{17}$$

Exogenous separations $\sigma\mu(a)$ reduce employment, and filled vacancies $\lambda_f v(a)$ increase employment. In Appendix D we describe in detail how to derive, from (17), the stationary measures of employment and vacancies:

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-(\sigma+\lambda_f)a}}{\bar{a} - \frac{1}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}, \text{ and} \tag{18}$$

$$\frac{v(a)}{v} = \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}. \tag{19}$$

The employment (vacancy) density is increasing and concave (decreasing and convex) in age a: for every age $a \in [0, \bar{a})$, there is a constant number of machines but older machines have a larger cumulative probability of having been matched since their entry. This feature distinguishes our model from standard search-vintage models where the distribution of vacant jobs is degenerate at zero and the employment density is decreasing in age (at a rate equal to the exogenous destruction rate σ), and convex.

With the vacancy distribution in hand, we arrive at the expression for the instantaneous rate at which unemployed workers meet idle machines of age a:

$$\lambda_w(a; \bar{a}, \lambda_f) = \lambda_w \frac{v(a)}{v} = m(\theta, 1) \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}, \tag{20}$$

which depends only on the pair $(\bar{a}, \lambda_f)$, given the relation between θ and λ_f, as seen in (3).

3 Analysis of the stationary equilibrium

We now proceed to show that, under some simple parametric conditions, there exists a unique steady state in the frictional economy. More specifically, we prove that the system of two key equilibrium equations—the job creation (JC) and the job destruction (JD) conditions —admits a unique solution for the pair $(\bar{a}, \lambda_f)$.

3.1 The job creation condition (JC)

The job creation condition states that a potential entrant makes zero profits from setting up a new machine. In Appendix E, we prove

Lemma 1 [shape of the (JC) curve]. *(a) The job creation condition (JC) describes a curve that is negatively sloped in $(\bar{a}, \lambda_f)$ space. (b) As $\bar{a} \to \infty$, the (JC) curve asymptotes towards*

$$\lambda_f^{\min} \equiv \frac{\bar{r}I}{1-\bar{r}I} \quad \frac{\bar{r}+\sigma}{1-\beta} \tag{21}$$

and, if A1 holds, $\lambda_f^{\min}$ is strictly positive. As $\lambda_f \to \infty$, the (JC) curve asymptotes to the scrapping age of the frictionless economy, $\bar{a}^{CE}$.

The (JC) curve is plotted in Figure 2. Part (a) of Lemma 1 follows from the fact that the value of new firms is increasing in the exit age $\bar{a}$ and in the rate at which firms find workers λ_f. A longer life-span of capital $\bar{a}$ increases the value of a new machine by raising the surplus of every potential match—recall the discussion after equation (14)—thus, more firms need to enter the labor market (i.e., λ_f needs to fall) to restore the zero-profit condition.[10] As a result, the job creation condition defines a curve in $(\bar{a}, \lambda_f)$ space that has *negative* slope.

Even if the lifespan of capital is infinite, vacant firms must meet workers at a rate that is bounded away from zero in order for the initial investment to pay off in expected terms. This is why the vertical asymptote value $\lambda_f^{\min}$ is increasing in I and in the effective discount rate $(\bar{r}+\sigma)$ and it is decreasing in $(1-\beta)$ as a rise in either $I, \bar{r}, \sigma$ or β makes it more difficult for the firm to recover its initial investment. In particular, if $\bar{r}I > 1$, this asymptote would be negative and, as we explain below, this may jeopardize the existence of the equilibrium. This situation is ruled out by the same condition guaranteeing existence of the frictionless equilibrium, assumption A1.

Similarly, even when vacant firms meet workers at an infinite contact rate λ_f, the exit age of capital, $\bar{a}$, must be bounded away from zero for the initial investment I to pay off. That is, a minimum life length is necessary to ensure that the free-entry condition can be satisfied with equality. The asymptote is the destruction age for the competitive solution $\bar{a}^{CE}$. Intuitively, as $\lambda_f \to \infty$, the matching frictions disappear for vacancies and the firms' entry problem converges to the competitive case (2) with solution $\bar{a}^{CE}$.

Origins of the vintage structure in equilibrium– This limiting result provides an additional motivation for our assumption that the investment cost I is irreversible. In the

[10] Even though the surplus of a match declines in λ_f as discussed above, in Appendix E we prove that this effect is always dominated.

search-vintage model where vacant firms can upgrade to the frontier capital without any investment cost, the only reason why matched firms hold on to their old machine is the search cost of hiring a new worker. Thus, as the matching friction is made weaker, separations occur earlier and earlier and in the limit, all capital is new. There, in other words, vintage capital is only present to the extent frictions are present. In this sense, our assumption is a more natural way to introduce frictions into the standard vintage capital growth model where investment decisions, and not labor market frictions, are at the origin of vintage effects.

3.2 The job destruction condition (JD)

The job destruction condition states that the productivity of the marginal match at the cutoff age $\bar{a}$ equals the flow value of the worker's outside option. It is convenient to proceed under an additional assumption, sufficient to characterize the slope of the (JD) condition:

A2 [elasticity of the matching function]: $\dfrac{m_1(v,u)\,v}{m(v,u)} > 1/2$

In Appendix F, we prove

Lemma 2 [shape of the (JD) curve]. *(a) If A0 and A2 hold, the job destruction condition (JD) describes a curve that is positively sloped in $(\bar{a}, \lambda_f)$ space. (b) As $\lambda_f \to \infty$, the (JD) curve asymptotes to $\bar{a}^{\max} = -\ln(\ell)/\phi > 0$.*

The value of being unemployed depends on the expected surplus from a match, and we know that the surplus function decreases in λ_f. Also, a higher λ_f decreases the unconditional meeting rate for workers $\lambda_w(a)$ by definition, for all a. But there is a counteracting "composition" effect on the value of search that is unique to our model with an endogenous vacancy distribution: a faster meeting rate for vacant firms shifts the vacancy density towards younger vintages with larger potential surpluses. In the proof of Lemma 2, we show that under assumption A2, the value of search is unambiguously decreasing in λ_f. To understand this result, suppose that $m(v,u)$ is Cobb-Douglas with vacancy share α and note that $\alpha = m_1(v,u)\,v/m(v,u)$. Then, one can write $\lambda_w = (\lambda_f)^{-\frac{\alpha}{1-\alpha}}$. The larger is α, the more will λ_w decline following an increase of λ_f. Thus, if α is large enough the decline of the unconditional meeting rate will overcome the counteracting shift in the vacancy distribution.

Since the value of search—the right-hand side of (JD)—falls with λ_f, $\bar{a}$ has to rise and augment the surplus on every job in order to reestablish equality in this equilibrium condition.[11]

[11] Note the effect of $\bar{a}$ on the value of search. First, the surplus is increasing with $\bar{a}$. However, as $\bar{a}$ goes up, it becomes relatively more likely to meet older vintages, and older vintages have lower surplus than younger ones. Once again, the latter effect is unambiguously dominated by the former effect under assumption A2 on the elasticity of the aggregate matching function with respect to vacancies.

We conclude that the (JD) curve has a positive slope in $(\bar{a}, \lambda_f)$ space (see Figure 2).

Part (b) of Lemma 2 is also intuitive: as the meeting friction worsens for workers ($\lambda_w \to 0$ as $\lambda_f \to \infty$), in the limit output on the marginal job equals the wage (like in the frictionless economy), which, in turn, equals the marginal value of leisure ℓ, since workers are forced by firms to be indifferent between participating and not participating in the matching process.

3.3 Existence and uniqueness

Based on our characterization of the (JC) and (JD) curves, we can now formally discuss existence and uniqueness of the steady-state equilibrium depicted in Figure 2.

Proposition 2 [frictional equilibrium]. *(a) If a unique stationary competitive equilibrium with production exists for the frictionless economy, an equilibrium with finite values of the pair $(\bar{a}, \lambda_f)$ exists for the economy with frictions. (b) If, in addition, A2 holds, then the equilibrium is unique.*

Proof. From Proposition 1, a unique stationary competitive equilibrium with production exists if the labor market is "viable", $\ell < 1$, and $\bar{r}I < 1$. Thus $\lambda_f^{\min}$ and $\bar{a}^{CE}$ are positive and finite, and the (JC) curve lies within the positive orthant. Since both curves are continuous in $(\bar{a}, \lambda_f)$, and $\bar{a}^{\max} \geq \bar{a}^{CE}$ the two curves intersect at least once in the positive orthant and an equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$ exists. Furthermore, under A2, by Lemma 2 the (JD) curve is monotonically increasing. Since the (JC) curve is monotonically decreasing, the intersection of the two curves is unique. ■

4 Technology-Policy Interaction

Capital-embodied technical change affects the equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$, but we argue that the adjustment following a technological acceleration can be of two distinct types, according to the initial value of $(\bar{a}, \lambda_f)$, i.e., the point in Figure 2 where the (JC) and (JD) curves intersect. Moreover, the location of the equilibrium is determined by policy: initial institutional discrepancies can lead to different types of adjustment. This is the *technology-policy interaction*, and we broadly identify these types with the U.S. and European economies.

4.1 Effects of technological change on the equilibrium

How does capital-embodied technological change γ affect job creation and job destruction, and the equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$? In Appendix G we prove

Lemma 3 [effects of capital-embodied technology]. *Under A0-A2, a rise in γ shifts both the (JC) curve and the (JD) curve downward, inducing a fall in $\bar{a}^*$, while the change in λ_f^* is ambiguous.*

An increase in the rate of technological obsolescence γ has three counteracting effects on the surplus through three different terms in (14). First, a standard capitalization effect increases the surplus: future output is discounted at a lower effective rate. Second, since the relative output of the marginal technology $e^{-\phi\bar{a}}$ shrinks compared to the frontier, the outside option of a worker falls and thus increases the match surplus (the outside option effect). Third, a higher γ makes output in an existing match fall faster with age, relative to the frontier. This decreases the match surplus (the obsolescence effect). The older a match, the stronger the obsolescence effect and the shorter the time period for which it will benefit from the first and the second forces at work: there is a critical age such that, for vintages younger (older) than this threshold, the surplus rises (falls) with γ.

Despite this non-monotonicity of the surplus with respect to γ, one can show (see Appendix G) that the (JC) and (JD) curves shift unambiguously. Thus, a rise in γ mainly acts as a "capital obsolescence" shock: for a given meeting rate λ_f, faster obsolescence leads to a lower exit age for firms, and thus the lifetime of firms $\bar{a}^*$ declines unambiguously with a higher γ. Whether labor market tightness goes up or down, however, is a quantitative question.[12]

4.2 Two types of labor market adjustment

Figure 3 portrays how the labor market adjustment to an acceleration in the rate of capital-embodied technical change depends on the initial location of the equilibrium. Economy F is initially (i.e., before the rise in γ) located in the vertical region of the (JC) curve and adjusts through a sharp decline in the scrapping age of capital $\bar{a}$ with little impact on λ_f. Economy R, instead, starts much closer to the flat region of the (JC) curve, and it will instead display only a moderate rise in job destruction (i.e., fall in $\bar{a}$), but a notable rise in λ_f.[13]

How do each of these two adjustment mechanisms translate into changes in unemployment, the labor share, and the vacancy-employment ratio?

Unemployment– In steady state, the flow into unemployment equals the flow out of

[12] In Aghion and Howitt (1994), the impact of a rise in γ on unemployment duration is also ambiguous. In contrast, Mortensen and Pissarides (1998) show that a rise in γ unambiguously raises both unemployment duration and incidence. This difference is attributable to the fact that Aghion and Howitt, as we do, model job set-up costs that have to be paid *before* entering the labor market, which strengthens the capitalization effect.

[13] The labels "F" and "R" stand for flexible and rigid, as explained in the next section where we introduce various labor market institutions in the model.

unemployment. That is,

$$\sigma\mu + \mu(\bar{a}) = \lambda_w u = m(\theta, 1)u. \tag{22}$$

It is convenient to use the fact that $\mu = 1 - u$ and restate (22) as

$$\frac{u}{1-u} = \frac{1}{m(\theta,1)}\left(\sigma + \frac{\mu(\bar{a})}{\mu}\right), \tag{23}$$

which is the product of unemployment duration and incidence.[14] The degree of endogenous job destruction $\mu(\bar{a})/\mu$, i.e., the fraction of jobs destroyed at $\bar{a}$, can be read from (18). A rise in $\bar{a}$ reduces unemployment, since endogenous job destruction is reduced. A rise in the firm's meeting rate λ_f (fall in θ) reduces the meeting probability for unemployed workers, which in turn increases unemployment duration.[15] Thus, one can read changes in unemployment incidence and duration directly from changes in $\bar{a}$ and λ_f, respectively.

In economy R, higher unemployment is due to longer unemployment duration, while in economy F, it shows up through higher incidence. However, as is clear from (23), a change in $\bar{a}$ has a small impact on unemployment, as long as endogenous job destruction due to capital obsolescence is negligible compared to separations due to the churning taking place routinely in the labor market and captured by σ. In the quantitative section we shall see that, for reasonable parameterizations, this is overwhelmingly the case.

Labor shares– The direct impact of a rise in γ on the labor share is negative: it reduces the equilibrium outside option of the worker, $e^{-\phi\bar{a}}$, that anchors the wage equation. In economy F, a shorter lifetime of capital $\bar{a}$ counteracts this direct effect. To the extent that the meeting rate of firms, λ_f, increases, the firm's outside option improves and the worker's value of search declines, and thus the labor share in each match falls.[16] This mechanism is reinforced by firms' outside options responding to a rise in γ; in the standard model, the equilibrium value of vacancies would remain at zero. In economy R large adjustments occur via the unemployment duration margin, and we therefore see a larger fall in labor shares because of changes in the outside options of firms and workers.

[14] Appendix D contains the closed form expression for equilibrium unemployment and its derivation.

[15] There is a counteracting effect of λ_f on equilibrium unemployment: as λ_f increases, vacant firms meet workers at a faster rate, so the employment distribution shifts towards younger machines, and there are relatively fewer machines at the exit age $\bar{a}$, which reduces unemployment incidence. We can show that, if assumption A2 holds, this effect is second order and unemployment increases with λ_f. To see this, multiply both numerator and denominator of (23) by λ_f and differentiate $u/(1-u)$ with respect to λ_f.

[16] There is also another effect following an increase in λ_f or a reduction in $\bar{a}$: the shift in the employment distribution towards younger vintages, which have a smaller labor share of output (recall that the labor share is one for the oldest firms). This additional effect reinforces the comparative statics for λ_f, while it counteracts the direct effect of $\bar{a}$.

The vacancy-employment ratio– Using equations (D.3) and (D.4) from Appendix D and rearranging terms, we obtain

$$\frac{v}{\mu} = \frac{\bar{a}\,(\sigma/\lambda_f) + \int_0^{\bar{a}} e^{-(\sigma+\lambda_f)a} da}{\bar{a} - \int_0^{\bar{a}} e^{-(\sigma+\lambda_f)a} da},$$

which is decreasing in λ_f: as the firm's meeting rate grows, vacancies are filled more quickly, so there will be fewer of them relative to employment. The effect of a decline in $\bar{a}$ is ambiguous, as both vacancies and employment fall. Therefore, we should expect a more pronounced fall in the vacancy-employment ratio in economy R relative to economy F.

To sum up, economy R—because its initial location is closer to the horizontal portion of the (JC) curve—will respond to an acceleration in capital-embodied technical change through a sharper increase in unemployment duration and a more severe fall in the labor share and in the vacancy-employment ratio, relative to economy F. Thus, at least qualitatively, economy F resembles the United States and economy R resembles Europe. But why would the initial positions of the United States and Europe be different? Our answer is their labor market policies differ. To see this, we introduce three policies: unemployment benefits, tax wedges, and firing costs.

4.3 Unemployment benefits and tax wedges

Let b denote the flow value of unemployment benefits paid to jobless workers, and let τ_f and τ_w, respectively, be the proportional payroll tax inflicted on firms and the proportional labor income tax borne by workers. In equation (8), the flow profit now reads $e^{-\phi a} - (1+\tau_f)\,w\,(a)$ since the payroll tax raises firms' labor cost; in equation (9), $\ell + b$ is now the flow value of unemployment; and in equation (10), the net wage now reads $(1-\tau_w)\,w\,(a)$. The equations for idle firm and worker are unchanged.

In Appendix H we derive the new pair of equilibrium equations and show that the free-entry condition remains exactly as in (JC), while the job destruction condition becomes

$$e^{-\phi\bar{a}} = (\ell + b)\,(1+\hat{\tau}) + \beta \int_0^{\bar{a}} \lambda_w(a;\bar{a},\lambda_f) S(a;\bar{a},\lambda_f) da, \qquad \text{(JD}')$$

where the term $1+\hat{\tau} \equiv (1+\tau_f)\,/\,(1-\tau_w)$ is the total tax wedge, i.e., the ratio between the labor costs to the employer and the net wage received by the worker. The surplus function $S\,(a;\bar{a},\lambda_f)$ is unchanged from 14. Clearly, the equilibrium vintage distributions of vacant and matched capital also remain unchanged. Thus, the two policies only affect one equilibrium condition, the (JD′) curve: the tax wedge acts exactly as a relative subsidy to unemployment.[17]

[17] As noted by Pissarides (1985) in the context of a simpler matching model, as long as $\ell + b > 0$, the worker

It is easy to extend the analysis of Section 3 to the case with benefits and taxes. Lemma 1 holds as before. Viability of the labor market now requires

A0′ [viability of labor market]: $(\ell + b)(1 + \hat{\tau}) < 1$

The (JD′) curve asymptotes to $\bar{a}^{\max} = -\ln\left[(\ell + b)(1 + \hat{\tau})\right]/\phi$, and assumption $A0'$ guarantees that this is a positive number; thus, existence and uniqueness of the stationary equilibrium can be proved exactly as in Proposition 2. As for policy, we have

Lemma 4 [effects of unemployment benefits and taxes]. *A rise in either b or τ_f or τ_w leaves the (JC) curve unaffected; under A0′ and A2, it shifts the (JD′) curve downward, inducing a fall in $\bar{a}^*$ and a rise in λ_f^*.*

Inspection of (JD′) reveals that an increase in any component of the triple (b, τ_f, τ_w) shifts the job destruction curve downward: since the policies increase the relative value of unemployment, for a given meeting rate λ_f, a worker-firm pair is indifferent between continuing the match and separating at a lower age of capital $\bar{a}$. The effects of taxes and benefits reinforce each other; e.g., the higher the tax rates, the more a given change in b shifts the (JD′) curve. Therefore, an economy with generous welfare benefits and heavy taxation—Europe—will behave similarly to economy R in response to a technology shock. An economy with stingy unemployment compensation and light payroll taxes—the United States—will mirror the response of economy F.

Technology-policy interaction: an interpretation– Faster embodied technical change acts as an obsolescence shock that requires a macroeconomic adjustment. Flexible economies adjust partially through the price and partially through the quantity of labor. In rigid economies, the value of unemployment (and hence the wage) is artificially high because of the generous benefits and, indirectly, through the heavy taxes, thus it is less elastic to changes in firms' productivity and in the market value of search. As a result, the adjustment to a shock requires massive changes in meeting rates and in unemployment duration, through a reduction in job creation rates and a fall in the employment-vacancy ratio. These features accord, qualitatively, with the different labor market experiences of the two regions described in Figure 1.

has some alternative return that is not taxed, so firms cannot pass the tax entirely onto the workers and $\hat{\tau}$ has real effects.

4.4 Firing costs

Suppose firms must pay a mandatory firing tax κ upon separating from their employee.[18] In particular, we assume that κ has to be paid as long as the firm and worker undertook production together: if the firm and worker "meet but do not match," no tax is due.

The firing cost changes the structure of equilibrium quite radically, since it delays the separation for matched firms, but not for vacancies. As a result, there will be two destruction cutoffs: one for vacant firms ($\hat{a}$) and one for matched firms ($\bar{a} > \hat{a}$). Solving the model now means characterizing the triplet $(\hat{a}, \bar{a}, \lambda_f)$ using three equilibrium conditions. For ease of exposition, in this section we abstract from other policies. Appendix K describes the model with all policies used in the quantitative exploration of Section 4.5.

The new value equations of the model are

$$\begin{aligned}
(r-g)V(a) &= \max\{\lambda_f [J_n(a) - V(a)] + V'(a), 0\}, && (24)\\
(r-g)J(a) &= \max\{e^{-\phi a} - w(a) - \sigma [J(a) - V(a) + \kappa] + J'(a), (r-g)[V(a) - \kappa]\}, && (25)\\
(r-g)U &= \ell + \int_0^{\hat{a}} \lambda_w(a) [W_n(a) - U] \, da, \text{ and} && (26)\\
(r-g)W(a) &= \max\{w(a) - \sigma [W(a) - U] + W'(a), (r-g)U\}, && (27)
\end{aligned}$$

where the subscript n denotes "newly formed" matches. Values for new matches differ from values for continuing jobs because the firing cost is not binding during the first negotiation between worker and firm. Appendix I shows that the first negotiation induces firms to extract a "hiring fee" from workers in order to get a prepayment of the future firing cost.[19] In particular, the worker prepays exactly a fraction β of the firing cost κ, i.e.,

$$J_n(a) - J(a) = -[W_n(a) - W(a)] = \beta\kappa. \tag{28}$$

Let $S(a)$ and $S_n(a) = S(a) - \kappa$ be the surplus of a continuing and new job, respectively. The new equilibrium condition is the destruction condition for a vacancy, or $S_n(\hat{a}) = 0$, which can be shown to read:

$$\int_{\hat{a}}^{\bar{a}} e^{-(r-g+\sigma)(\tilde{a}-\hat{a})} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}}\right] d\tilde{a} - \kappa = 0. \tag{VD$''$}$$

Thus, at $\hat{a}$ any new meeting yields a surplus—the value of production until destruction age $\bar{a}$, net of firing costs κ—equal to zero. Equation (VD$''$) gives an implicit, strictly increasing function relating $\hat{a}$ to $\bar{a}$ (and to structural parameters): $\hat{a}(\bar{a})$. The higher is κ, the more distant is $\hat{a}$ from $\bar{a}$, since matched firms delay separations to avoid the payment of κ.

[18] There is a long tradition of modelling employment protection this way within both competitive and frictional labor market models; see Ljungqvist (2002) for a survey.

[19] To be precise, we have a two-tier model, where the first tier is instantaneous.

The updated job creation condition is reformulated as

$$I = (1-\beta)\,\lambda_f \int_0^{\hat{a}} e^{-(r-g)a} S_n(a;\hat{a},\bar{a},\lambda_f)da. \qquad (\text{JC}'')$$

There are two differences with respect to equation (JC): a vacancy will last until age $\hat{a}$, and the relevant surplus is $S_n(a;\hat{a},\bar{a},\lambda_f)$ which in Appendix I is shown to satisfy

$$S_n\left(a;\hat{a},\bar{a},\lambda_f\right) = \int_a^{\hat{a}} e^{-\left(r-g+\sigma+\lambda_f(1-\beta)\right)(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}} - (r-g+\sigma)\,\kappa\right] d\tilde{a}. \qquad (29)$$

The new job destruction condition is $e^{-\phi\bar{a}} + (r-g)\kappa = (r-g)\,U$, since there is an additional gain to continue production: delaying the payment of κ. Thus, we have

$$e^{-\phi\bar{a}} + (r-g)\kappa = \ell + \beta \int_0^{\hat{a}} \lambda_w(a;\hat{a},\lambda_f) S_n(a;\hat{a},\bar{a},\lambda_f)da, \qquad (\text{JD}'')$$

where $\lambda_w(a;\hat{a},\lambda_f)$ has exactly the same form as in (20) with $\hat{a}$ in place of $\bar{a}$ because the oldest vacancies that workers can potentially meet are now of age $\hat{a}$.

The three equations (VD$''$), (JC$''$), (JD$''$) and unknowns $(\hat{a},\bar{a},\lambda_f)$ represent the new equilibrium system to be solved. Given the monotone relation between $\hat{a}$ and $\bar{a}$ implied by (VD$''$) we can still represent the equilibrium in the $(\bar{a},\lambda_f)$ space. Substitute for $\hat{a}$ and the (JC$''$) curve is negatively sloped. Its vertical asymptote lies in the positive orthant as long as the following assumption holds:

A1$''$ [recoverability of the investment cost]: $\bar{r}\left(I + \kappa\frac{r-g+\sigma}{r-g}\right) < 1$.

The firing cost lowers the maximum investment cost that a firm can afford. Let the horizontal asymptote of the (JC$''$) curve be $\overline{\overline{a}}^{CE}$. It can also be shown that the (JD$''$) curve has an asymptote $\overline{\overline{a}}^{\max} = -\ln\left[\ell - (r-g)\,\kappa\right]/\phi$ (as $\lambda_f \to \infty$) that is positively sloped under the following condition:

A3 [size of the firing cost]: $(r-g)\,\kappa < \ell$.

We now state global existence of equilibrium of the model with firing costs under a set of conditions very similar to those in Proposition 2. Uniqueness and some useful comparative statics results are also provided for κ small enough (see Appendix J for the proofs).

Proposition 3 [equilibrium with firing costs]: *(a) If A0, A1$''$ and A3 hold, and* $\overline{\overline{a}}^{\max} \geq \overline{\overline{a}}^{CE}$*, an equilibrium with finite values of the pair* $(\bar{a},\lambda_f)$ *exists. In a neighborhood of* $\kappa = 0$*, the following is true: (b) if, in addition, A2 holds, then the equilibrium is unique; (c) under A0-A3, a rise in* γ *shifts both the (JC$''$) curve and the (JD$''$) curve downward; (d) under A0-A3, a rise in* κ *shifts both the (JC$''$) curve and the (JD$''$) curve upward.*

The comparative statics with respect to γ, studied in Lemma 3, are robust to the introduction of a firing cost.[20] Our extension of the standard search model also inherits its qualitative comparative statics with respect to firing costs: as in Pissarides (2000, chapter 9), κ delays separations, $\bar{a}$ increases, and unemployment incidence falls. The effect on λ_f, i.e., on unemployment duration is ambiguous: workers' wages increase, since firms' outside options are reduced by the firing fee, but the rise in $\bar{a}$ induces lower unemployment which, in turn, tends to reduce wages.

From these comparative statics results and the discussion following Lemma 4 on how policies change the location of the (JD$''$) curve, it appears that the firing cost—by shifting the (JD$''$) curve upward, i.e., along the steeper portion of the (JC$''$) locus—could partially mitigate the policy-technology interaction discussed in Section 4.3. However, it is also clear that κ amplifies the effects of a rise in γ: by reducing effective discounting, γ makes the firing cost larger in present value terms. Therefore, whether our bundle of policies amplifies or dampens the technological shock is a quantitative question. The next section pursues this question.

4.5 The U.S.-Europe comparison: quantitative analysis

We now calibrate the model economy to explore the following questions: can our model account quantitatively for the differential behavior of unemployment in the United States and Europe over the past thirty years? Can it generate the rise in unemployment through a surge in unemployment duration and a stable separation rate? Is the model quantitatively consistent with the additional facts we documented on the relative evolution of the labor share, the vacancy rate, and the age of capital equipment?

In particular, we calculate the steady-state responses to an increase in the rate of embodied technological change γ for two model economies differing with respect to the institutional quadruplet $(b, \tau_f, \tau_w, \kappa)$, and calibrated to the United States and Europe.

4.5.1 Calibration

In the calibration procedure we aim to match the key technological, institutional, and labor market variables in the United States and Europe. Overall, we have 15 parameters to calibrate, $(\gamma;\ b, \kappa, \tau_f, \tau_w, \sigma;\ \delta, \psi, \omega, \ell, r, A, \alpha, \beta, I)$. We assume that, except for the rate of embodied technical change, γ, all parameters remain fixed over time. Furthermore, our calibrated versions

[20] The reason for assuming a small κ is that we were not able to find a global condition for κ under which the (JD$''$) curve is downward-sloping. In particular, even though a rise in $\bar{a}$ raises the surplus, it also influences $\hat{a}$, which can lead to a decrease in the surplus for small enough values of a. The (JD$''$) curve is an integral over the different values for a, however, and we always found the curve to be downward-sloping in our numerical work.

of the European and U.S. economies are the same, except for the institutional quadruplet $(b, \tau_f, \tau_w, \kappa)$ and the separation rate σ.

Technology– The rate of physical decay δ is set to 2 percent, matching an average age of capital of 11.5 years in the United States, as reported by the Bureau of Economic Analysis (2004) for the mid-1960s (Figure 1.D). We set $\psi = 0.008$, the estimate of annual disembodied growth in the United States for 1954-1993 computed by Hornstein and Krusell (1996). Greenwood et al. (1997) measure the speed of embodied technical change through the (inverse of the) rate of decline of the quality-adjusted relative price of capital. The relative price of new investment in the United States decreased at a 2 percent rate before the mid-seventies and at 4.5 percent in the 1990s, implying an acceleration in capital-embodied growth (Gordon, 1990; Cummins and Violante, 2002). As demonstrated in equation (B.1) of Appendix B, the relative price of efficiency units of capital changes at the rate $g - \gamma$. Given the observed average output growth rate $g = 0.02$, the pre-1970s 2 percent decline implies a γ of 4 percent. From $g - \gamma \equiv \psi - (1 - \omega)\gamma$, we then obtain $\omega = 0.30$. Thus, one needs to set γ to 7.7 percent to generate a post-1990 decline in the relative price of capital of 4.5 percent per year.

Institutions– The OECD Employment Outlook (1996) computes unemployment benefit replacement rates (as a fraction of average wages) of around 10 percent in the United States and of 40 percent or higher in Europe (Chart 2.2, page 29). However, this is a lower bound: typically, European countries offer generous long-term social assistance programs for the unemployed. Hansen (1998) calculates replacement ratios that include social assistance benefits of up to 75 percent of average wages in some European countries (Hansen 1998, Graph 3, page 29).[21] Thus, we set $b = 0.05$ for the United States and $b = 0.33$ for Europe. The OECD Employment Outlook (1999, Table 2.A.3) computes firing costs for a large set of countries: in the United States, κ is estimated to be roughly zero, whereas in Europe it varies by tenure length. We summarize the variety of different European legislations with a firing cost of one month per year of tenure. Based on average tenure and average wage in the European economy, we set $\kappa = 0.45$. OECD (2004) allows a separate computation of payroll and income taxes for the United States and a set of European countries. For the United States, we set $\tau_f = 0.17$ and $\tau_w = 0.08$; for Europe, we set $\tau_f = 0.21$ and $\tau_w = 0.24$.[22]

Labor markets– We normalize $\ell = 0$ and choose r to match an annual interest rate of 4%

[21] These replacement rates are calculated for a 40-year-old single male production worker. Ljungqvist and Sargent (1998, Table 3, page 523) report similar evidence from a different source.

[22] We compute these tax rates as unweighted averages of Austria, Belgium, Denmark, Finland, France, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland, to be consistent with our data on unemployment, labor shares, and vacancy rates for Europe.

(Cooley, 1995). A value for $\beta = 0.89$ delivers a labor income share of 0.70 in the U.S. economy. The setup cost I affects vacancy creation, so we choose $I = 2.53$ to match an average vacancy duration of 4 weeks (Hall, 2005, Table 2). We assume a Cobb-Douglas parametric form for the matching function, $m = Av^{\alpha}u^{1-\alpha}$. We set the matching elasticity $\alpha = 0.5$, an average of the values reported in the comprehensive survey of empirical estimates of matching functions by Petrongolo and Pissarides (2001, Table 3), and we set the scale parameter A of the matching function to reproduce an average unemployment duration of approximately 8-9 weeks in the United States, as reported by Abrahams and Shimer (2002).

We now use the unemployment rate to determine the sole remaining parameter: the exogenous separation rate σ. Unemployment rates were roughly around 4 percent both in Europe and in the United States in the 1960s: unemployment was even somewhat lower in Europe (see Figure 1A). Given the institutional quadruplet $(b, \tau_f, \tau_w, \kappa)$, for the United States we set σ to 0.22 so that the implied total separation rate, together with the average unemployment duration, generates an unemployment rate of 4 percent. Given our assumed differences for the institutions in Europe and the United States, the same U.S. separation rate will generate a much higher initial unemployment rate in Europe, about 9 percent. We therefore need a lower separation rate for Europe, $\sigma = 0.06$, to match the same initial 4 percent unemployment rate. This calibration implies that, in the initial steady state, unemployment duration in the European economy is already three times as high as in the U.S. economy. This implication is broadly consistent with the evidence in Machin and Manning (1999, Figure 2) who report that in 1970 the incidence of long-term unemployment (fraction of spells longer than 6 months) was less than 10 percent in the United States, but over 20 percent in Spain and Germany, over 40 percent in France and Netherlands, and over 70 percent in Belgium. In other words, given the similar unemployment rates and the longer durations in Europe, separation rates in Europe must have been higher than in the United States.[23]

4.5.2 Results

Figure 4 reports the key equilibrium outcomes (unemployment rate, unemployment duration, separation rate, average age of capital, vacancy rate, and labor share) in the two economies for a range of embodied productivity growth rates.

As we raise γ from 4 percent to 7.7 percent, the unemployment rate increases by just one percentage point in the U.S. economy and by over 4 percentage points in Europe. The latter

[23]Except for rescaling the level of the unemployment rate, the assumed difference in exogenous quit rates has no bearing on any of the results in Figures 4 and 5. If we use the same value for the exogenous separation rate in both economies, the results of the quantitative experiments are virtually identical to the ones reported in this paper, and are available upon request from the authors.

represents roughly half of the observed increase, as evident from Figure 1.A. As in the data, the entire change in European unemployment is due to a decline in the job finding rate: mean unemployment duration rises from 27 weeks to almost 50 weeks.

The model matches the sharp fall in the European vacancy-employment ratio of Figure 1.C from a (normalized) value of 1 to 0.6. The fall in the U.S. vacancy-employment ratio is much milder. Moreover, the labor share in the model decreases by approximately 3 percentage points in both regions, which lines up well with the U.S. experience, but it represents less than half of the decline in the Europe. Finally, the model predicts a decline in the average age of capital, as a result of the faster obsolescence, of 2.7 years, consistent with the U.S. data in Figure 1.D.

Figure 5 compares individual policies and their interaction. First, the unemployment benefit/tax policies seem most important both for influencing unemployment and for amplifying the effect of technological change. Second, and more strikingly, the technology-policy interaction is much starker when the three policies are considered together: as γ increases, if one estimated the total role of policy by merely summing the effects of the individual policies, one would only account for less than one third of the total technology-policy interaction predicted by the model with all policies jointly considered.[24] We conclude that, first, it would be inaccurate to point to one particular institution as the culprit; second, looking ahead, reforming any one institution could reduce dramatically the elasticity of the unemployment rate to obsolescence shocks.

4.6 Discussion of related results in the literature

The view that a common shock and different policies account for the diverging labor market experiences of the United States and Europe dominates the literature. Cross-country regression studies have concluded that shock-policy interactions help explain the observed variation in the unemployment rate over time and across countries (Blanchard and Wolfers 2000; Bertola, Blau, and Kahn 2001). A major shortcoming of these empirical implementations of the Krugman (1994) hypothesis, however, is that they are not explicit about the underlying economic mechanism.

Ljungqvist and Sargent (1998) offer the first rigorous model of how a common technological shock might interact with the degree of generosity of welfare states. A related explanation is set forth by Marimon and Zilibotti (1999). In these two models, the technology-shock-policy interaction operates entirely through the *labor supply* side. These authors essentially argue that unemployment in Europe went up because it was more beneficial for the jobless workers to collect unemployment insurance than to work at a low wage, given that technological change

[24] The equations in Appendix K show this policy nonlinearity clearly. For example, as $\kappa = b = 0$, taxes have no effect; but, if both benefits and firing costs are positive, taxes can have a major effect.

made their skills obsolete (as in Ljungqvist and Sargent) or made it increasingly difficult to match with existing job vacancies (as in Marimon and Zilibotti).

Our model, in contrast, is one where workers accept every job offer, but in the aftermath of the obsolescence shock firms reduce *labor demand* as part of the adjustment process. To paraphrase the discussion above: jobless workers did not work on the low-wage jobs because the latter were not created.[25] Taken together, this set of papers highlights the complementarity between technological shocks and policies along two parallel and equally important margins: labor supply and labor demand. We also showed that a labor-demand-based story has the right implications for the labor share and the vacancy-employment ratio. In contrast, the McCall (1970) job-search model used by Ljungqvist and Sargent is silent on these dimensions.

Our paper is the first to highlight the interaction between *capital-embodied* technical change and labor market institutions, but others before us have studied models where the shock-policy interaction occurs from the labor demand side. Bertola and Ichino (1995) argue that a rise of economic uncertainty in the 1970s coupled with severe firing costs made firms cautious in hiring and reduced job creation. Caballero and Hammour (1998) suggest that the uprise of the labor movement in the 1970s triggered an "appropriation" shock that changed the division of quasi rents away from capital towards labor and, in response, firms adopted ever more labor-saving technologies reducing labor demand. Mortensen and Pissarides (1999) build a version of their matching model where labor markets for workers with different skill levels are segmented. They also study the interaction between a mean-preserving spread of the productivity distribution across skills, intended to capture skill-biased technical change, and labor market policies, such as unemployment benefits and firing taxes. Finally, den Haan, Haefke, and Ramey (2005) study the quantitative implications of interest rate shocks within a calibrated version of the traditional Mortensen-Pissarides framework with various labor market policies.

5 Concluding remarks

The past twenty years have been marked by rapid productivity advancements embodied in new capital goods. We have made an attempt at understanding how this type of technological change affects frictional labor markets. Despite the increased complexity introduced into the Diamond-Mortensen-Pissarides model by an explicit vintage capital structure, our analysis shows that one can maintain analytical tractability in characterizing the main features of the equilibrium and its comparative statics with respect to the relevant technology and policy parameters.

[25] Consistently with this view, Rogerson (2004) argues that the dismal growth in European employment is due to excessive regulations that hampered the growth of the service sector, where many low-wage jobs are potentially located.

We have used our framework to investigate the diverging labor-market experiences of Europe and the United States in the past thirty years: an acceleration in embodied technological change interacted with common labor-market policies to sharply reduce labor demand. Our approach complements recent work on European and U.S. labor markets that focuses on labor supply (e.g., Ljungqvist and Sargent, 1998). Obviously, interpreting the European and U.S. labor-market outcomes in terms of "labor demand" or "labor supply" is not mutually exclusive. In a model with elements of both vintage human capital and vintage physical capital, a technological acceleration will also reduce labor supply by worsening the rate of skill obsolescence. The next generation of investigations on the European unemployment puzzle should bring together supply and demand forces, allowing an evaluation of their respective strengths.

Finally, there are some important connections between our results and an emerging set of studies pointing to two shortcomings of the standard matching model. First, the matching model does not readily generate short-run volatility in the vacancy-unemployment ratio (see, e.g., Hall, 2005; Shimer, 2005; and Hornstein, Krusell, and Violante, 2005b). One lesson from these studies (Hagedorn and Manovskii, 2006, in particular) is that significant volatility requires large values of a worker's utility when unemployed—the value of leisure and unemployment benefits ($\ell + b$ in our model). This leads to a low wage elasticity to variations in productivity; thus productivity changes have a strong effect on profits and hence on firm inflow as well as unemployment. Although we focus on steady states rather than short-run dynamics, we also emphasize that in the presence of strict labor-market policy (e.g., high unemployment benefits b), productivity changes (here, growth in capital-embodied technology) have a significant impact on quantities: market tightness falls sharply as a result of significant withdrawal of firm entry. Thus, our interpretation of the recent European experience builds on similar mechanisms as those discussed in the recent literature on short-run dynamics.

Second, our vintage capital model does not generate much wage dispersion; for example, the 90-10 log-wage differential is 6 percent. Even allowing for the fact that workers are homogeneous in our economy, this is small relative to the U.S. economy. Katz and Autor (1999) report that the 90-10 log-wage differential that is left as a residual in a typical wage regression, after controlling for observable characteristics of the workers, including fixed effects to capture "unobserved ability," is around 60 percent. As explained in Hornstein, Krusell, and Violante (2006), this striking feature is not specific to our particular vintage economy, but applies to a very large class of plausibly calibrated search and matching models. In a nutshell, the key reason is that, given the workers' search behavior central to these models, durations of unemployment spells as short as those observed in actual economies must imply that workers perceive a very small amount of dispersion in the wage distribution; otherwise they would wait longer for a

better offer. Introducing on-the-job search in a model of the sort considered here would break the relationship between wage dispersion and the value of search during unemployment and would have some potential for incorporating significant wage inequality into the analysis of the diverging labor-market experiences of Europe and the United States.

Appendix

A Data Appendix for Figure 1

Panel A of Figure 1 (standardized unemployment rate) is based on Blanchard and Wolfers (2000) and available at `http://econ-www.mit.edu/faculty/blanchar/harry_data/`. The data set is constructed from raw OECD data. The "Europe" aggregate is an unweighted mean of: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland. The U.K. is excluded since its unemployment experience and its labor market institutions are more similar to the U.S. economy. The data are only available at 5-year intervals: the 1960 data-point is an average of 1960-1964, the 1965 is an average of 1965-1969, and so on, until 1995 which is reported by the authors to be an average of 1995 and 1996 only.

Panel B (labor share) is based on Blanchard and Wolfers (2000), except the 1995 entry for Austria, Denmark, Ireland, and Portugal (not part of the original Blanchard-Wolfers data) which comes from the raw OECD data, averaging out 1995 and 1996.

Panel C (vacancy-employment ratio) is the ratio of job vacancies to total civilian employment. The numerator is computed from the "number of registered vacancies" by country, a set of series available in the OECD Main Economic Indicators. In the United States vacancies come in the form of a "help-wanted" advertising index. The denominator is computed from the OECD Labor Force Statistics. Due to data limitations, only Austria, Belgium, Finland, France, Germany, Netherlands, Norway, Sweden, and Switzerland have complete time series spanning 1960-1996. The series for Spain starts in 1977, the series for Portugal in 1974, and the series for Denmark in 1970. Data on vacancies are unavailable for Greece, Ireland, and Italy. Since vacancies are reported as an index, the level of the ratio is meaningless; hence the normalization in 1960.

Panel D (average age of capital for the U.S.) is based on the data in Table 2.10 labelled "Historical-Cost Average Age at Year-end of Private Fixed Assets; Equipment, Software, and Structures; by Type" available in the Fixed Assets Tables from the Bureau of Economic Analysis official web site.

B Rendering the equilibrium stationary

Aggregate output at time t can be written as

$$Y(t) = \int_0^{\bar{a}} z(t)\, k(t,a)^{\omega}\, \mu(t,a)\, da,$$

where $\mu(t,a)$ is the distribution of employment across vintages at time t. Along a balanced growth path $\mu(a)$ does not depend on time t, $z(t) = z_0 e^{\psi t}$ and $k(t,a) = k_0 e^{\gamma(t-a)-\delta a}$. With the normalization $z_0 = k_0 = 1$, aggregate output becomes

$$Y(t) = e^{(\psi+\omega\gamma)t} \int_0^{\bar{a}} e^{-\phi a} \mu(a)\, da,$$

with $\phi \equiv \omega(\gamma+\delta)$. Therefore, on the balanced growth path, $Y(t)$ must grow at the constant rate $g \equiv \psi + \omega\gamma$. The aggregate resource constraint reads

$$Y(t) = C(t) + I(t)\, e_f(t),$$

where $C(t)$ is aggregate consumption and $I(t) e_f(t)$ is aggregate investment. The cost of a new machine is $I(t)$ and $e_f(t)$ is the measure of new entrant firms. Along the balanced growth path, $C(t)$ grows at rate g, $e_f(t) = e_f$ and $I(t) = Ie^{gt}$. Since the amount $I(t) e_f$ of foregone consumption translates into $X(t) \equiv k(t,0) e_f$ efficiency units of new investments, the *price of an efficiency unit of new capital* at time t (relative to the price of the final good, normalized to 1) is

$$p(t) \equiv \frac{I(t)}{k(t,0)} = Ie^{(g-\gamma)t}, \tag{B.1}$$

implying that along the balanced growth path, the relative price of an efficiency unit of capital grows at rate $g - \gamma = \psi - (1-\omega)\gamma$, a number that can be either positive or negative.

C Proof of Proposition 1 (frictionless equilibrium)

Part (a): The right-hand side of (2) is strictly increasing in $\bar{a}$ because the relative productivity of workers operating on the marginal firm is decreasing in $\bar{a}$, by definition, and therefore wages will be be lower and profits higher. In particular, the right-hand side of (2) increases from 0 to $1/\bar{r}$ as $\bar{a}$ goes from 0 to infinity. These facts mean that there exists a unique steady state exit age $\bar{a}^{CE}$ whenever $I < 1/\bar{r}$, the condition stated in assumption A1.

Part (b): With risk neutrality, the planner maximizes the discounted value of future aggregate consumption subject to the constraint that, in every period t, the total number of machines in operation– i.e., all the firms entered until time t–cannot exceed the aggregate labor force:

$$\max_{\{e_f(t), \hat{a}(t)\}} \int_0^\infty e^{-(r-g)t} \left[\int_0^{\hat{a}(t)} e_f(t-a) e^{-\phi a} da - e_f(t) I \right] dt$$
$$\text{s.t.} \int_0^{\hat{a}(t)} e_f(t-a) da \leq 1, \text{ for all } t.$$

The planner controls the measure $e_f(t)$ of entrant firms and the maximal age $\hat{a}(t)$ of vintages which are operating at t. The Lagrangian stated in terms of the contribution of each different vintage is:

$$\begin{aligned} \mathcal{L} &= \int_0^\infty e^{-(r-g)t} e_f(t) \left[\int_0^{\bar{a}(t)} e^{-(r-g+\phi)a} da - I \right] dt \\ &\quad - \int_{-\bar{a}(0)}^\infty e^{-(r-g)t} e_f(t) \left[\int_0^{\bar{a}(t)} \varphi(t+a) e^{-(r-g)a} da \right] dt + \int_0^\infty e^{-(r-g)t} \varphi(t) dt , \end{aligned}$$

where $\varphi(t)$ is the multiplier on the labor endowment constraint at t, and $\bar{a}(t)$ denotes the retirement age of vintage t. The first-order conditions with respect to $e_f(t)$ and $\bar{a}(t)$ read, respectively,

$$\begin{aligned} \int_0^{\bar{a}(t)} e^{-(r-g+\phi)a} da &= I + \int_0^{\bar{a}(t)} e^{-(r-g)a} \varphi(t+a) da, \\ e^{-\phi \bar{a}(t)} &= \varphi(t + \bar{a}(t)) . \end{aligned}$$

The first condition states that the planner will add new jobs until the benefits (the present value of additional output) from the last job equal the cost of purchasing a machine I and the shadow cost due to the fact that creating a new firm requires the destruction of another (older firm), given the fixed

amount of labor available. The second condition equates the value of keeping a job created at t alive until age $\bar{a}(t)$ to the shadow value of creating one additional job at time $t + \bar{a}(t)$.

In steady state, the time arguments can be omitted and $\bar{a}(t) = \bar{a}$, for every t. Moreover, $e_f = 1/\bar{a}$ guarantees that the distribution is stationary and that all the available labor is employed. From the second condition, we obtain that $\varphi = e^{-\phi\bar{a}}$. This expression is easily interpretable: φ is the multiplier on the labor force constraint, and $e^{-\phi\bar{a}}$ is the value of slackening this constraint, i.e., the marginal contribution of an extra unit of labor to production (recall that this is also the equilibrium wage rate). We then arrive at

$$I = \int_0^{\bar{a}} e^{-(r-g+\phi)a} \left[1 - e^{-\phi(\bar{a}-a)}\right] da,$$

which is the key equilibrium condition (2) in the decentralized frictionless economy.

D Derivations of employment distributions

Consider the measure of firms of vintage a matched at time t. Over a short time interval of length Δ, the approximate change in this measure is

$$\mu(t + \Delta, a) = \mu(t, a - \Delta)(1 - \Delta\sigma) + \Delta\lambda_f v(t, a - \Delta).$$

Subtracting $\mu(t, a)$ from both sides and dividing by Δ we obtain

$$\frac{\mu(t + \Delta, a) - \mu(t, a)}{\Delta} = -\frac{\mu(t, a) - \mu(t, a - \Delta)}{\Delta} - \sigma\mu(t, a - \Delta) + \lambda_f v(t, a - \Delta).$$

Taking the limit for $\Delta \to 0$, one obtains

$$\mu_t(t, a) = -\mu_a(t, a) - \sigma\mu(t, a) + \lambda_f v(t, a).$$

In steady state, these measures do not change with t, and we obtain the differential equation (17).

Given the differential equation for employment (17), we can easily determine that

$$\mu(a) = \frac{v(0)\lambda_f}{\sigma + \lambda_f}\left[1 - e^{-(\sigma+\lambda_f)a}\right], \quad \text{and} \tag{D.1}$$

$$v(a) = \frac{v(0)}{\sigma + \lambda_f}\left[\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}\right]. \tag{D.2}$$

Thus, the total number of vacancies, v, satisfies

$$v = \int_0^{\bar{a}} v(a)da = \frac{v(0)}{\sigma + \lambda_f}\left\{\bar{a}\sigma + \frac{\lambda_f}{\sigma + \lambda_f}\left[1 - e^{-(\sigma+\lambda_f)\bar{a}}\right]\right\}. \tag{D.3}$$

Integrating both sides of the equation $v(a) + \mu(a) = v(0)$ over the support $[0, \overline{a})$, we conclude that total employment μ satisfies $\mu = v(0)\bar{a} - v$, or

$$\mu = \frac{v(0)\lambda_f}{\sigma + \lambda_f}\left\{\bar{a} - \frac{1}{\sigma + \lambda_f}\left[1 - e^{-(\sigma+\lambda_f)\bar{a}}\right]\right\}. \tag{D.4}$$

Equations (18) and (19) in the main text come from dividing (D.1) by (D.4) and (D.2) by (D.3), respectively.

To obtain (22), note that over a short time period of length Δ the change in unemployment is

$$u(t+\Delta) = u(t)\left[1 - \int_0^{\bar{a}} \Delta\lambda_w(a)da\right] + \Delta\sigma\int_0^{\bar{a}} \mu(t,a)da + \int_0^{\Delta} \mu(t,\bar{a}-x)dx.$$

The first two terms on the right-hand side are standard: assuming a Poisson process, these flows are approximately linear in the length of the interval since the interval is small. The third term sums all those matches that will reach $\bar{a}$ by the end of the period and will therefore separate. Subtracting $u(t)$ on both sides, dividing by Δ, taking limits as Δ approaches 0, and assuming steady state yields equation (22). To find the $\lim_{\Delta\to 0}\left[\int_0^{\Delta}\mu(\bar{a}-x)dx\right]/\Delta$, use l'Hôpital's rule.

Solving (22) for u, and substituting in for $\mu(\bar{a})/\mu$, we arrive at

$$u = \frac{1+\sigma\left(\frac{\bar{a}}{1-e^{-(\sigma+\lambda_f)\bar{a}}} - \frac{1}{\sigma+\lambda_f}\right)}{1+[\sigma+m(\theta,1)]\left(\frac{\bar{a}}{1-e^{-(\sigma+\lambda_f)\bar{a}}} - \frac{1}{\sigma+\lambda_f}\right)}. \tag{D.5}$$

Having found the unemployment rate u, the measure of entrant firms $v(0)$ is implied directly by equation (D.4), using the fact that $\mu = 1-u$.

E Proof of Lemma 1 (shape of the job creation curve)

Part (a): We show that the RHS of equation (JC) is increasing in $\bar{a}$ and λ_f, which implies that the (JC) curve is downward sloping in the $(\bar{a},\lambda_f)$ space.

Straightforward integration of (14), the equation defining the surplus, yields

$$S(a;\bar{a},\lambda_f) = \frac{e^{-\phi a}\left(1-e^{-\rho_2(\bar{a}-a)}\right)}{\rho_2} - \frac{e^{-\phi\bar{a}}\left(1-e^{-\rho_1(\bar{a}-a)}\right)}{\rho_1}, \tag{E.1}$$

with $\rho_0 \equiv r-g+\sigma$, $\rho_1 \equiv \rho_0+(1-\beta)\lambda_f$, and $\rho_2 \equiv \rho_1+\phi$. The surplus function is decreasing in a and increasing in $\bar{a}$. It follows that the RHS of (JC) is increasing in the exit age $\bar{a}$.

To show that the RHS of (JC) is increasing in λ_f, rewrite the (JC) equation as

$$I = e^{-\phi\bar{a}}(1-\beta)\lambda_f\int_0^{\bar{a}} e^{-(r-g)a}\left\{\int_0^{\bar{a}-a} e^{-\left(\rho_0+(1-\beta)\lambda_f\right)\tilde{a}}\left[e^{\phi(\bar{a}-a-\tilde{a})}-1\right]d\tilde{a}\right\}da. \tag{E.2}$$

We now show that the integral of the function $f(\tilde{a};\lambda_f) \equiv \lambda_f e^{-\left(\rho_0+\lambda_f(1-\beta)\right)\tilde{a}}$ with respect to the weighting function $h(\tilde{a}) \equiv \left[e^{\phi(\bar{a}-a-\tilde{a})}-1\right]$ is increasing in λ_f. The function f is increasing (decreasing) with respect to λ_f for $\tilde{a} < (>) \frac{1}{(1-\beta)\lambda_f}$. The integral of the function f, however, is increasing with λ_f, as

$$\int_0^{\bar{a}-a} f(\tilde{a};\lambda_f)\,d\tilde{a} = \left[1-e^{-\left(\rho_0+(1-\beta)\lambda_f\right)(\bar{a}-a)}\right]\frac{\lambda_f}{\rho_0+(1-\beta)\lambda_f}.$$

Therefore,

$$\frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}-a} f(\tilde{a};\lambda_f)\,d\tilde{a} > 0.$$

The integral of f with respect to h is also increasing with λ_f, since the weighting function h is positive and monotonically decreasing in $\tilde{a}$:

$$\begin{aligned}\frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}-a} f\left(\tilde{a};\lambda_f\right)h\left(\tilde{a}\right)d\tilde{a} &= \int_0^{a_+} f_{\lambda_f}\left(\tilde{a};\lambda_f\right)h\left(\tilde{a}\right)d\tilde{a}+\int_{a_+}^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right)h\left(\tilde{a}\right)d\tilde{a}\\ > \int_0^{a_+} f_{\lambda_f}\left(\tilde{a};\lambda_f\right)h\left(a_+\right)d\tilde{a}&+\int_{a_+}^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right)h\left(a_+\right)d\tilde{a}=h\left(a_+\right)\int_0^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right)d\tilde{a}>0,\end{aligned}$$

with $a_+=\min\left\{\frac{1}{(1-\beta)\lambda_f},\bar{a}-a\right\}$.

Part (b): Taking the limit of (E.2) as $\bar{a}\to\infty$, we obtain

$$I=\frac{(1-\beta)\,\lambda_f^{\min}}{r-g+\sigma+(1-\beta)\,\lambda_f+\phi}\frac{1}{r-g+\phi}\Rightarrow\lambda_f^{\min}=\frac{(r-g+\phi)\,I}{1-(r-g+\phi)\,I}\left(\frac{r-g+\phi+\sigma}{1-\beta}\right),$$

which is expression (21), given the definition $\bar{r}\equiv r-g+\phi$.

Integrating equation (E.2) yields

$$I=\frac{(1-\beta)\,\lambda_f}{\rho_2}\left[\frac{1-e^{-(r-g+\phi)\bar{a}}}{r-g+\phi}-\frac{\rho_2}{\rho_1}e^{-\phi\bar{a}}\frac{1-e^{-(r-\phi)\bar{a}}}{r-\phi}+e^{-\rho_2\bar{a}}\frac{1-e^{g\bar{a}}}{\sigma+(1-\beta)\,\lambda_f}\frac{\phi}{\rho_1}\right]. \tag{E.3}$$

Taking the limit of this expression as $\lambda_f\to\infty$, we obtain

$$\begin{aligned}I &= \frac{1-e^{-(r-g+\phi)\bar{a}^{\min}}}{r-g+\phi}-e^{-\phi\bar{a}^{\min}}\frac{1-e^{-(r-g)\bar{a}^{\min}}}{r-g}\\ &= \int_0^{\bar{a}^{\min}} e^{-(r-g+\phi)a}\left[1-e^{-\phi\left(\bar{a}^{\min}-a\right)}\right]da \quad\Rightarrow\quad \bar{a}^{\min}=\bar{a}^{CE},\end{aligned}$$

where $\bar{a}^{CE}$ is the age cut-off of the frictionless economy, implicitly defined by (2).

F Proof of Lemma 2 (shape of the job destruction curve)

Part (a): The (JD) curve is implicitly defined by the equation

$$1=\ell e^{\phi\bar{a}}+\beta\int_0^{\bar{a}}\lambda_w(a;\bar{a},\lambda_f)e^{\phi\bar{a}}S(a;\bar{a},\lambda_f)da. \tag{JD}$$

We show that the RHS of this expression is increasing in $\bar{a}$ and decreasing in λ_f, which implies that the (JD) curve is upward-sloping in $(\bar{a},\lambda_f)$ space.

The RHS of (JD) is increasing in $\bar{a}$: Let $\tilde{S}\equiv e^{\phi\bar{a}}S$. It is convenient to take the derivative of the integrand and express it in terms of elasticities as

$$\left[\frac{\partial\lambda_w}{\partial\bar{a}}\frac{\bar{a}}{\lambda_w}+\frac{\partial\tilde{S}}{\partial\bar{a}}\frac{\bar{a}}{\tilde{S}}\right]\frac{\tilde{S}\lambda_w}{\bar{a}}. \tag{F.1}$$

The elasticity of the density λ_w is given by

$$\frac{\partial\lambda_w}{\partial\bar{a}}\frac{\bar{a}}{\lambda_w}=-\frac{\sigma\bar{a}+\lambda_f\bar{a}e^{-\left(\sigma+\lambda_f\right)\bar{a}}}{\sigma\bar{a}+\lambda_f\int_0^{\bar{a}}e^{-\left(\sigma+\lambda_f\right)a}da}<0.$$

Thus, the first term in (F.1) is negative, but its absolute value is less than one since $e^{-(\sigma+\lambda_f)a} > e^{-(\sigma+\lambda_f)\bar{a}}$ for $a \leq \bar{a}$. We will now show that the elasticity of the modified surplus function $\tilde{S}$ with respect to $\bar{a}$ is greater than or equal to one. It will therefore follow that the integral in (JD) is increasing in $\bar{a}$. We proceed in two steps. First, we show that the elasticity of $\tilde{S}$ with respect to $\bar{a}$ is increasing in a for any given $\bar{a}$. Second, we show that this elasticity is greater than one at $a = 0$.

The elasticity of $\tilde{S}$ with respect to $\bar{a}$ is given by

$$\frac{\partial \tilde{S}}{\partial \bar{a}} \frac{\bar{a}}{\tilde{S}} = \frac{\phi \bar{a}}{1 - H(\bar{a} - a)}, \quad \text{with } H(x) \equiv e^{-\phi x} \frac{(1 - e^{-\rho_1 x})/\rho_1}{(1 - e^{-\rho_2 x})/\rho_2}, \tag{F.2}$$

and $x \equiv \bar{a} - a$. The sign of the derivative of the function H is given by

$$sign(H') = sign\left(\rho_2 \gamma e^{-(\rho_2+\phi)} \left[\int_0^x e^{\phi y} dy - \int_0^x e^{\rho_2 y} dy\right]\right).$$

Since $\rho_2 = \rho_1 + \phi$ and since, by assumption, $\rho_1, \phi > 0$, the function H is decreasing in x. Therefore the elasticity is increasing in a.

Now, for the elasticity in (F.2) evaluated at $a = 0$ to be greater than or equal to 1, it must be that

$$\bar{a}\phi > 1 - e^{-\phi \bar{a}} \frac{(1 - e^{-\rho_1 \bar{a}})/\rho_1}{(1 - e^{-\rho_2 \bar{a}})/\rho_2},$$

or, equivalently,

$$\frac{(1 - e^{-\rho_1 \bar{a}})/\rho_1}{(1 - e^{-\rho_2 \bar{a}})/\rho_2} = \frac{\int_0^{\bar{a}} e^{-\rho_1 a} da}{\int_0^{\bar{a}} e^{-\rho_2 a} da} > e^{\phi \bar{a}} (1 - \bar{a}\phi),$$

which is true as for any $z > 0$, $e^z (1 - z) \leq 1$, whereas the LHS exceeds one since $\rho_1 = \rho_2 - \phi < \rho_2$.

The RHS of (JD) is decreasing in λ_f under A2: We rewrite equation (F.1) as

$$e^{-\phi \bar{a}} = \ell + \beta [m(\theta, 1) \lambda_f] \int_0^{\bar{a}} \left[\frac{\lambda_w(a; \bar{a}, \lambda_f)/\lambda_f}{m(\theta, 1)}\right] \tilde{S}(a; \bar{a}, \lambda_f) da.$$

It is immediate that the two terms under the integral, the modified density λ_w and the modified surplus function $\tilde{S}$, are decreasing in λ_f. Substituting λ_f with $m(\theta, 1)/\theta$ from (3), the term pre-multiplying the integral becomes $\beta m^2(\theta, 1)/\theta$. Proving that this term is decreasing in λ_f is equivalent to showing that it is increasing in θ. Differentiating, we obtain that its derivative is positive when

$$\frac{1}{2} < \frac{m_1(\theta, 1)\theta}{m(\theta, 1)} = \frac{m_1(v, u) v}{m(v, u)},$$

where the equality follows from the linear homogeneity of m and the fact that its derivative is homogeneous of degree zero. This is the condition in assumption A2 stating that the elasticity of the matching function with respect to vacancies should be smaller than one half.

Part (b): For λ_f large, the density λ_w converges to a uniform density on $[0, \bar{a}]$, $\lim_{\lambda_f \to \infty} \lambda_w(a) = \frac{1}{\bar{a} + 1/\sigma}$, and the surplus function converges to zero, $\lim_{\lambda_f \to \infty} S(a; \bar{a}, \lambda_f) = 0$. Therefore equation (JD) becomes $1 = \ell e^{\phi \bar{a}^{\max}} \Rightarrow \bar{a}^{\max} = -\ln(\ell)/\phi$.

G Proof of Lemma 3 (effects of capital-embodied technology)

Recall that $g \equiv \psi + \omega\gamma$ and $\phi \equiv \omega(\gamma + \delta)$; thus γ enters both g and ϕ. Let us start from the (JC) curve. The RHS of (JC) is increasing in γ, because the function $f(\gamma) = e^{g(\gamma)a} S(a; \bar{a}, \lambda_f, \gamma)$ is increasing in γ. Since ρ_2 does not depend on γ, using (E.1) the derivative of f with respect to γ is

$$\frac{\partial f}{\partial \gamma} = \left(x - \frac{1 - e^{-\rho_1 x}}{\rho_1} \right) \frac{\omega e^{ga - \phi\bar{a}}}{\rho_1},$$

with $x = \bar{a} - a$. Notice that $\partial f / \partial \gamma = 0$ at $x = 0$, and that the term in brackets is increasing in x, and therefore $\partial f / \partial \gamma > 0$ for $a \in [0, \bar{a}]$. Since the RHS of (JC) is increasing in γ and $\bar{a}$, the (JC) curve shifts downward as γ increases.

In the (JD) equation, multiply both sides by $e^{\phi\bar{a}}$. Now, the RHS of (JD) is increasing in γ because $\tilde{S} \equiv e^{\phi(\gamma)\bar{a}} S(a; \bar{a}, \lambda_f, \gamma)$ is increasing in γ. The proof is exactly as for the (JC) curve because γ enters the same way in g and in ϕ. Since $\tilde{S}$ is increasing in γ for all $a \in [0, \bar{a}]$, the RHS of (F.1) is increasing in γ. Since the RHS of (F.1) is increasing in γ and $\bar{a}$, the (JD) curve shifts downward as γ increases.

H Derivation of the equilibrium conditions in the economy with unemployment benefits and taxes

We begin by defining a new surplus function $S_\tau(a)$, denoting the *after-tax* surplus from the match:

$$(r - g) S_\tau(a) = \max\{ e^{-\phi a} - (\tau_f + \tau_w) w(a) - \sigma S_\tau(a) - \lambda_f (1 - \beta_\tau) S_\tau(a) - (r - g) U + S'_\tau(a), 0 \}. \quad \text{(H.1)}$$

The novelty is that the wage does not cancel out in the surplus equation, due to the presence of taxes. Let $\tau \equiv \tau^f + \tau^w$ be the total tax wedge. Joint surplus maximization implies that at every instant a fraction β_τ of the total surplus $S_\tau(a)$ of a type a match goes to the worker and the rest goes to the firm, implying

$$W(a) - U = \beta_\tau S_\tau(a) \text{ and } J(a) - V(a) = (1 - \beta_\tau) S_\tau(a), \quad \text{(H.2)}$$

where β_τ is defined as

$$\beta_\tau \equiv \left(\frac{1 - \tau_w}{1 + \tau_f - \beta\tau} \right) \beta. \quad \text{(H.3)}$$

The wage that solves the Nash bargaining is

$$w(a) = (r - g) U + \beta_\tau \left[(r - g + \sigma) S_\tau(a) - S'_\tau(a) \right]. \quad \text{(H.4)}$$

Substituting this expression for the wage into $(H.1)$ and rearranging terms, we arrive at

$$\begin{aligned} (r - g + \sigma) S_\tau(a) \;=\; & \max \left\{ \left(\frac{1 + \tau_f - \beta\tau}{1 + \tau_f} \right) e^{-\phi a} - \lambda_f (1 - \beta) S_\tau(a) \right. . \\ & \left. - \left(\frac{1 + \tau_f - \beta\tau}{1 - \tau_w} \right) (r - g) U + S'_\tau(a), 0 \right\} \end{aligned} \quad \text{(H.5)}$$

The solution of this first-order linear differential equation is the function:

$$S_\tau(a) = (1 + \tau_f - \beta\tau) \int_a^{\overline{a}} e^{-\rho_1(\tilde{a} - a)} \left[\frac{e^{-\phi\tilde{a}}}{1 + \tau_f} - \frac{(r - g) U}{1 - \tau_w} \right] d\tilde{a}, \quad \text{(H.6)}$$

where we have used the boundary condition associated with the fact that $S_\tau(\bar{a}) = 0$ and the usual definition $\rho_1 \equiv r - g + \sigma + (1-\beta)\lambda_f$. Substituting the optimal separation rule

$$e^{-\phi\bar{a}} = \left(\frac{1+\tau_f}{1-\tau_w}\right)(r-g)U \tag{H.7}$$

back into $(H.6)$ yields

$$S_\tau(a;\bar{a},\lambda_f) = \left(\frac{1+\tau_f-\beta\tau}{1+\tau_f}\right)\int_a^{\bar{a}} e^{-\rho_1(\tilde{a}-a)}\left(e^{-\phi\tilde{a}} - e^{-\phi\bar{a}}\right)d\tilde{a} = \left(\frac{1+\tau_f-\beta\tau}{1+\tau_f}\right)S(a;\bar{a},\lambda_f) \tag{H.8}$$

for the after-tax surplus as a function of the pre-tax surplus. Using $(H.8)$ in the free-entry condition $V(0;\bar{a},\lambda_f) = I$ delivers exactly the same (JC) condition as before, since $(1-\beta_\tau)(1+\tau_f-\beta\tau)/(1+\tau_f) = (1-\beta)$. Substituting (9) and (H.2) into (H.7), and using the fact that $\beta_\tau(1+\tau_f-\beta\tau)/(1-\tau_w) = \beta$, we arrive at the new optimal separation condition (JD$'$) in the main text.

I Derivation of the equilibrium conditions in the economy with firing cost

The surplus of an ongoing match is $S(a) \equiv J(a) + W(a) - U - (V(a) - \kappa)$. Nash bargaining yields $W(a) - U = \beta S(a)$ and $J(a) - V(a) + \kappa = (1-\beta)S(a)$. Note the similarity to (6), with the only change being the firing cost κ entering the definition of the surplus. For new matches, the surplus is defined as $S_n(a) = S(a) - \kappa$ since the firing tax κ is not due in case of disagreement. Combining the solution of the Nash bargaining on new and continuing matches, one obtains

$$W_n(a) - W(a) = -\left[J_n(a) - J(a)\right] = -\beta\kappa,$$

which shows that $\beta\kappa$ is the hiring fee paid by the worker to the firm upon entry in the match.

In this model, we have two destruction cutoffs. Vacant machines are destroyed at age $\hat{a}$ where $V(\hat{a}) = 0$, implying $S_n(\hat{a}) = 0$, whereas filled jobs are destroyed at age $\bar{a}$ when $S(\bar{a}) = 0$, with $\hat{a} < \bar{a}$.

The differential equation for the surplus is now split in two ranges:

$$(r-g+\sigma)S(a) = \begin{cases} \max\left\{e^{-\phi a} - \lambda_f(1-\beta)\left[S(a)-\kappa\right] - (r-g)(U-\kappa) + S'(a), 0\right\} & \text{if } a \in [0,\hat{a}), \\ \max\left\{e^{-\phi a} - (r-g)(U-\kappa) + S'(a), 0\right\} & \text{if } a \in [\hat{a},\bar{a}]. \end{cases} \tag{I.1}$$

To see this, note that as long as $a \in [0,\hat{a})$, $V(a) \geq 0$, whereas if $a \geq \hat{a}$, $V(a) = 0$. Over $[\hat{a},\bar{a}]$,

$$S(a) = \int_a^{\bar{a}} e^{-(r-g+\sigma)(\tilde{a}-a)}\left[e^{-\phi\tilde{a}} - (r-g)(U-\kappa)\right]d\tilde{a}, \tag{I.2}$$

with the terminal condition

$$e^{-\phi\bar{a}} = (r-g)U + (r-g)\kappa. \tag{I.3}$$

Using equation $(I.3)$ in $(I.2)$, evaluating the latter equation at $\hat{a}$, and exploiting the fact that $S(\hat{a}) - \kappa = 0$ yields our first equilibrium condition characterizing the optimal destruction of a vacancy:

$$\kappa = \int_{\hat{a}}^{\bar{a}} e^{-(r-g+\sigma)(\tilde{a}-\hat{a})}\left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}}\right]d\tilde{a}. \tag{I.4}$$

This equation, stated as equation (VD″) in the main text, represents an implicit function $\hat{a}(\bar{a})$, which only depends on primitive parameters (in particular, it does not depend on λ_f).

For ease of notation, let $\rho_1 \equiv r - g + \sigma + \lambda_f(1-\beta)$. The solution to $(I.1)$ now is

$$\begin{aligned} S(a) &= \int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}} + \lambda_f(1-\beta)\kappa \right] d\tilde{a} + e^{-\rho_1(\hat{a}-a)} S(\hat{a}) \\ &= \int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}} \right] d\tilde{a} + \kappa \left[e^{-\rho_1(\hat{a}-a)} + \lambda_f(1-\beta) \frac{1 - e^{-\rho_1(\hat{a}-a)}}{\rho_1} \right], \end{aligned}$$

where the second line uses the vacancy destruction condition $S(\hat{a}) = \kappa$. To derive the job creation and the job destruction conditions, we need $S_n(a)$. Since $S_n(a) = S(a) - \kappa$, we have

$$\begin{aligned} S_n(a; \hat{a}, \bar{a}, \lambda_f) &= \int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}} \right] d\tilde{a} + \kappa \left[e^{-\rho_1(\hat{a}-a)} + \lambda_f(1-\beta) \frac{1 - e^{-\rho_1(\hat{a}-a)}}{\rho_1} - 1 \right] \\ &= \int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}} - (r - g + \sigma)\kappa \right] d\tilde{a}. \end{aligned}$$

To obtain (JC″) and (JD″) in the main text, it is enough to follow our derivations in Section 2.1.2. Finally, when $\kappa = 0$, the surplus function becomes exactly as in (14) and, from (I.4), $\hat{a} \to \bar{a}$. Thus, we get back our baseline model.

J Proof of Proposition 3 (equilibrium with firing costs)

Part (a): The proof that the (JC") curve is negatively sloped follows the argument in Lemma 1, Appendix E. The difference is that the relevant weighting function now is $h(\tilde{a}) = e^{\phi(\hat{a}-a-\tilde{a})} - e^{-\phi(\bar{a}-\hat{a})} - e^{\phi\hat{a}}(r - g + \sigma)\kappa$. Using the implicit relationship between $\bar{a}$ and $\hat{a}$ in $(I.4)$, it is straightforward to show that this weighting function is always positive in the range of interest, i.e., $\tilde{a} \in [0, \hat{a} - a]$.

The asymptote $\overline{\overline{a}}^{CE}$ of the (JC") curve, as $\lambda_f \to \infty$, is determined implicitly by the equation

$$I = \frac{1 - e^{-(r-g+\phi)\hat{a}\left(\overline{\overline{a}}^{CE}\right)}}{r - g + \phi} - \frac{e^{-\phi\bar{a}} - e^{-\left[(r-g)\hat{a}\left(\overline{\overline{a}}^{CE}\right) + \phi\overline{\overline{a}}^{CE}\right]}}{r - g} - \kappa(r - g + \sigma) \frac{1 - e^{-(r-g)\hat{a}\left(\overline{\overline{a}}^{CE}\right)}}{r - g}, \quad \text{(J.1)}$$

where $\hat{a}\left(\overline{\overline{a}}^{CE}\right)$ is the implicit function $(I.4)$ evaluated at $\bar{a} = \hat{a}\left(\overline{\overline{a}}^{CE}\right)$.

Now, take the limit of the (JC") equation as $\bar{a} \to \infty$. After some algebra, we see that one needs to find a zero to the following expression in $\lambda_f^{\min}$:

$$\begin{aligned} &\left(\lambda_f^{\min}\right)^2 (1-\beta) \left[\frac{(\bar{r} - \phi)(1 - \bar{r}I)}{\bar{r} - \phi + \sigma} - \kappa\bar{r} \right] + \lambda_f^{\min} \left[(\bar{r} - \phi)(1 - \bar{r}I) - (\bar{r} + \sigma) \left(\kappa\bar{r} + \frac{\bar{r}(\bar{r} - \phi) I}{\bar{r} - \phi + \sigma} \right) \right] \\ &- \frac{r + \sigma}{1 - \beta} \frac{\bar{r}(\bar{r} - \phi) I}{\bar{r} - \phi + \sigma}. \end{aligned}$$

For this to equal zero for only one value of $\lambda_f^{\min}$, the term multiplying $\left(\lambda_f^{\min}\right)^2$ has to be positive, i.e.,

$$\frac{(\bar{r} - \phi)(1 - \bar{r}I)}{\bar{r} - \phi + \sigma} - \kappa\bar{r} > 0,$$

which determines the sufficiency of assumption A1". Note that, if $\kappa = 0$, this condition coincides with assumption A1 of the baseline model.

We can now compute the asymptote of the (JD") curve as $\lambda_f \to \infty$. It is easy to see that $\overline{\overline{a}}^{\max} = -\ln\left[\ell - (r-g)\,\kappa\right]/\phi$, which will be strictly positive, if A3 holds. As long as $\overline{\overline{a}}^{\max} > \overline{\overline{a}}^{CE}$, a condition equivalent to that needed for Proposition 2, we can establish global existence of an equilibrium, since the curves are continuous in $(\bar{a}, \lambda_f)$.

Parts (b) and (c): It suffices to note that the model with $\kappa = 0$ is a special case of the $\kappa > 0$ model: in the special case, $\bar{a} = \hat{a}$. Thus, all the derivatives of the (JD") and the (JC") conditions (with respect to $\bar{a}$, λ_f, and γ, in particular), since these conditions are continuous functions, are as in the baseline model. Furthermore, their signs will not change in a neighborhood of $\kappa = 0$.

Part (d): First, note that deriving the surplus function with respect to κ yields

$$\partial S_n/\partial\kappa = -\left[(r-g+\sigma)\int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)}d\tilde{a}\right] < 0$$

and note that, along the (VD") curve, we have $\partial\hat{a}/\partial\kappa = -\left[e^{-\phi\hat{a}} - e^{-\phi\bar{a}}\right]^{-1} < 0$. For what follows, it is also useful to determine the sign of the term

$$\begin{aligned}
&\frac{\partial S_n(a)}{\partial\kappa} + \frac{\partial S_n(a)}{\partial\hat{a}}\frac{\partial\hat{a}}{\partial\kappa} \\
= \;& -(r-g+\sigma)\int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)}d\tilde{a} - \left[e^{-\phi\hat{a}} - e^{-\phi\bar{a}}\right]^{-1} e^{-\rho_1(\hat{a}-a)}\left[e^{-\phi\hat{a}} - e^{-\phi\bar{a}} - (r-g+\sigma)\,\kappa\right] \\
= \;& -(r-g+\sigma)\int_a^{\hat{a}} e^{-\rho_1(\tilde{a}-a)}d\tilde{a} - e^{-\rho_1(\hat{a}-a)} + (r-g+\sigma)\,e^{-\rho_1(\hat{a}-a)}e^{\phi\hat{a}}\left(\frac{\kappa}{1-e^{-\phi(\bar{a}-\hat{a})}}\right).
\end{aligned}$$

The first two terms are negative. Consider the third term and take the limit as $\kappa \to 0$. Using the (VD") to substitute κ out, we have that

$$\begin{aligned}
\lim_{\kappa\to 0}\left(\frac{\kappa}{1-e^{-\phi(\bar{a}-\hat{a})}}\right) &= e^{-\phi\hat{a}}\lim_{(\hat{a}-\bar{a})\to 0}\frac{\frac{1-e^{-(r-g+\sigma+\phi)(\bar{a}-\hat{a})}}{r-g+\sigma+\phi} - \frac{e^{-\phi(\bar{a}-\hat{a})}-e^{-(r-g+\sigma+\phi)(\bar{a}-\hat{a})}}{r-g+\sigma}}{1-e^{-\phi(\bar{a}-\hat{a})}} \\
&= e^{\phi\bar{a}}\frac{\left[1+\frac{\phi}{r-g+\sigma} - \frac{r-g+\sigma+\phi}{r-g+\sigma}\right]}{\phi} = 0,
\end{aligned}$$

where the second line uses l'Hôpital rule. We conclude that, in a neighborhood of $\kappa = 0$,

$$\frac{\partial S_n(a)}{\partial\kappa} + \frac{\partial S_n(a)}{\partial\hat{a}}\frac{\partial\hat{a}}{\partial\kappa} < 0.$$

Now consider the (JC") equation. Differentiate with respect to λ_f and κ, fixing $\bar{a}$, recall that $S_n(\hat{a};\hat{a},\lambda_f) = 0$, and obtain

$$0 = \left\{\lambda_f\int_0^{\hat{a}} e^{-(r-g)a}\left[\frac{\partial S_n(a)}{\partial\kappa} + \frac{\partial S_n(a)}{\partial\hat{a}}\frac{\partial\hat{a}}{\partial\kappa}\right]da\right\}d\kappa + \left\{\int_0^{\hat{a}} e^{-(r-g)a}\frac{\partial\left[\lambda_f S_n(a)\right]}{\partial\lambda_f}da\right\}d\lambda_f.$$

We have just shown that the term multiplying $d\kappa$ is negative. In part (b) we established that the term multiplying $d\lambda_f$ is positive. As a result, $d\lambda_f/d\kappa > 0$ along the (JC") curve. Thus, the (JC") curve shifts upward in response to a rise in κ.

Turning to the (JD") equation, fix $\bar{a}$, differentiate with respect to λ_f and κ and recall that $S_n(\hat{a};\hat{a},\lambda_f)=0$ so that we obtain

$$0=\left\{-(r-g)+\beta\int_0^{\hat{a}}\left\{\lambda_w(a)\left[\frac{\partial S_n(a)}{\partial\kappa}+\frac{\partial S_n(a)}{\partial\hat{a}}\frac{\partial\hat{a}}{\partial\kappa}\right]\right\}da\right\}d\kappa+\left[\beta\int_0^{\hat{a}}\frac{\partial\lambda_w(a)S_n(a)}{\partial\lambda_f}da\right]d\lambda_f.$$

The two terms multiplying $d\kappa$ are both negative. The term multiplying $d\lambda_f$ is negative. Then, $d\lambda_f/d\kappa<0$ along the (JD") curve, and the (JD") curve shifts upward in response to a rise in κ.

K Equilibrium conditions in the model with (b,τ_f,τ_w,κ)

Recall the definition $\tau\equiv\tau_f+\tau_w$. The destruction condition for vacancies is given by the implicit equation

$$\left(\frac{1+\tau_f-\beta\tau}{1+\tau_f}\right)\kappa=\int_{\hat{a}}^{\bar{a}}e^{-(r-g+\sigma)(\tilde{a}-\hat{a})}\left[e^{-\phi\tilde{a}}-e^{-\phi\bar{a}}\right]d\tilde{a}.$$

The job creation and job destruction conditions, respectively, read

$$\begin{aligned}I &= (1-\beta)\lambda_f\int_0^{\hat{a}}e^{-(r-g)a}S_n(a;\hat{a},\bar{a},\lambda_f)da,\\ e^{-\phi\bar{a}}+(r-g)\kappa &= (\ell+b)\left(\frac{1+\tau_f}{1-\tau_w}\right)+\beta\int_0^{\hat{a}(\bar{a};\kappa,\tau)}\lambda_w(a;\hat{a},\lambda_f)S_n(a;\hat{a},\bar{a},\lambda_f)da,\end{aligned}$$

where the definition of the pre-tax surplus for new firms $S_n(a;\hat{a},\bar{a},\lambda_f)$ is given by

$$S_n(a;\hat{a},\bar{a},\lambda_f)=\int_a^{\hat{a}}e^{-\rho_1(\tilde{a}-a)}\left[e^{-\phi\tilde{a}}-e^{-\phi\bar{a}}-(r-g+\sigma)\left(\frac{1+\tau_f}{1+\tau_f-\beta\tau}\right)\kappa\right]d\tilde{a},$$

and the expression for the equilibrium distribution of vacancies is that of the model with firing costs.

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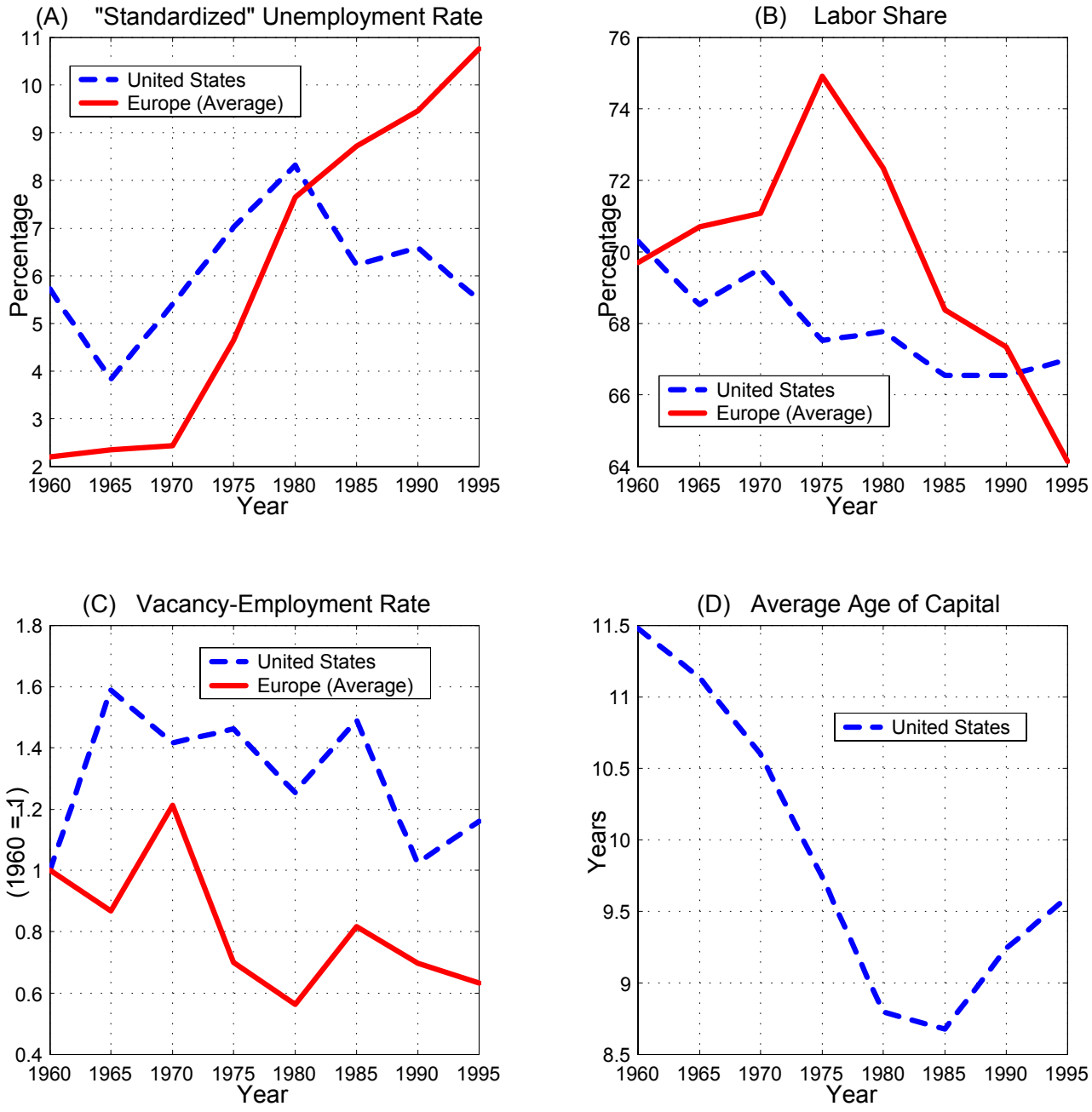

Figure 1: Panels A, B, and C depict the OECD standardized definition of the unemployment rate, the labor share of output, the vacancy-employment ratio for the U.S. and for an average of 15 European countries, respectively. Panel D plots the evolution of the age of capital (equipment and structures) in the U.S. economy. See Appendix A.1 for a detailed description of the data sources and the methodology.

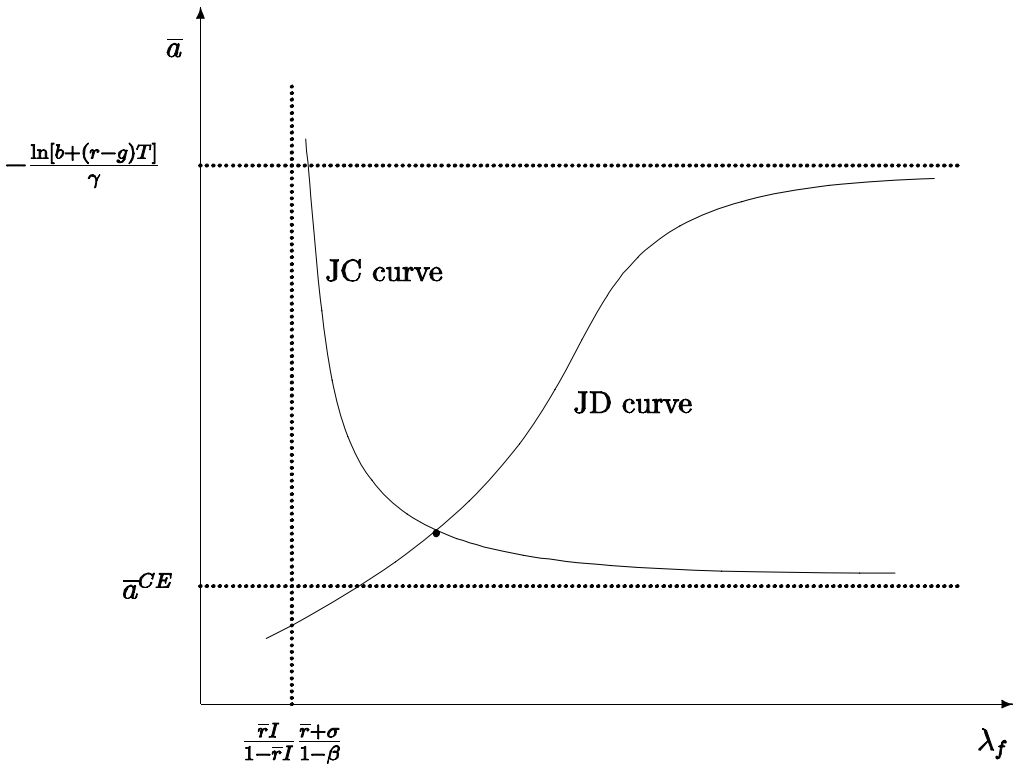

Figure 2: Job Creation and Job Destruction conditions, plotted in the $(\lambda_f,\ \bar{a})$ space.

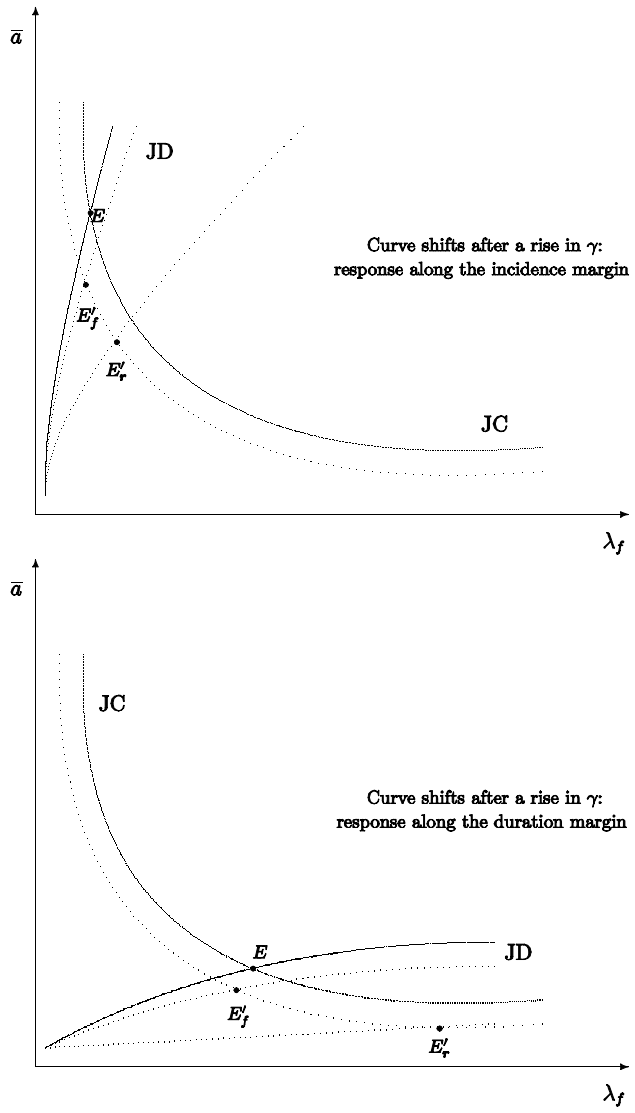

Figure 3: Qualitative comparative statics with respect to γ. The labels E_F and E_R refer to the initial (pre-shock) equilibrium in the flexible and rigid economy, respectively. The label E'_F and E'_R refer to the final equilibrium. The initial location of the (JD) curve is determined by the size of the policies. The initial location of the (JD) curve is determined by the size of the policies.

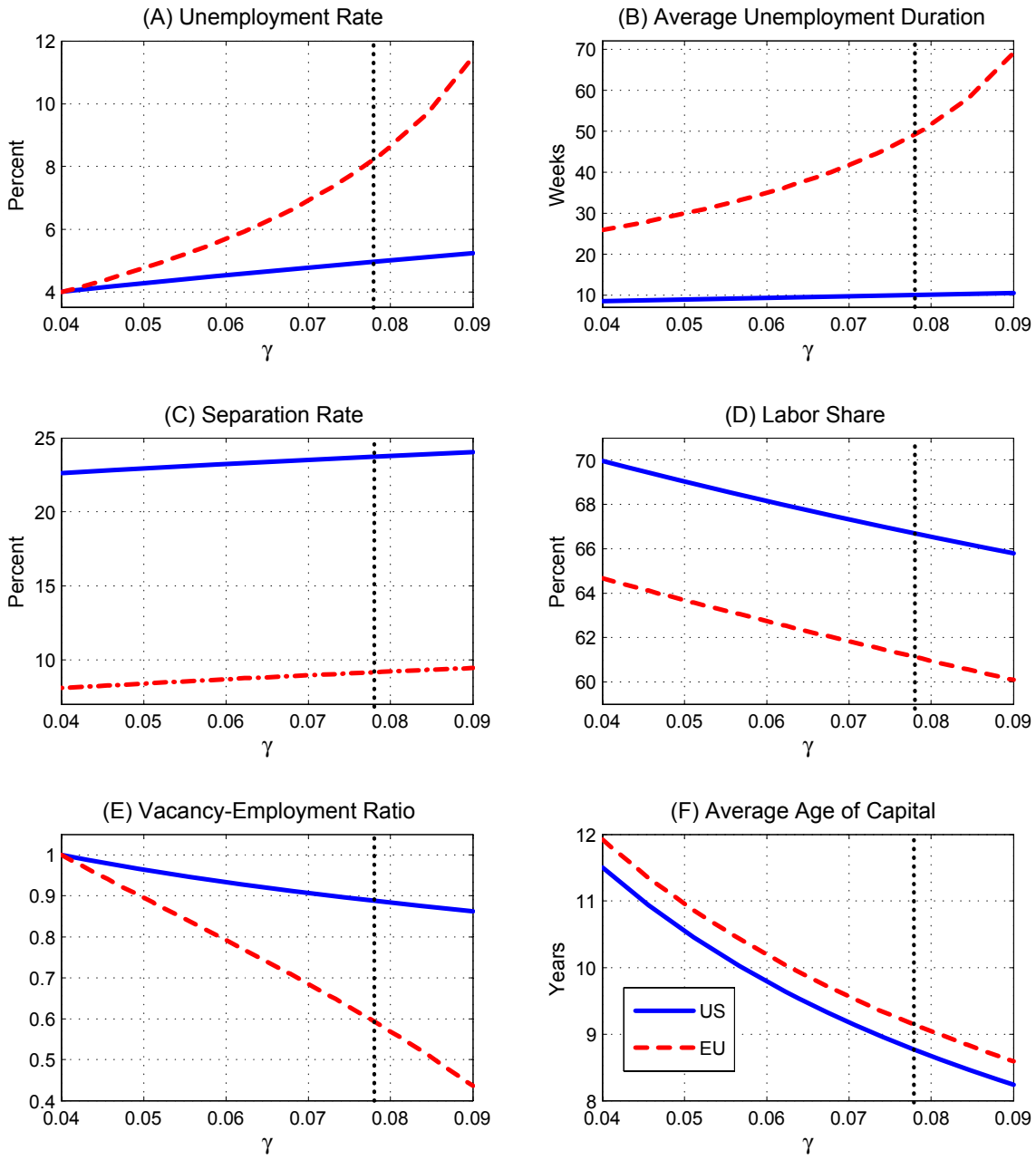

Figure 4: Experiment where the common shock is a rise in the rate of embodied technical change γ and the economies differ in the level of policies $(b, \tau_f, \tau_w, \kappa)$. The two economies are calibrated to the United States and Europe. The vertical dotted line indicates the new long-run level of embodied technical change (7.7 percent per year).

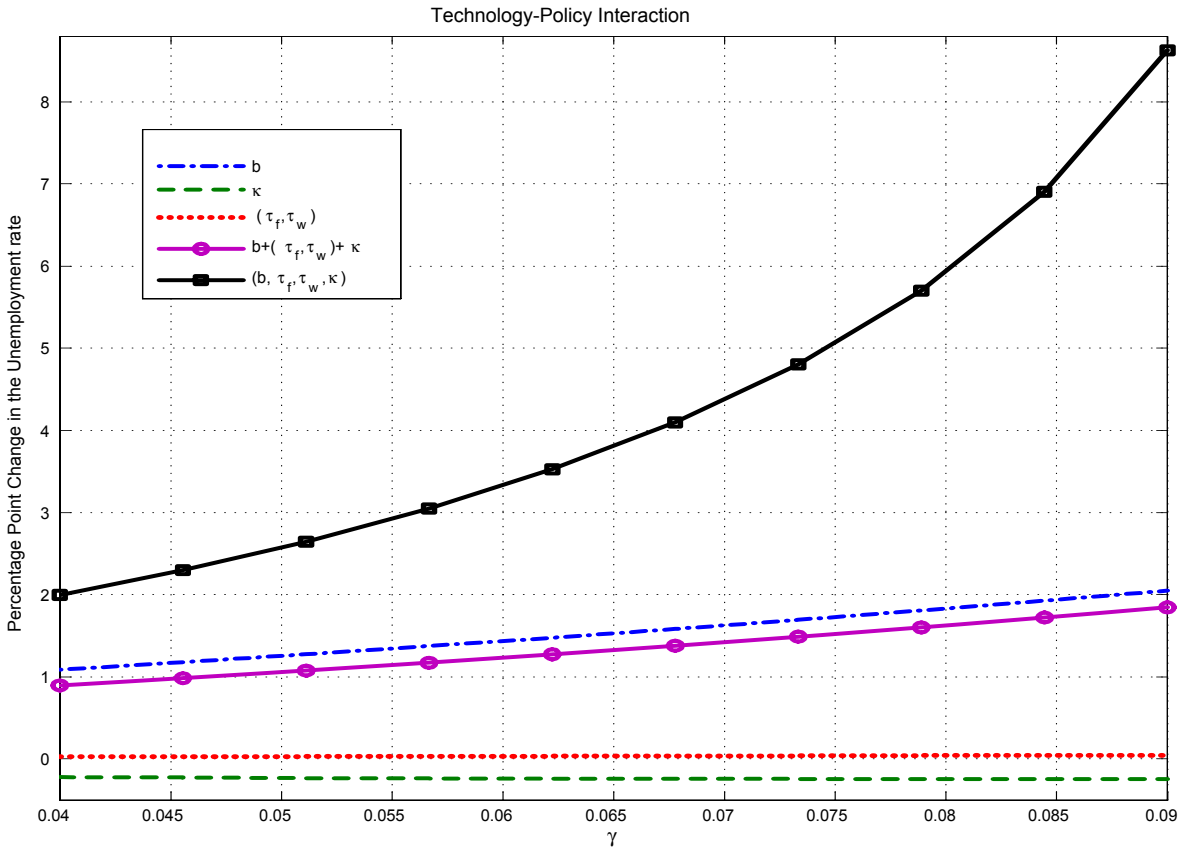

Figure 5. The lines labelled b, κ, and (τ_w, τ_f) plot the change in the unemployment rate due to each single policy (unemployment benefits, firing cost and taxes), as a function of the rate of embodied technical change γ. The line labelled $b + (\tau_w, \tau_f) + \kappa$ sums up the three individual policy effects. The line labelled $(b, \kappa, \tau_w, \tau_f)$ plots the change in the unemployment rate due to all policies *jointly* considered.