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FINANCIAL FRAGILITY AND OVER-THE-COUNTER MARKETS

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This paper studies the interaction between financial fragility and over-the-counter markets. In the model, the financial sector is composed of a large number of investors divided into different groups, which are interpreted as financial institutions, and a large number of dealers. Financial institutions and dealers trade assets in an over-the-counter market *à la* Duffie *et al.* (2005) and Lagos and Rocheteau (2009). Investors are subject to privately observed preference shocks, and financial institutions use the balanced team mechanism, proposed by Athey and Segal (2013), to implement an efficient risk-sharing arrangement among its investors. I show that when the market is more liquid, in the sense that the search friction is mild, the economy is more likely to have a unique equilibrium and, therefore, is not fragile. However, when the search friction is severe, I provide examples with run equilibria—where investors announce low valuation of assets because they believe everyone else in their financial institution is doing the same. In terms of welfare, I find that, conditional on bank runs existing, the welfare impact of the search friction is ambiguous. The reason is that, during runs, trade is inefficient and, as a result, a friction that reduces trade during runs has the potential to improve welfare. This result is in sharp contrast with the existing literature which suggests that search friction has a negative impact on welfare.

KEYWORDS: Decentralized trade, search, trade volume, bid-ask spreads, liquidity, liquidity insurance, financial fragility, bank-run, dynamic mechanism design.

JEL CLASSIFICATION: D82, E58, G01, G21.

1. INTRODUCTION

IN DEVELOPED FINANCIAL SYSTEMS, investors participate in asset markets as part of financial institutions which trade assets (often over the counter) on their investors' behalf and provide liquidity (withdrawal options) to investors. Examples of such institutions are money market mutual funds, bank conduits and asset-backed commercial paper programs in general. An empirical literature suggests that large outflows from financial institutions during the 2007-08 crisis were due to runs. In particular, Schmidt *et al.* (2014) find evidence of runs within investors of money market mutual funds in the spirit of the equilibrium bank-run discussed in Diamond and Dybvig (1983), where

strategic complementarity plays an important role in withdrawal decisions.[1] However, the existing literature does not address whether the observed run episodes are connected to the over-the-counter market structure. Establishing this connection is important to provide policy makers with a better understanding of which financial markets are fragile and which institutions are prone to runs.

In this paper, I build a model embedding the main ideas of financial fragility discussed in the Diamond and Dybvig (1983) literature, into a dynamic model of over-the-counter (OTC) markets. The model allows me to study the connection between this market structure and the existence of runs in the financial sector. I find that run equilibria are more likely in OTC markets with severe search frictions. Importantly, I also find that small increases in search frictions might be beneficial to agents if runs occur with positive probability in equilibrium. This result differs from the standard models of decentralized asset markets, where welfare always decreases with increases in search frictions. The reason I obtain a different result is that, during normal times, the trades that occur increase welfare; but during runs, some trades decrease welfare because they do not represent investors' true valuation but rather a panic behavior. As a result, by increasing search frictions, welfare can go up or down depending on which effect dominates. I study numerical examples and find this to be the case for several parameters.

The model builds on Lagos and Rocheteau (2009), hereafter LR,[2] and extends it in two ways: investors are divided in groups and their preference shocks are privately observed. The basic environment is the following. Time is infinite and there are two types of agents: investors and dealers. There is an asset in fixed supply. Investors derive utility from holding this asset and dealers do not. All agents can transfer utility using a linear technology. Investors are organized in groups, which I label financial institutions, and within the same financial institution they can trade assets for utility in any period of time. Financial institutions do not trade with each other, only with dealers. Dealers, on the other hand, participate in a competitive inter-dealer market where they trade assets among themselves. In every period there is a random match between financial institutions and dealers, and their terms of trade are established using Nash bargaining. Investors receive privately observed preference shocks over time following a Markov chain. Financial institutions use the balanced team mechanism, proposed by Athey and Segal (2013), to implement an efficient risk-sharing arrangement against the preference shocks of their investors.[3]

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[1]See Kacperczyk and Schnabl (2010), Gorton and Metrick (2012), Schmidt *et al.* (2014), Kacperczyk and Schnabl (2013), Covitz *et al.* (2013), and Foley-Fisher *et al.* (2015) for empirical evidence on runs in the 2007-08 financial crisis.

[2]Lagos and Rocheteau (2009) extends Duffie *et al.* (2005) by allowing investors to hold any positive amount of assets instead of only 0 or 1. However, it also simplified the environment so there is no trade between investors.

[3]The balanced team mechanism extends the AGV-Arrow mechanism to dynamic environments. The AGV-Arrow mechanism was initially proposed by Arrow (1979) and d'Aspremont and Gérard-Varet (1979). See Fudenberg and Tirole (1991) for details and results.

I show that a truth-telling equilibrium exists if the bargaining power of dealers is sufficiently low. I impose this condition so financial institutions have no incentives to misreport during the Nash bargaining with dealers. The advantage of using Nash bargaining is tractability, since it makes the payments in the bargaining linear on the gains from trade. The truth-telling equilibrium implements the same allocation of Lagos and Rocheteau (2009), where preference shocks are publicly observed. Moreover, this allocation is constrained Pareto efficient when a dealer's bargaining power is zero.

Once I establish conditions for existence and efficiency of truth-telling equilibrium, I study conditions for its uniqueness. I show that the truth-telling equilibrium is the unique equilibrium if the financial institutions' probability of meeting a dealer is high enough. The literature on over-the-counter markets associates this probability with the degree of search friction in the economy. When it is low, search frictions are severe and the equilibrium outcome is close to autarky; when it is high, search frictions are small and the equilibrium outcome is close to Pareto efficient. My result highlights another desirable feature of having a high probability of meeting a dealer: it eliminates multiple equilibria, leading to a stable financial sector.

When the probability of meeting a dealer is not high, however, non-truth-telling equilibria also exist. I compute equilibria where the economy switches back and forth between a "run" and a "no-run" state following a Markov chain of sunspots. In the "run" state, investors in a subset of the financial institutions announce low valuation for holding the asset independently of their true preference shock. I verify numerically that in a large range of the parameter space this outcome is supported as a perfect Bayesian equilibrium of the game associated with the balanced team mechanism. I conclude then that multiplicity is not a particular outcome of a knife edge case of the parameter space, but rather a robust feature of the balanced team mechanism in the model with incomplete information.

But what generates incentives for a run in equilibrium? The answer lies in the liquidity insurance provided by the balanced team mechanism. This mechanism can be interpreted in the following way. Investors agree to buy/sell assets from other financial institution investors in case they have the desire to sell/buy assets and it is costly (or unfeasible) to trade it with a dealer—a form of liquidity insurance that is also embedded in the contract offered by several real-life financial institutions. The price an investor pays/receives for these assets is based on the expected welfare impact of his own announcement. The reason it must be based on the expected welfare impact and not on the realized welfare impact is to make the payments budget-balanced (that is why it is called the *balanced* team mechanism). When investors truthfully announce their preference shocks, this liquidity insurance improves the allocation of assets in the financial institution by equalizing the marginal utility of consumption of asset dividends. But this insurance also has another effect. The

amount of assets purchased by a particular investor is increasing in the other investors' desire to sell the asset. Hence, when an investor believes for sure other investors will announce they want to sell their assets, he expect to have a lot of assets and, since there is decreasing marginal utility from holding assets, he expects a low marginal utility from holding more of those assets. However, the price he pays does not react to this low marginal utility because it is based on the expected impact of his announcement, not the realized one. As a result, under this belief, he may prefer to untruthfully announce a desire to sell his own assets as a way to avoid buying assets at a price that is higher than the marginal benefit he gets from buying those assets. And, of course, if all investors have this same belief, they all prefer to announce the desire to sell their assets—confirming their initial beliefs in equilibrium and generating a self-fulfilling run on the financial institution.

The runs have several implications for market outcomes. The price of the asset drops in the run states. Trade volume initially spikes, following a "fire sale" by the financial institutions under a run, then collapses. The average bid-ask spread can go up or down with runs, depending on the fraction of financial institutions under a run, where the bid-ask spread in the model is the difference between the price in a financial institution-dealer meeting and the price in the inter-dealer market. When about half of the financial institutions are under a run, the average bid-ask spread increases in the run state. Otherwise, it decreases. Further, I investigate how these effects change with changes in the probability of finding a dealer. I find that increasing this probability generates a larger price decline, larger spike, and faster collapse of trade volume, and stronger moves in bid-ask spreads.

I also use the numerical examples to study the welfare implications of the model. I find that welfare is decreasing in the fraction of financial institutions under a run. This is intuitive. When there are more financial institutions under a run, we have more investors misrepresenting their type and, therefore, a worse allocation of assets. Moreover, I find that in some regions of the parameter space welfare is also decreasing in the probability of finding a dealer. The intuition for this result is twofold. There is an effect on the extensive margin and one on the intensive margin. The financial institutions under a run sell assets at a price below the average valuation of their investors. They sell it because their investors are misreporting their preference shocks. If the probability of finding a dealer is higher, then in the same time interval more financial institutions under a run have a trade opportunity. So the fraction of financial institutions inefficiently selling their assets is higher. This is the extensive margin effect. On the intensive margin, when the probability of finding a dealer is higher, financial institutions put more weight in their short-term valuation when deciding how much to buy/sell of the asset. This effect has been extensively discussed in Lagos and Rocheteau (2009). And it implies that a financial institution under a run inefficiently sells more of its assets if the probability of finding a dealer is higher.

The fact that increasing the probability of finding a dealer can decrease welfare in a run equilibrium creates a dilemma. As stated before, if this probability is high enough, the truth-telling equilibrium is the unique equilibrium. Therefore, implementing policies to increase this probability is desirable because it can eliminate all run equilibria, and implement the equilibrium outcome associated with the highest welfare. On the other hand, if it does not do so, it can make agents in the economy worse off if they keep playing the run equilibrium.

This paper contributes to the large literature, started by Diamond and Dybvig (1983), which takes a mechanism design approach to the bank problem and interprets financial fragility as the multiplicity of equilibria in such mechanisms. This literature includes, Wallace (1988), Peck and Shell (2003), Green and Lin (2003), Andolfatto *et al.* (2007), Ennis and Keister (2009), Cavalcanti and Monteiro (2011), and Andolfatto *et al.* (2014) among others. There is also a related literature that investigates the connection between Diamond-Dybvig banks and markets, which includes Allen and Gale (2000) and Allen and Gale (2004). My main contribution to these literatures that build on Diamond and Dybvig (1983) is to embed the ideas regarding financial fragility into a dynamic model of over-the-counter markets. And, by doing so, I provide a way to relate this market structure to the fragility of the financial system.

More closely related in terms of the environment, there are papers that study financial crises, or selling pressures, in over-the-counter markets. For example, Lagos *et al.* (2011) studies financial crises in the context of an over-the-counter market where dealers provide liquidity to the economy and there is an aggregate shock which lowers the investors' valuation of the asset. Feldhütter (2012) proposes and estimates a structural model, which is a variation of Duffie *et al.* (2005), where there is also an aggregate shock that lowers the investors' valuation of the asset. In these papers, the financial crisis is associated with the moment in which the aggregate shock hits the economy. My contribution is to show that "lower valuation" can be generated endogeneously, as an equilibrium outcome and to study how the market structure relates with the existence of those equilibria.

Trejos and Wright (2014) generates multiple equilibria in a generalized version of Duffie *et al.* (2005) where preferences are separable but not quasi-linear. My paper complements this result by showing how the multiplicity can be the outcome of runs against financial institutions.

This paper is also related to the mechanism design literature. In particular, it is related to papers that illustrate how an optimal direct mechanism can implement perverse outcomes. See Demski and Sappington (1984) and Postlewaite and Schmeidler (1986) for some of the early papers on this issue. And to the best of my knowledge, this is the first application of the Athey and Segal (2013) balanced team mechanism to contracts in financial markets.

The rest of the paper is organized as follows. In section 2, I study a simple model of a financial

institution and provide intuition for the run equilibria with only one period and without markets. In section 3, I introduce the environment of the general model. In section 4, I introduce the financial institution problem, its optimal asset allocation, and the balanced team mechanism. In section 5, I define equilibrium and state the main theoretical results. In section 6, I provide the numerical examples. In section 7, I offer a discussion of my findings in terms of policy implications.

2. A SIMPLE MODEL OF FINANCIAL INSTITUTIONS

In this section, I discuss a simple one-period model of financial institutions—a simplified version of the full model that provides intuition for the financial institution structure and the existence of run equilibria.

The environment is the following. There is one period and $N \in \mathbb{N}$ agents called investors. There is an asset and each investor starts the period with the positive endowment $\bar{A} > 0$ of it. Investors are *ex-ante* identical. In the beginning of the period, $N_h > 0$ investors turn to be of type θ_h and $N_l = N - N_h > 0$ turn to be type θ_l. In this simplified version, there is no aggregate uncertainty, so the number of agents of each type is known in advance to all investors. The model follows a transferable utility framework, and investors derive utility from holding the asset.[4] The utility of an investor of type θ is $u(a; \theta) + m$, where a is the asset holding and m is the transfer of utility. Note that m can be either positive or negative. For each θ, the utility function $u(\cdot; \theta)$ is twice continuous differentiable, concave and $u'(a; \theta_h) > u'(a; \theta_l) \geq 0$ for all strictly positive a. Investors in the same financial institution can commit with contractual arrangements among them. The only frictions they face are the private information and the limited participation to the centralized market.

The Pareto efficient outcome

Investors in this economy are able to transfer utility using a linear technology. An implication of this assumption is that, if an allocation is Pareto efficient, the allocation of assets maximizes *ex-ante* expected utility of investors. Therefore, a Pareto efficient allocation solves

$$(1) \quad \max_{a \in \mathbb{R}_+^2} \{N_h u(a_h; \theta_h) + N_l u(a_l; \theta_l); \text{ subject to } N_h a_h + N_l a_l = N\bar{A}\},$$

where a_h denotes the assets allocated to an investor of type θ_h and a_l the assets allocated to an investor of type θ_l. With some abuse of notation, for the rest of the section I call $a = (a_h, a_l)$ the solution to problem (1). The outcome a cannot be directly implemented because types are private

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[4]Assuming investors derive utility from holding the asset is a reduced form meant to capture different reasons investors have for holding an asset. Duffie *et al.* (2005) discuss some of these different reasons which include: liquidity needs, financing costs, hedges benefits, and/or tax advantages.

information. Fortunately, the incentive compatibility constraint does not bind in this environment. A mechanism can be set up using transfers of utils so investors have incentive to truthfully reveal their type. I explain this mechanism in the next subsection.

A money market mutual fund mechanism

In order to implement the Pareto efficient outcome, I will focus on a simple mechanism resembling the operation of a money market mutual fund (MMMF). It is worth mentioning that there are different ways to implement this outcome. I use this particular mechanism because the intuition I build from it can be carried out to the general version of the model I discuss later.

The mechanism works as follows. At the onset, before knowing their types, investors deposit $\bar{A} - a_l$ assets in the MMMF. Note that $\bar{A} - a_l$ is non-negative since $u'(a; \theta_h) > u'(a; \theta_l)$ for all strictly positive a. This deposit gives equal shares of the total fund portfolio, $N(\bar{A} - a_l)$, to all investors. The MMMF then commits to buying the investor's shares at the price $p = u'(a_h; \theta_h) \geq u'(a_l; \theta_l)$ in terms of utility. The purchase of the shares is paid with utils from the investors who decide to hold their shares until the end of the period. If all investors decide to sell their shares of the MMMF instead of holding them, there is no investor to actually make the transfers of utility that pays for the shares. In this case, I say that the MMMF could not meet its obligations, and the assets are equally divided among investors.[5] I refer to the action of not selling (or holding) the shares in the MMMF as action s_h and the action of selling the shares as action s_l.

There are alternative interpretations of this mechanism besides the one of a money market mutual fund. For example, one can interpret the financial institution as an asset-backed commercial paper (ABCP) program. The program issues ABCPs in order to finance the purchase of the $N(\bar{A} - a_l)$ assets. Investors then have two options. They either liquidate these papers and get the payment $p(\bar{A} - a_l)$ in terms of utility, or they roll them over to the end of the period and become the residual claimant on the portfolio in the ABCP program. Another interpretation is of life insurers issuing extendable funding agreement backed notes (XFBS). These notes give investors the option to extend their investment and stay as the residual claimant on the portfolio in the program, or to not extend and get paid a predetermined amount. These financial institutions are considered part of the so-called shadow bank system, which suffered with runs during the financial crisis in 2007-08.[6]

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[5]Note that, alternatively, the mechanism could ask all investors at the onset to deposit not only assets, but also utils. This alternative would be consistent with the fact that MMMFs hold a diverse portfolio with liquid (cash, treasuries, etc) and illiquid (asset-backed securities) assets. The utils in my model represent the liquid assets at the MMMF, while the asset represents only the illiquid assets.

[6]See Covitz *et al.* (2013) for the institutional details of ABCP programs and Foley-Fisher *et al.* (2015) for the institutional details of XFBS programs. Both these papers found evidence of runs in 2007 in the beginning of the financial crisis. Additionally, Schmidt *et al.* (2014) found evidence of runs against money market mutual funds in 2008.

Equilibrium

The mechanism above is associated with a Bayesian game of incomplete information. Investor n's strategy consists of an action $s^n(\theta^n) \in \{s_h, s_l\}$ as a function of his type $\theta \in \{\theta_h, \theta_l\}$. The payoffs of an investor n implied by an action profile $s = (s^1, \ldots, s^N) \in \{s_h, s_l\}^N$ are

$$v_n(s_h; \theta^n, s^{-n}) = u(\bar{A} + (N - n_h)(\bar{A} - a_l)/n_h; \theta^n) - p(N - n_h)(\bar{A} - a_l)/n_h$$

$$v_n(s_l; \theta^n, s^{-n}) = \begin{cases} u(a_l; \theta^n) + p(\bar{A} - a_l) & ; \quad \text{if} \quad n_h > 0 \\ u(\bar{A}; \theta^n) & ; \quad \text{if} \quad n_h = 0 \end{cases}$$

where n_h is the number of investors holding their shares (action s_h) given the action profile s, and s^{-n} is the action profile of investors other than n. For the remainder of this section, I use $n_h(s)$ and $n_h(s^{-n})$ to denote the number of investors holding their shares (action s_h) in the profiles s and s^{-n} respectively. An equilibrium of the model is a strategy profile $\{s^n(\theta^n)\}_n$ constituting a Bayesian-Nash equilibria of this game.

Truth-telling equilibrium

An outcome of the MMMF mechanism is the solution of the planer problem (1) if, and only if, all θ_h investors hold their shares (action s_h), while all θ_l investors sell it (action s_l). For this reason, I call a strategy s truth-telling if $s(\theta_h) = s_h$ and $s(\theta_l) = s_l$. Truth-telling is a Bayesian-Nash equilibrium of the game. To see this note that, under truth-telling strategies, the asset allocated to an investor who takes action s_h is

$$\bar{A} + \frac{(N - n_h)(\bar{A} - a_l)}{n_h} = \frac{N_h \bar{A} + (N - N_h)(\bar{A} - a_l)}{N_h} = \frac{N\bar{A} - N_l a_l}{N_h} = \frac{N\bar{A} - N_l a_l}{N_h} = a_h,$$

where the last equality comes from the resource constraint in problem (1). Combining the above equation with the definition of v_n, we find that $v_n(s_h; \theta, s^{-n}) = u(a_h; \theta_h) - p(a_h - \bar{A})$. Thus,

$$v_n(s_h; \theta, s^{-n}) = u(a_h; \theta_h) - p(a_h - \bar{A}) = \max_{a \in \mathbb{R}_+} \{u(a; \theta_h) - p(a - \bar{A})\},$$

where the last equality is implied by $u'(a_h; \theta_h) = p$ and the first order condition of the Pareto problem. The above equation implies that $v_n(s_h; \theta_h, s^{-n}) \geq u(a_l; \theta_h) - p(a_l - \bar{A}) = v_n(s_l; \theta_h, s^{-n})$. It is analogous to show that s_l is a best response for an investor of type θ_l.

Numerical example with runs

The MMMF mechanism implements the Pareto efficient outcome. However, in some cases, it also implements an untruthful equilibrium where every investor announces type θ_l. I call this

equilibrium a run. To illustrate the possibility of a run, consider the following numerical example. The economy has $N = 10$ investors. The number of type θ_h investors is $N_h = 9$, and the number of type θ_l investors is $N_l = 1$. The utility of an investor of type θ_h is $2\sqrt{c} + m$ and the utility of an investor of type θ_l is just m. That is, investors of type θ_l do not derive utility from the consumption of fruits. Lastly, $\bar{A}$ is normalized to be $N_h/N = 0.9$. Under this normalization the Pareto efficient outcome has $a_h = 1$ and $a_l = 0$. Each investor deposits 0.9 assets at the MMMF in the onset of the period, and the price they can sell their shares for is $p = \partial(2\sqrt{a})/\partial a|_{a=1} = 1$.

Figure 1 displays the indifference curves of an investor of type θ_h, associated with actions s_h and s_l, given different actions profiles of other investors. The blue curves are associated with the investor not selling their portfolio to the MMMF—action s_h. The highest level blue curve gives his payoff if $n_h(s^{-n}) = 8$. The other blue curves gives his payoff if $n_h(s^{-n}) = 2,\ 1$ and 0. The red curves give his payoff for action s_l when $n_h(s^{-n}) = 0$, and $n_h(s^{-n}) > 0$. Regarding a type θ_l investor, selling their portfolio (action s_l) is a strictly dominant action. To see this, note that his payoff is $v_n(s_l; \theta_l, s^{-n}) = 0.9 > -0.9(10 - n_h)/n_h = v_n(s_h; \theta_l, s^{-n})$, if $n_h(s^{-n}) > 0$. And it is $v_n(s_l; \theta_l, s^{-n}) = 0.0 > -0.9 \times 9 = v_n(s_h; \theta_l, s^{-n})$, if $n_h(s^{-n}) = 0$. In either case, he is strictly better off by taking action s_l.
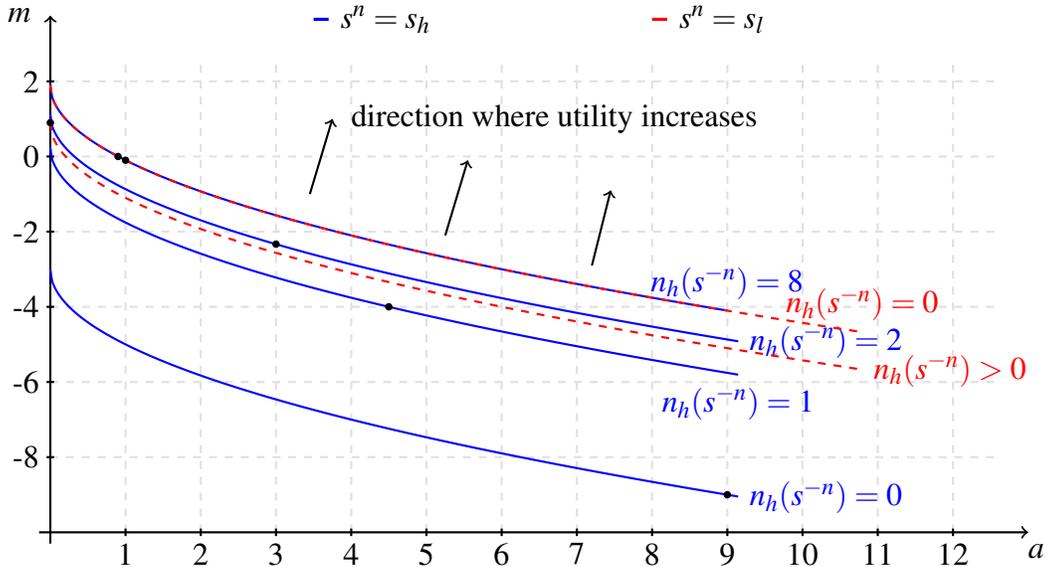

Figure 1: Payoff of type θ_h investor

If an investor n of type θ_h believes that at least other $n_h(s^{-n}) = 2$ investors will not sell their portfolio to the MMMF, his best response is not to sell his portfolio, action $s_h = 0$. However, once his belief of $n_h(s^{-n})$ goes below 2, his best response is to sell his portfolio, action $s_l = 0.9$. This strategic complementary generates an equilibrium where every investor tries to sell their portfolio,

the MMMF doesn't have funds to cover its obligations and, as a result, assets are equally shared among investors—a form of a self-fulfilling run against the money market mutual fund.

Introducing markets

The simple model presented so far may look very particular, and hard to match to real-life financial institutions who, in practice, trade assets in financial markets. But is not actually hard to embed it in a model with multiple financial institutions who trade assets in a financial market. Consider the following extension of the model. Assume there is a non-atomic measure one of financial institutions labeled by $c \in [0, 1]$. Each financial institution is a replica of the financial institution defined previously. That is, each financial institution is composed by N investors, whose types, preferences and endowments are as specified before.

The important component of this extension is that the trade will be over the counter, and, therefore, it will require search. One approach used in the literature to model the search aspect of OTC markets is to assume that the access to the asset market is uncertain. Specifically, with probability $\alpha \in [0, 1]$, a financial institution accesses a centralized competitive market for the asset. The parameter α captures the degree of search friction in the economy—higher is α, smaller is the search friction. The market access is independent across financial institutions. The price for the asset in terms of utility in the centralized market is p_m. Each financial institution takes the price of the asset in the market as given. If a financial institutions does not access the centralized market, as before, its investors can only trade assets among themselves.

Now that financial institutions have access to trade in a market, the MMMF mechanism proposed before must be adjusted. The adjusted mechanism is simple: if the financial institution access the market, it implements the Walrasian demand for each investor separately; otherwise, it implements the allocation discussed in the previous section. That is, in the onset, every investor deposit all their assets in the MMMF. Once types are realized, as before, investors can either sell their shares or keep it. If the financial institution have access to the market, it trades in order to make $u'(a_h; \theta_h) = p_m$ for all investors who decided to keep their shares; and $u'(a_l; \theta_l) = p_m$ for all investors who decided to sell their shares. And the transfers in terms of utility to investors are $p_m(\bar{A} - a_h)$ and $p_m(\bar{A} - a_l)$. If the MMMF does not access the market, it implements the same allocation as before.

It is interesting to notice that, when $\alpha = 0$, the model is basically the same as the one studied in the previous subsection—there will be a continuum of financial institutions, but they are completely independent and can be studied separately. Moreover, the payoff of the game associated with the MMMF mechanism is continuous in α. As a result, by continuity, if a run equilibrium exist for $\alpha = 0$, then it will also exist for some parameters of $\alpha \in [0, 1)$. On the other hand, when $\alpha = 1$,

the model is equivalent to a Walrasian model. Then we can show that truth-telling is the unique equilibrium—there is no runs. With this simple example we can already see how the search friction matters for which markets and financial institutions are vulnerable to runs or not. The advantage of extending the model to a dynamic setting is that we can study the dynamic of such equilibria and better understand its properties.

Comparison with Diamond and Dybvig (1983)

The model presented in this section has some interesting similarities with Diamond and Dybvig (1983). Both models use mechanisms that resemble contracts observed in the bank/shadow bank industry in order to provide liquidity insurance against preference shocks. But these contracts also have an inefficient equilibrium: a form of run that leads to lower welfare.[7]

The advantage of using this model instead of the classic Diamond and Dybvig (1983) is that, once combined with some recent developments in mechanism design, the model can be easily integrated with the literature of over-the-counter markets. This integration allows us to discuss several features of financial markets such as asset price, trade volume, and bid-ask spreads. Besides, it provides conditions on the market structure that may suggest when multiple equilibria exist and, therefore, the financial system is fragile. In the remainder of the paper, I develop the general version of the model, provide theoretical and numerical results to discuss the mentioned variables and the welfare implications of the model.

3. GENERAL ENVIRONMENT

Time is discrete and infinite. There is a non-atomic measure of investors and a non-atomic measure of dealers. Investors are divided in groups of size $N \in \mathbb{N}$, labeled financial institutions, and stay in the same financial institution forever. The measure of financial institutions is normalized to one, which implies a measure N of investors, and the measure of dealers is $\alpha \in (0,1)$. There is a non-perishable asset and each investor holds an endowment $\bar{A} > 0$ of it at period zero.

The model has transferable utility and, as in the previous section, investors derive utility from holding the asset. Investors' period utility is $u(a; \theta) + m$, where a is the investor asset holdings, θ is the investor's preference type, and m is the transfer of utility to the investor, which can be either positive or negative. For each θ, the utility function $u(\cdot; \theta)$ is twice continuous differentiable, strictly

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[7]In the Diamond-Dybvig environment, more sophisticate contracts are able to prevent run equilibria under very general assumptions. For example, Andolfatto *et al.* (2014) proposes an indirect mechanism that is able to prevent runs under the same assumptions of the original Diamond-Dybvig model. It is also possible to prevent runs in the version of my model presented in this section. In the general version presented in the next few sections, whether a more sophisticated mechanism is able to eliminate multiple equilibria is an open question. Studying this question is an interesting avenue for future research, but it is out of the scope of this paper.

increasing, strictly concave, $\lim_{a \to 0} u'(a; \theta) = \infty$, and $\lim_{a \to \infty} u'(a; \theta) = 0$. Preference types are privately observed, independent across investors, have finite support $\Theta \subset \mathbb{R}$, and follow a Markov chain with transition F. The transition F has a unique ergodic distribution, π^*, and I assume for simplicity that the initial θs are drawn from π^*. This assumption is not needed for the theoretical results, but it makes it easier to compute equilibria. Dealers do not hold assets and their period utility is just the transfer from investors. Both investors and dealers maximize expected utility and have inter-temporal discount $\beta \in [0, 1)$. Investors in the same financial institution can commit toward future transfers, the only friction is the private information regarding preference shocks.

Agents observe a sunspot variable $x_t \in \mathbb{S} := \{R, NR\}$ in the beginning of period t. The process $\{x_t\}_t$ follows a Markov chain with transition Q. The letters R and NR extend to run and not to run. I consider sunspot equilibria where investors use this variable to coordinate their actions.

In every period there is a random match between financial institutions and dealers. Without loss of generality, I assume that every dealer is matched with a financial institution. Since there is a measure $\alpha \in (0, 1)$ of dealers, this implies that a financial institution meets a dealer with probability α. Dealers have access to an inter-dealer competitive market where they can buy/sell the assets that they trade with the financial institutions. I denote the price of assets in terms of utility in the inter-dealer market by $\boldsymbol{p} = \{p_t\}$.

The asset distribution is a state variable in the economy. Thus, in equilibrium, the price can also be a function of the asset distribution. With quasi-linear preferences, however, it can be shown that there are equilibria where the price is independent of the asset distribution. For simplicity, I focus on this class of equilibria and, as a result, the price in a period t is a function of the only aggregate shock affecting the economy—the history of sunspot realizations $x_0, x_1, \ldots, x_t$.
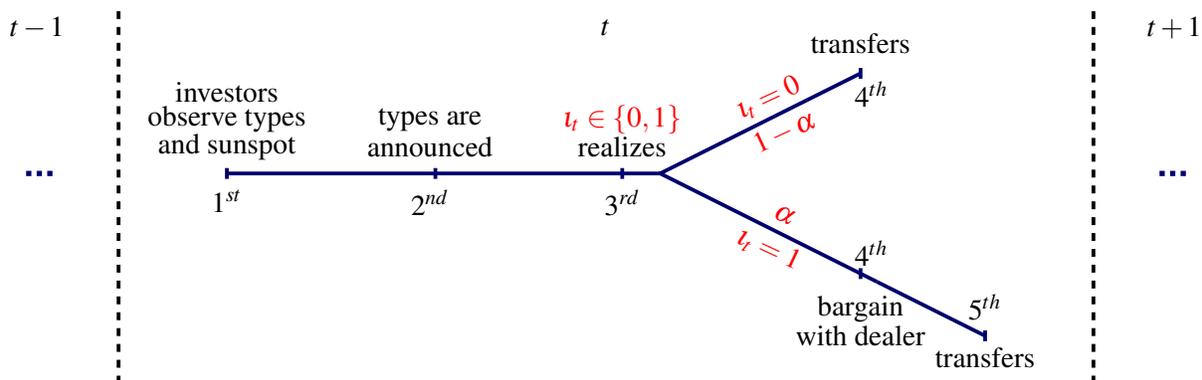

Figure 2: Sequence of actions

Figure 2 depicts the sequence of actions. In the beginning of the period, investors observe their preference type and sunspot realization. Then they simultaneously announce their type to the

financial institution. After announcements, the financial institution either meets a dealer ($\iota_t = 1$) with probability α, or it does not meet a dealer ($\iota_t = 0$). The dealer can buy/sell assets for the financial institution in the inter-dealer market at the price p_t. But the dealer bargains with the financial institution over the gains from trade using Nash bargaining, where the bargaining power of the dealer is $\eta \in [0,1)$. $\boldsymbol{\phi} = \{\phi_t\}_t$ denotes the transfer from the financial institution to the dealer resulting from the bargaining. During the bargaining, dealers observe the financial institution aggregate asset holdings and the most recent type announcements of investors in the financial institution.[8] After the bargaining, transfers of assets and utilities are made.

The environment is characterized by a family $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$. The section 2 environment is a variation of this one where there are no dealers ($\alpha = 0$), investors completely discount the future ($\beta = 0$), and preference shocks follow the distribution described in that section.

4. THE FINANCIAL INSTITUTION PROBLEM

The financial institution problem is to maximize the ex-ante utility of its own investors, taking as given what other agents in the economy are doing. Investors face two frictions. They may not meet a dealer to trade in the inter-dealer market when they need, and their investment decisions are distorted because dealers extract part of the gains from trade in the bargaining process. A contractual arrangement that reallocates assets across investors in the financial institution attenuates these two frictions. In the periods where the financial institution does not meet a dealer, it efficiently reallocates assets among the financial institution investors. And in periods where the financial institution does meet a dealer, it minimizes the amount of assets traded with him, which reduces the rent he extracts. The difficulty in implementing such an arrangement is that preference types are private information. That is why the balanced team mechanism, proposed by Athey and Segal (2013), is useful in this setting. It implements the efficient allocation of assets among the investors in perfect Bayesian equilibrium (PBE). Pretty much in the same way the money market mutual fund contract did in the simple model of section 2.

The balanced team mechanism has two components. There is the efficient asset policy for the financial institution and the transfer scheme that implements this policy outcome in PBE. In the remainder of this section, I discuss these two components in detail.

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[8]The Nash bargaining between financial institutions and dealers keeps the model tractable. However, since investors have private information, it cannot be applied in general. For this reason, I assume in all the propositions in the paper that η is in a neighborhood of zero.

The assumption that dealers observe the announcement vector is not essential for the theoretical results (though it may change the neighborhood of $\eta = 0$ for which the results apply). But it simplifies the computation of equilibria because you only have to verify investors' deviations when checking for Bayesian perfect equilibrium.

The assumption that asset holdings are observed is usual and many financial institutions are required by regulators to keep public records of their portfolio.

Asset policy

In this subsection, I characterize the asset allocation that maximizes welfare in the financial institution for a given price process—all under the assumption that investors' types are truthfully announced. Let me start with notation. Label $\boldsymbol{\theta}_t = (\theta_t^1, \ldots, \theta_t^N) \in \Theta^N := \boldsymbol{\Theta}$ the period t announcement vector. The period t financial institution state is a triple $h_t = (x_t, \boldsymbol{\theta}_t, \iota_t) \in H := \mathbb{S} \times \boldsymbol{\Theta} \times \{0,1\}$ of sunspot realization, announcement vector, and status of meeting with the dealer ($\iota_t = 1$) or not ($\iota_t = 0$). The history of state realizations up to period t is denoted by $h^t = (h_0, h_1, \ldots, h_t)$. Throughout the paper, subscript t denotes a variable realization in period t and superscript t denotes a vector with variable realizations from period zero up to period t. Define the function $U : \mathbb{R}_+ \times \boldsymbol{\Theta} \to \mathbb{R}$ as the maximum aggregate period utility of a financial institution with total assets A and vector type $\boldsymbol{\theta}$. That is,

$$(2) \quad U(A; \boldsymbol{\theta}) = \max_{a \in \mathbb{R}_+^N} \left\{ \sum_n u(a^n; \theta^n); \text{ subject to } \sum_n a^n = A \right\}.$$

An asset policy is a sequence $\boldsymbol{a} = \{a_t\}_t$, where $a_t = (a_t^1, \ldots, a_t^N) : H^t \to \mathbb{R}_+^N$ denotes the amount of assets allocated to each financial institution investor in period t contingent on the history h^t. An asset policy is feasible if $(1 - \iota_t)[A_t(h^t) - A_{t-1}(h^{t-1})] = 0$ for all histories h^t, where $A_t(h^t) = \sum_n a_t^n(h^t)$ is the aggregate asset holdings in period t and $a_{-1}^n := \bar{A}$ for all n. This condition means that the financial institution can only adjust its aggregate asset holdings when it meets a dealer, but it can adjust the asset holdings within its own investors at any period. Label Γ the set of all feasible asset policies. The welfare induced by $\boldsymbol{a} \in \Gamma$ is

$$(3) \quad W(\boldsymbol{a}) = \mathbb{E} \sum_t \beta^t \left[U(A_t; \boldsymbol{\theta}_t) - p_t(A_t - A_{t-1}) - \phi_t(A_{t-1}; x^t, \boldsymbol{\theta}_t) \right],$$

where ϕ_t denotes the dealer's fee, which is determined by Nash bargaining. Note that ϕ_t depends only on what is observed by the dealer: the aggregate asset holdings of the financial institution, the history of sunspots and the current announcement vector. The financial institution asset allocation problem is

$$(4) \quad \max_{\boldsymbol{a} \in \Gamma} W(\boldsymbol{a}).$$

In order to characterize the dealers fee, ϕ_t, it is useful to write problem (4) recursively. The relevant state variables for a financial institution in period t are the aggregate asset holding A, the history of sunspots x^t, and the current vector type $\boldsymbol{\theta}_t$. Let $V_t(A; x^t, \boldsymbol{\theta}_t)$ denote the financial institution value function in the end of period t. That is, after any possible trade with a dealer has occurred. By

construction, $V_t(A;x^t,\boldsymbol{\theta}_t)$ satisfies the functional equation

$$(5) \quad V_t(A;x^t,\boldsymbol{\theta}_t) = \mathbb{E}_t\sum_{T=1}^{\infty}(1-\alpha)^{T-1}\alpha\{\sum_{s=0}^{T-1}\beta^s U(A;\boldsymbol{\theta}_{t+s})+$$
$$\beta^T \max_{\hat{A}\in\mathbb{R}_+}\left[V_{t+T}(\hat{A};x^{t+T},\boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A}-A) - \phi_{t+T}(A;x^{t+T},\boldsymbol{\theta}_{t+T})\right]\}.$$

$\phi_t(A;x^t,\boldsymbol{\theta}_t)$ is determined by the Nash bargaining between the financial institution and the dealer, where the dealer's bargaining power is η and the financial institution's bargaining power is $1-\eta$. Therefore, it is given by

$$(6) \quad \phi_t(A;x^t,\boldsymbol{\theta}_t) = \underset{\phi\in\mathbb{R}_+}{\operatorname{argmax}}\left\{\phi^{\eta} \max_{\hat{A}\in\mathbb{R}_+}\left[V_t(\hat{A};x^t,\boldsymbol{\theta}_t) - p_t(\hat{A}-A) - V_t(A;x^t,\boldsymbol{\theta}_t) - \phi\right]^{1-\eta}\right\}.$$

The solution of the above problem implies that

$$(7) \quad \phi_t(A_t,x^t,\boldsymbol{\theta}_t) = \eta \max_{\hat{A}\in\mathbb{R}_+}\left[V_t(\hat{A};x^t,\boldsymbol{\theta}_t) - p_t(\hat{A}-A_t) - V_t(A;x^t,\boldsymbol{\theta}_t)\right].$$

Note that the dealer in this model is passive. He benefits from the rent he extracts from the financial institution, but there is no decision-making besides the one embedded in the Nash bargaining. Replacing (7) in (5) we obtain

$$(8) \quad V_t(A;x^t,\boldsymbol{\theta}_t) = \mathbb{E}_t\sum_{T=1}^{\infty}(1-\alpha)^{T-1}\alpha\{\sum_{s=0}^{T-1}\beta^s U(A;\boldsymbol{\theta}_{t+s}) + \eta\beta^T V_{t+T}(A;x^{t+T},\boldsymbol{\theta}_{t+T})+$$
$$(1-\eta)\beta^T \max_{\hat{A}\in\mathbb{R}_+}\left[V_{t+T}(\hat{A};x^{t+T},\boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A}-A)\right]\}.$$

Analogous to Lagos and Rocheteau (2009), the functional equation in (8) is equivalent to one in which the financial institution has all the bargaining power when trading with the dealer, but it only meets a dealer with probability $\hat{\alpha} = \alpha(1-\eta)$. Under this formulation, $V_t(A;x^t,\boldsymbol{\theta}_t)$ can be written as

$$(9) \quad V_t(A;x^t,\boldsymbol{\theta}_t) = \mathbb{E}_t\sum_{T=1}^{\infty}(1-\hat{\alpha})^{T-1}\hat{\alpha}\{\sum_{s=0}^{T-1}\beta^s U(A;\boldsymbol{\theta}_{t+s})+$$
$$\beta^T \max_{\hat{A}\in\mathbb{R}_+}\left[V_{t+T}(\hat{A};x^{t+T},\boldsymbol{\theta}_{t+T}) - p_{t+T}(\hat{A}-A)\right]\}.$$

And the associated sequential problem is

$$(10) \quad \max_{\boldsymbol{a}\in\Gamma}\hat{W}(\boldsymbol{a}) = \hat{\mathbb{E}}\sum_t\beta^t\left[U(A_t;\boldsymbol{\theta}_t) - p_t(A_t-A_{t-1})\right],$$

where $\hat{\mathbb{E}}$ denotes the expectation of future meetings with dealers using the probability measure induced by $\hat{\alpha} = \alpha(1-\eta)$ instead of α.

A solution to problem (10) doesn't always exist because the problem is unbounded for some prices (for instance, if p is identically zero). However, if it does exist, it is unique since $u(\cdot, \theta)$ is strictly concave for all θ. For now, I assume that p is such that a solution exists and label it a_p^*. Note that a_p^* is a function of the price p, but I omit this subscript throughout the text to keep the notation simple. The first order conditions of problem (10) are given by

(11) $\quad p_t - \hat{\mathbb{E}}\{\beta^{d_k} p_{t+d_k} \mid h^t\} = \hat{\mathbb{E}}\{\sum_{d=0}^{d_k-1} \beta^d U'(A_t; \boldsymbol{\theta}_{t+d}) \mid h^t\}$ whenever $\iota_t = 1$,

(12) $\quad [\forall\, n, m]:\ u'(a_t^{*n}, \theta_t^n) = u'(a_t^{*m}, \theta_t^m)$ and

(13) $\quad \lim_{k \to \infty} \hat{\mathbb{E}}\{\beta^{t_k} p_{t_k} A_{t_k}^*\} = 0;$

where $A_t^* = \sum_n a_t^{*n}$ is the aggregate asset holdings of the financial institution, t_k is the time period in which the financial institution meets with a dealer for the kth time, and $d_k = t_k - t_{k-1}$ is the time interval between the meetings with dealers. Equation (11) is the condition that the expected cost of buying one additional unit of asset and holding it until the next meeting with a dealer is equal to the expected benefit of holding this asset for the same period of time. Equation (12) is the condition that at any period in time the marginal utility of holding an asset should be equalized across investors in the same financial institution. And equation (13) is the usual transversality condition. It is straightforward to show that equations (11) to (13) provide necessary and sufficient conditions for a feasible asset policy $a^* \in \Gamma$ to solve problem (10) and, therefore, the financial institution problem.

Transfers

The transfers in the balanced team mechanism are designed so investors internalize the impact of their announcements on the other financial institution investors. Let

(14) $\quad v^n(a^{*t}, h^t) = u(a_t^{*n}(h^t), \theta_t^i) - p_t(x^t)[a_t^{*n}(h^t) - a_{t-1}^{*n}(h^{t-1})] - \phi_t(A_{t-1}^*; x^t, \boldsymbol{\theta}_t)/N$

denote the period utility of an investor n given the policy a^* and history h^t. It is implicit in equation (14) that investors pay the cost associated with their changes in asset holdings at the market price, $p_t(x^t)$, and share the cost associated with the dealer's fee equally. There are different, in fact, ways to formulate how these costs are shared. For instance, one could have each investor paying an equal share of the aggregate cost, *i.e.*, $\{p_t(x^t)[A_t^*(h^t) - A_{t-1}^*(h^{t-1})] + \phi_t(A_{t-1}^*; x^t, \boldsymbol{\theta}_t)\}/N$. I chose the formulation above because the associated payoffs converge to those in a competitive Walrasian equilibrium when α converges to one and η to zero. This convergence guarantees uniqueness of equilibrium in a neighborhood of these parameters as discussed in section 5.

Given a history h^t, let $\psi_t^{*n}(h^t)$ be the period t utility, implied by the policy a^*, of all investors

in the financial institution other than investor n. That is,

$$(15) \quad \psi_t^{*n}(h^t) = \sum_{i \neq n} v^i(a^{*t}, h^t).$$

$\psi_t^{*n}(h^t)$ is the term that investor n must internalize in order to have incentives to truthfully reveal his type. Athey and Segal (2013) refer to the mechanism associated with the transfer $\psi_t^{*n}(h^t)$ as the team mechanism. It is the equivalent to the Vickey-Groves-Clarke (VGC) mechanism for a dynamic setting. A major problem with this transfer is that it is not budget balanced. In order to make it budget balanced, instead of working directly with $\psi_t^{*n}(h^t)$, the balanced team mechanism uses the impact of the announcement in the expected present value of $\psi_t^{*n}(h^t)$, which Athey and Segal (2013) call the incentive term of the agents. Formally, the investor n incentive term is

$$(16) \quad \gamma_t^n(h^{t-1}, x_t, \theta_t^n) = \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} \psi_s^{*n}(h^s) \mid h^{t-1}, x_t, \theta_t^n\right] - \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} \psi_s^{*n}(h^s) \mid h^{t-1}, x_t\right].$$

And the transfer in the balanced team mechanism to investor n in a period t given history h^t is

$$(17) \quad \tau_t^{*n} = \underbrace{p_t\left[a_{t-1}^{*n} - a_t^{*n}\right] - \frac{\phi_t}{N}}_{\text{Market component}} + \underbrace{\gamma_t^n - \frac{\sum_{i \neq n} \gamma_t^i}{N-1}}_{\text{Insurance component}}.$$

Let $\boldsymbol{\tau}_p^*$ denote the sequence of transfers defined by equation (17). The balance team mechanism is given by the pair $\boldsymbol{\mu}_p^* = \{\boldsymbol{a}_p^*, \boldsymbol{\tau}_p^*\}$ of asset policy and transfer. The mechanism $\boldsymbol{\mu}_p^*$ is associated with a price $\boldsymbol{p}$, but I omit the subscript for simplicity when possible. The transfer has a market component and an insurance component, as depicted in equation (17). The market component reflects the costs in the inter-dealer market of changes in asset holdings, $p_t\left[a_{t-1}^{*n} - a_t^{*n}\right]$, and the rent imposed by the dealer to the financial institution, ϕ_t. The insurance component has two parts. The first one, the incentive term γ_t^n, makes the investor internalize the welfare impact of his announcement so he has incentives to truthfully reveals his type. The second part, $\sum_{i \neq n} \gamma_t^i / (N-1)$, charges investor n the cost of the incentive term of the other investors so the mechanism is budget balanced.

The financial institution game

The price $\boldsymbol{p}$, the dealer fees $\boldsymbol{\phi}$, and the balanced team mechanism $\boldsymbol{\mu}^*$ are associated with a dynamic game of incomplete information. I label it the financial institution game and focus on its perfect Bayesian equilibria (PBE). In this game, investors' strategies are sequences of type announcements as functions of sunspots, past histories and type realizations. Formally, the strategies are sequences $\boldsymbol{\sigma} = \{\sigma_t\}_t \in \boldsymbol{\Sigma}$, where $\sigma_t(x^t, h^{t-1}, \theta_t^n) \in \boldsymbol{\Theta}$ and $\boldsymbol{\Sigma}$ denotes the set of investors' strategies.

5. EQUILIBRIUM

In the previous section, I described the financial institution problem, the implied financial institution game, and the outcome of the bargaining between financial institutions and dealers, all taking as given the price p in the inter-dealer market. But in equilibrium, the price must be such that excess demand in this market equals zero. In this section, I provide a definition of equilibrium for the whole economy taking into account market clearing in the inter-dealer market.

Let $\{\boldsymbol{\sigma}^{c,n}\}_{c,n}$ denote the strategy profile in financial institution c. The transition probability of types, F, the probability of meeting with a dealer, α, and the distribution of sunspots, Q, generate a sequence of measures $\boldsymbol{\psi} = \{\psi_t\}_t$ over the space of histories H^t. These measures are defined in the usual way. The excess demand for assets in the centralized market in period t is

$$(18) \quad ED_t(x^t) := \int\int\sum_n a_t^n(\tilde{h}^t(h^t))d\psi_t(h^t|x^t)dc - N\bar{A},$$

where $\tilde{h}_c^t(h^t)$ is constructed by replacing the realizations of types in h^t by the announcements associated with the strategy profile $\{\boldsymbol{\sigma}^{c,n}\}_{c,n}$ played in each coalition. It is worth mentioning that $\boldsymbol{\sigma}^{c,n}$ must be measurable in c, otherwise the integral in (18) is not well defined. Note also that the excess demand for assets is a function of the sunspot history x^t. Excess demand in period t is zero if $ED_t(x^t)$ equals zero for all realized histories of sunspots.

DEFINITION 1: An equilibrium is a triple $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}\}$ such that: (i) for every financial institution c, the strategy profile $\{\boldsymbol{\sigma}^{c,n}\}_n$ is a PBE of the financial institution game associated with the price sequence, $\boldsymbol{p}$, and the balanced team mechanism, $\boldsymbol{\mu}_{\boldsymbol{p}}^*$; (ii) the dealer's fee, ϕ_t, is the solution to the Nash bargaining between financial institution and dealer given in equation (7); and (iii) the excess demand in the inter-dealer market is zero in every period t and for every sunspot history x^t.

Existence and uniqueness of truth-telling equilibrium

A truth-telling strategy is a strategy $\boldsymbol{\sigma} \in \boldsymbol{\Sigma}$ such that $\sigma_t(x^t, h^{t-1}, \theta_t^n) = \theta_t^n$ for all periods t and $(x^t, h^{t-1}, \theta_t^n)$. And a truth-telling equilibrium is an equilibrium $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}\}$ such that the perfect Bayesian equilibrium played in every financial institution is in truth-telling strategies.

In order to understand the existence of a truth-telling equilibrium in this model, it is useful to draw a parallel between the model I develop here and Lagos and Rocheteau (2009). This model extends LR in two ways. First, investors are organized in financial institutions of size $N \in \mathbb{N}$, thus, $N = 1$ is the particular case of LR. But this extension alone does not change LR model in any substantial way if there was complete information regarding the preference shocks. If that was the case, this extension would only relabel the utilities in LR to the sum of the utilities of the

members in each financial institution. The second extension is the private information of preference shocks. With private information, the financial institutions become an interesting object because a mechanism is needed in order to induce investors to truthfully announce the preference shocks they receive over time.

I build on these observations to show that a truth-telling equilibrium exists for small enough η. First I construct the equilibrium price of the complete information case, which is just a relabeling of LR. For η equal to zero, all the conditions of Athey and Segal (2013) apply and the balanced team mechanism has a truth-telling equilibrium when taking as given any price and, in particular, given the equilibrium price of the complete information case. The balance team mechanism does not distort the optimal asset policy, thus it must be the same of the complete information case and the market clearing condition is satisfied. Therefore, the constructed price, the associated balanced team mechanism and truth-telling strategies for all financial institutions must constitute an equilibrium of the economy with private information for the case of η equal to zero. Moreover, the associated outcome is constrained Pareto efficient as it was in Lagos and Rocheteau (2009). Since when η is equal to zero, truth-telling is a strict best response against truth-telling and payoffs are continuous in η, we can conclude that there is also a truth-telling equilibrium in a neighborhood of η equal to zero. These results are summarized in the following proposition. Proofs are in the appendix.

PROPOSITION 1: *Given u, N, α, F, and Q, there exists $\bar{\eta} \in (0,1)$ such that for all $\eta \in [0,\bar{\eta}]$ the economy $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$ has a truth-telling equilibrium. Additionally, the associated equilibrium outcome is constrained Pareto efficient if, and only if, η is equal to zero.*

An implication from the proof of proposition 1 is that, in a truth-telling equilibrium, prices and asset allocations are the same of the unique equilibrium of an alternative economy with complete information. Therefore, equilibrium outcomes of the private information model form a super-set of the outcome of the complete information model. But under what conditions is truth-telling also the unique equilibrium of the private information model? In other words, under what conditions do the outcomes of the complete and incomplete information environments coincide?

With complete information, the liquidity insurance embed in the asset allocation only relies on observed variables. Therefore, there is no reason for investors to announce their types, and the only possible sources of multiplicity in the model are the bargaining with the dealer and the trades in the inter-dealer market. Regarding the bargaining with the dealer, with complete information Nash bargaining has a unique outcome. Regarding the inter-dealer market, this market is competitive so agents take prices as given, they don't act strategically. Hence, there is no strategic complementary and the usual assumptions that spur uniqueness in general equilibrium models apply. As a result,

the model with complete information generates a unique equilibrium.

With incomplete information, the Nash bargaining protocol does not apply unless η is small, but even under this assumption one cannot rule out multiplicity. The reason is that the balanced team mechanism may generate multiple equilibria in the same way that the model in section 2 does. However, when η is small, this mechanism's only goal is to provide insurance to investors against the risk of needing to trade an asset and not finding a dealer to do so. Hence, one can conjecture that if the probability of finding a dealer is high enough, the need for this insurance is low and the same market forces that spur uniqueness in the complete information case also spur uniqueness in the incomplete information one. I find this conjecture to be true.

PROPOSITION 2: *Given u, N, F, and Q, there exists $\bar{\alpha}$, $\bar{\eta} \in (0,1)$ such that for all $\alpha \in [\bar{\alpha}, 1]$ and $\eta \in [0, \bar{\eta}]$ the economy $\mathscr{E} = \{u, N, \alpha, \eta, F, Q\}$ has a truth-telling equilibrium and it is unique.*

6. EXAMPLES OF NON-TRUTH-TELLING EQUILIBRIA

In the previous section we saw that, if search frictions are small enough, the unique equilibrium will have investors playing truth-telling strategies and those outcomes are the same as the model without private information. But what happens to the economy if that is not the case and other equilibria arise? In this section, I study numerical examples and compute different equilibria of the model in order to provide a better understanding of what happens with market variables in non-truth-telling equilibria. In particular, I compute equilibria that resemble a financial crises—where investors run against their financial institutions in the same spirit of the runs in section 2.

Computation

I compute equilibria in the following way. I first guess strategy profiles for the financial institutions. Then I compute the prices and asset policy that clear the market given those profiles. Finally, I verify that the guessed strategy profiles constitute a perfect Bayesian equilibrium.

The financial institution game is a dynamic game of incomplete information. In this type of game, strategies can be very complicated objects (even to guess) since they are functions of all past histories. For this reason, I consider only strategy profiles where announcements are functions of the investors' current type and sunspot. Specifically, in a fraction $\upsilon \in [0,1]$ of the financial institutions, investors follow a run strategy $\boldsymbol{\sigma}^r$, where

$$(19) \quad \sigma_t^r(x^t, h^{t-1}, \theta_t^n) = \begin{cases} \theta_t^n & ; \quad \text{if } x_t = NR \\ \theta_L & ; \quad \text{if } x_t = R \end{cases}$$

for all t and $(x^t, h^{t-1}, \theta_t^n)$. And in a fraction $1 - \upsilon$, investors follow truth-telling strategies.

In order to verify that a strategy profile is a PBE, you need to find beliefs that are consistent with Bayesian updates, given other investors' strategies, and check if each strategy is a best response given those beliefs. Since the game is dynamic, it is also necessary to keep track of those beliefs over time. This makes verifying if a strategy profile is an equilibrium challenging. For example, if the sunspot realization is R from period t to $t+5$ and all investors in a financial institution are following the run strategy $\boldsymbol{\sigma}^r$, then the beliefs over the distribution of types in period $t+6$, consistent with Bayesian updates, will be the composition of the Markov chain of types for five periods $F^5 = F \circ F \circ \cdots \circ F$. This is the case because, under $\boldsymbol{\sigma}^r$ strategy, announcements are only informative when the sunspot is NR, so investors truthfully announce their types. Therefore, to check if the strategy profile $\{\boldsymbol{\sigma}^r\}_n$ is a PBE one would have to check if it is a best response given beliefs F^l, where $l = 1, \ldots,$ is the number of consecutive periods with sunspot realization R.

There are two cases in which keeping track of beliefs is not necessary. First, if types are *i.i.d.* over time, the distribution of types is the same in every period so beliefs coincide with the true distribution. Second, when the state R is an absorbing state for the sunspot, after the first realization of R, announcements are independent of types. Therefore, the beliefs over types are irrelevant to identify whether a strategy is a best response or not. In this paper, I study the *i.i.d.* case.

Parametrization

There are $N = 3$ investors per financial institution and two types, $\theta_L = 0.7$ and $\theta_H = 1.0$. The utility function is a constant relative risk aversion $u(a; \theta) = \theta \frac{a^{1-\delta}-1}{1-\delta}$ with parameter $\delta = 5.0$. The discount factor β equals 0.9. The endowment of assets per investor is $\bar{A} = 1.0$. The bargaining power of the dealers is set to $\eta = 0.1$. In order to understand the impact of search frictions, I compute numerical examples with α going from 0.025 to 0.175 in increments of 0.025. In a similar way, the fraction υ of the financial institutions following the run strategy is computed from 0.05 to 0.95 in increments of 0.025. Preference shocks are *i.i.d.* with $F(\theta_L, \theta_L) = F(\theta_H, \theta_L) = 0.1$. And the sunspot distribution is $Q(NR, NR) = 0.99$ and $Q(R, R) = 0.97$. I simulate the model for 200 periods. In the simulation, x_t equals R from period 37 to 111 and from period 130 to 136. The initial distribution of asset holdings is set to be what would be the distribution after a long period of time in which x_t equals NR. Prices, trade volume, and welfare are relative to the values in the friction-less economy where α equals 1 and η equals 0. In this case, the over-the-counter market is equivalent to a competitive Walrasian market, hence there is a unique equilibrium and those variables are well defined. In figures 3 to 6, the first graph displays the variables as a function of υ, the fraction of financial institutions under a run, for α fixed at 0.1. And the second graph displays the variables as a function of α, the probability of meeting with a dealer, for υ fixed at 0.5. It is worth mentioning that there is nothing special in these parameters and I had simulated the economy

with several others. The qualitative results I obtained seem to be quite general.
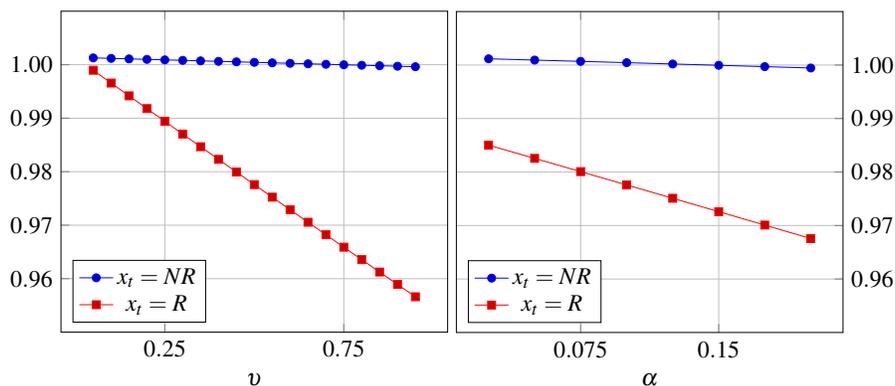
Numerical examples

Figure 3: Prices

Figure 3 depicts the price for different values of υ and α. Since preferences are quasi-linear and strategies are only function of types and sunspots, the equilibrium price process has a simple structure taking only two values. One for periods in which the sunspot realization is $x_t = NR$ and one for periods in which the sunspot realization is $x_t = R$. The price in periods with runs is lower than the price in periods without runs, which is intuitive. Besides, both prices are decreasing in υ. The reason the price in periods with runs is decreasing in υ is straightforward. Due to a higher number of financial institutions with investors announcing low valuation for the asset, more financial institutions try to sell assets to dealers, which creates a selling pressure and reduces price. The reason the price in periods with no runs is also decreasing on υ is that financial institutions anticipate that the price will collapse if there is a run. This generates the possibility of capital losses and, as a consequence, the price must go down in advance. And a higher υ implies a higher capital loss and the more the price responds in advance. Interestingly, the increase in the fraction of financial institutions in which investors run not only decreases prices. It also increases price volatility since the price in periods with runs decreases more with υ than the price in periods with no runs. The same is true regarding increases in α, as we can see from the second graph in figure 3. The result that higher α, or lower search frictions, could reduce prices were already present in LR. This numerical examples show that it can also increase the price volatility in the presence of runs.

Figure 4 depicts a simulation of trade volume over the 200 periods for different values of υ and α. When the sunspot $x_t = R$ hits the economy, there is an initial spike in trade volume, which is accompanied by a following contraction. This spike is due to a fire sale effect. Since several financial institutions try to sell assets at the same time, price collapses and some financial institutions end up buying assets, which increases volume. After this initial reallocation of assets, the financial

Figure 4: Trade volume
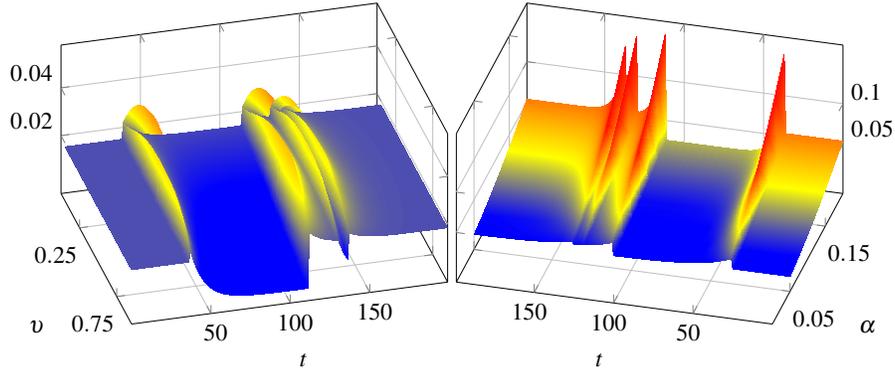
institutions under a run stop trading, which in turn decreases trade volume. The spike in trade volume also occurs in recovery periods when the economy switches from $x_t = R$ to $x_t = NR$. In these periods investors stop the run and financial institutions try to buy back some of the assets sold in the run periods. As a result, the asset price goes up and there is an initial spike in trade volume that then bounces back to the average in periods where $x_t = NR$. The first graph in figure 4 shows that the higher the fraction of financial institutions under a run, the higher is the collapse in trade volume for $x_t = R$. The second graph shows that the spikes in trade volume are more substantial when search frictions are smaller.

The bid-ask spread is a traditional measure of liquidity used in financial markets. In the context of this model, I define it as the average difference between the price faced by the financial institutions when buying/selling assets to dealers and the price faced by dealers in the inter-dealer market. The unit price for a financial institution buying assets from a dealer is $p_t + \phi_t/dA_t$, where dA_t is the total amount of asset bought. Similarly, the unit price for a financial institution selling assets to the dealer is $p_t - \phi_t/dA_t$, where dA_t is the total amount of asset sold. Therefore, the bid-ask spread is defined as $\phi_t/p_t dA_t$.

Figure 5 depicts the average bid-ask spread for each period in the simulation. The direction in which runs impact the bid-ask spread is not uniform, as we can see from the first graph in the figure. If the fraction of financial institutions under a run is either small or large, a run decreases the average bid-ask spread. On the other hand, for intermediary values, a run increases the average bid-ask spread. The intuition is that financial institutions under a run want to sell their assets and, as a consequence, the price collapses. Then the financial institutions that are not under a run have huge gains in buying those assets at this lower price. Dealers take advantage of those financial institutions by charging them a high bid-ask spread. If there are too many financial institutions under a run, then there are not many financial institutions to take advantage of the lower prices and dealers are able to charge this high bid-ask spread. If there are too little financial institutions under a run, then

Figure 5: Bid-ask spread
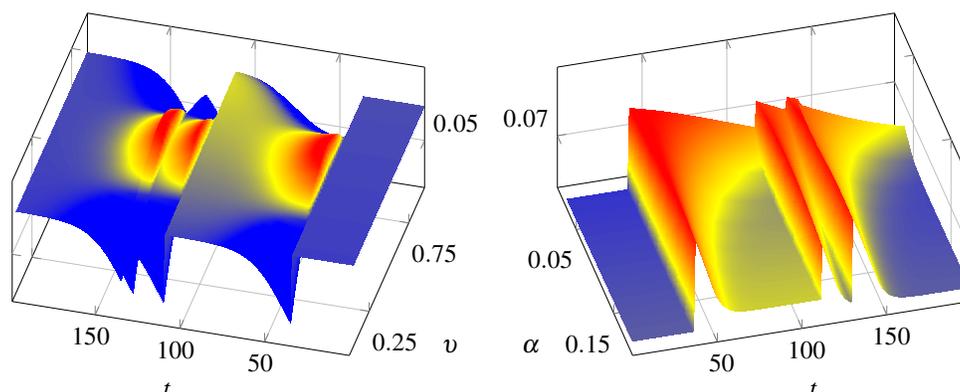
the price doesn't collapse so there is no margin for dealers to increase the bid-ask spreads. The intermediary case is where a run causes the bid-ask spreads to go up. The same analysis apply for when the economy switches back from a run state to a no run state. From the second graph in figure 5 we can see that if search frictions are smaller (higher α), these effects fade away faster over time. Which is typical in this class of over-the-counter market models.

From a welfare perspective, a run has two effects and they are both due to misallocation of assets in the economy. The first effect is within a financial institution under a run. The assets in this financial institution are not well allocated because investors with high valuation of the asset end up holding the same amount as investors with low valuation of the asset since they all announce the lower type θ_L. The second effect is across financial institutions. When investors in a fraction υ of the financial institutions run, these financial institutions sell assets in response to the announcements of low valuation. But the financial institutions that end up buying those assets do not necessarily have investors with higher valuation than the investors in the financial institutions facing a run. It just happens that their investors are not misrepresenting their types. So, for example, you could have a financial institution with all investors of type θ_H selling assets and ending up with a smaller asset position than a financial institution with only one investor of type θ_H.

Figure 6 depicts the period aggregate welfare in the simulation. Both effects of runs discussed above makes the welfare decreasing in the fraction of financial institutions under a run in periods where $x_t = R$, as we can see from the first graph. However, the welfare in the periods of run is also decreasing in α. In other words, a reduction in search friction lowers welfare in periods of runs. The reason is if the search friction is smaller (α higher) it is easier for financial institutions to sell assets. But it is inefficient for financial institutions under a run to do so and, as a result, the welfare can go down during periods of runs.

Search friction has an ambiguous effect on period welfare. In periods with no runs, the welfare

Figure 6: Period aggregate welfare
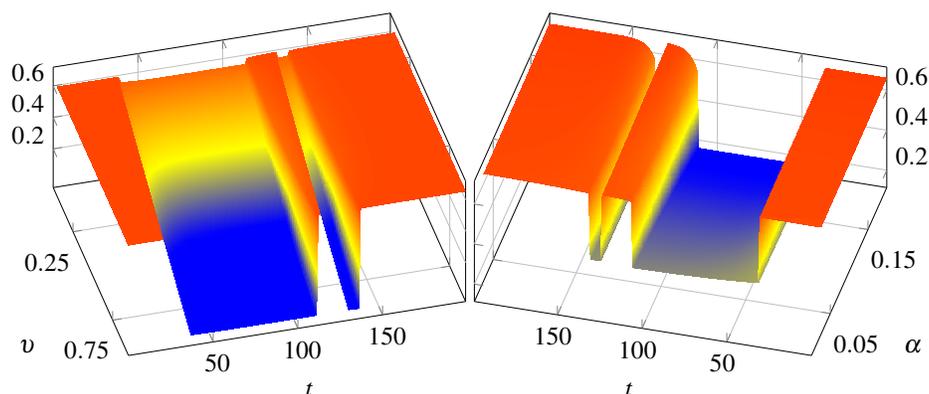
is increasing in α because it allow investors to sell their assets if they need it. And in periods with runs, the welfare can be decreasing in α because financial institutions sell their assets inefficiently. But what effect dominates? In other words, is the expected welfare increasing or decreasing in α?
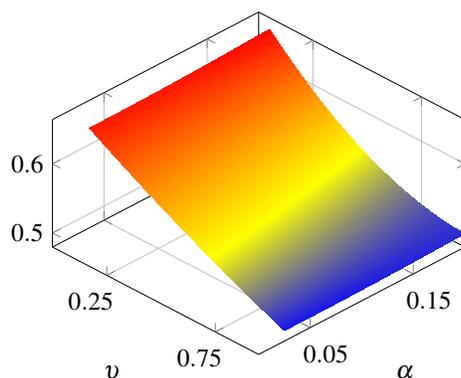
Figure 7: Average aggregate welfare

Figure 7 shows the expected aggregate welfare of the economy in the long run. I find that depending on the fraction of financial institutions playing the run equilibrium, υ, the expected welfare can be increasing or decreasing in α. For low or high values of υ, the welfare is increasing in α. While for intermediary values of υ the welfare is decreasing in α. Proposition 2 states that, as long as the dealer's bargaining power is not too high, if the search frictions is small, truth-telling is the unique equilibrium. This proposition could lead someone to think that a good way to improve welfare is by reducing search frictions and, consequently, eliminating all the run equilibria. These numerical simulations show that such policy could have unintended consequences. If the reduction in search friction is not enough to completely eliminate the run equilibria, it may reduce welfare by allowing more financial institutions to inefficiently sell their assets during episodes of runs.

7. DISCUSSION

In this paper I identify a connection between runs and over-the-counter markets. I show that, in the model, financial institutions are more vulnerable to runs when search frictions are severe. It is interesting to notice that during the 2007-08 financial crisis the institutions that suffered runs were engaged in trade of asset-backed securities (ABSs), which were mostly traded over the counter. Annual issuance of ABS went from $10 billion in 1986 to $893 billion in 2006, as reported by Agarwal *et al.* (2010). And a growing shadow bank sector has purchased most of these assets. As a result, in 2007 the financial sector featured a large number of financial institutions operating in a market with severe over-the-counter market frictions. My model suggests that these features are important elements to understand the runs during the period.

The most common prescription for enhancing financial stability is to regulate the contracts offered by financial institutions. For example, recently, the Securities and Exchange Commission (SEC) announced a set of proposals to enhance financial stability, which includes a recommendation for the MMF board of directors to impose fees and gate payments in times of heavy redemption activity.[9] And Cochrane (2014) calls for an narrow bank sector and the ban on run-prone contracts.

There are two downsides of directly regulating contracts. First, each type of financial institution serves a different type of investor and, therefore, requires a different contract. As a result, the regulation needs to be specific to the type of institution. That is, we need one particular regulation for commercial banks, one for mutual funds, one for structured investment vehicles, etc. Which results in a complex regulatory system doomed to feature loopholes and regulatory arbitrage possibilities. The second downside is that, even if we are willing to write complex regulations to every type of financial institution, it is not clear what regulations we should impose. Even a glimpse through the Diamond-Dybvig literature shows that the optimal contract depends on several details of the environment. When you consider models other than Diamond-Dybvig, the possible regulations grow exponentially. Besides, more often than not, regulations have a welfare cost. In which case, the optimal regulation also depends on how the policymaker evaluates welfare.

My results suggest a different avenue in which a policymaker can enhance financial stability. Instead of focusing on the particular contract that financial institutions offer, it can intervene on the market for the underlying assets. That is, if we reduce trade frictions enough, we enhance financial stability with no need to regulate individual institutions. And we can interpret some of the policies set in place during the 2007-08 financial crisis as a step in this direction. The Federal Reserve responded to the financial crisis by implementing a number of programs designed to support the liquidity of financial institutions. An example of this kind of program is the Asset-Backed

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[9]See SEC (2013).

Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF). In the words of the Board of Governors of the Federal Reserve System, "The AMLF was designed to provide a market for ABCP that MMMFs sought to sell." and "These institutions used the funding to purchase eligible ABCP from MMMFs. Borrowers under the AMLF, therefore, served as conduits in providing liquidity to MMMFs, and the MMMFs were the primary beneficiaries of the AMLF."[10] This seems to suggest an attempt to increase the chances of an MMMF to find a buyer for their ABCP. In light of the model, I interpret this as an attempt to increase the probability of a MMMF of meeting a dealer. My results suggest that such policies have the potential of eliminating runs on financial institutions and stabilizing the financial sector. But it also points out to a risk; it can increase asset misallocation during a crisis, and ultimately decrease welfare.

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[10]See `http://www.federalreserve.gov/newsevents/reform_amlf.htm`

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APPENDIX A—PROOF OF PROPOSITION 1

PROOF: Proposition 1 has two parts. The first one is existence of truth-telling equilibrium, and the second one is the Pareto efficiency of this equilibrium when $\eta = 0$. Let's start with the first part. The proof has the following structure. I consider an alternative economy where the bargaining power of dealers is $\hat{\eta} = 0$ and the probability of finding a dealer is $\hat{\alpha} = \alpha(1 - \eta)$. I solve a planner's problem for this alternative economy and write its first order conditions. I use the Lagrange multiplier of the planner's problem to construct a price sequence for my original economy. Finally, I show that, for η small, the associated balanced team mechanism features a truth-telling equilibrium.

The planner's problem is to choose an asset allocation $\boldsymbol{a} = \{a_t\}_t$, where $a_t = (a_t^1, \ldots, a_t^N)$: $H^t \to \mathbb{R}_+^N$, that maximizes expected aggregated utility of investors. Note that I assume that the asset allocation is symmetric across agents. This assumption is without loss of generality with respect to finding a constrained Pareto efficient outcome since preferences are quasi-linear. Any differences in Pareto weights would be adjusted by transfers of utility, not distorting the asset allocation.

Let $\hat{\boldsymbol{\psi}} = \{\hat{\psi}_t\}_t$ denote the sequence of measures over the space of histories, H^t, implied by the transition probability of types, F, the probability of meeting with a dealer, $\hat{\alpha} = \alpha(1 - \eta)$, and the distribution of sunspots, Q. An asset allocation $\boldsymbol{a} = \{a_t\}_t$ is feasible if for all h^t it satisfies

$$(20) \quad \int \sum_n a_t^n(h^t) \, d\hat{\psi}_t(h^t) = \bar{A} \text{ and}$$

(21) $\sum_n a_t^n(h^t) = \sum_n a_{t-1}^n(h^{t-1})$ whenever ι_t equals zero,

where $a_{-1}^n = \bar{A}$ for all n. Equation (20) is a resource constraint and equation (21) imposes that financial institutions only adjust their aggregate asset holdings when ι_t is one. Let $\mathscr{F}$ denote the set of all feasible allocations. Note that the feasible set for the planner, $\mathscr{F}$, is different than the feasible set for a financial institution, Γ, because the financial institution does not face a resource constraint. Financial institutions take the price in the inter-dealer market as given, and assume that, at those prices, they can demand any quantity.

The aggregate welfare of the economy implied by an asset allocation $\boldsymbol{a} \in \mathscr{F}$ is

(22) $W(\boldsymbol{a}) = \sum_t \beta^t \int \sum_n u(a_t^n(h^t), \theta_t^n) d\hat{\psi}_t(h^t) = \hat{\mathbb{E}}\{\sum_t \beta^t \sum_n u(a_t^n(h^t), \theta_t^n)\}.$

Since preferences are quasi-linear, an asset allocation is constrained Pareto efficient if, and only if, it achieves the maximum aggregate welfare among all feasible allocations. It is easy to show that such allocation exists and it is unique. With some abuse of notation, label this allocation $\boldsymbol{a}$.

Note that $\boldsymbol{a}$ is independent of the sequence of sunspots shocks x^t. To see this, note that by construction x^t has no effect on any primitive, it is merely a sunspot. So, if $\boldsymbol{a}$ were a function of x^t, it would be as if we were randomizing among different allocations. And, since the object function is concave, the expected allocation would achieve a higher value of the object function than $\boldsymbol{a}$, which would contradict the fact that $\boldsymbol{a}$ itself maximizes the objective function.

Since $\boldsymbol{a}$ maximizes (22), there exists a sequence of Lagrange multipliers, $\boldsymbol{\lambda} = \{\lambda_t\}_t$, such that for all histories h^t with $\iota_t = 1$, and all n, the asset allocation $a_t^n(h^t)$ satisfies

(23) $\lambda_t = \hat{\mathbb{E}}\{\sum_{d=0}^{d_k-1} \beta^d u'(a_t^n; \boldsymbol{\theta}_{t+d}^n) \mid h^t\},$

where d_k is the time interval between two meetings with the dealer. And $\boldsymbol{\lambda}$ satisfy the transversality condition

(24) $\lim_{t \to \infty} \beta^t \lambda_t = 0.$

The existence of the Lagrange multipliers is a standard result and can, for instance, be derived from theorem 1, section 8.3, and theorem 1, section 8.4, in Luenberger (1969).

Define the price sequence $\boldsymbol{p} = \{p_t\}_t$ as

(25) $p_t = \lambda_t - \hat{\alpha}\beta\sum_{d=0}^{\infty} \beta^d \lambda_{t+1+d}.$

Our equilibrium candidate will be $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}\}$, where $\boldsymbol{p}$ is constructed from (25), $\boldsymbol{\phi}$ is

constructed solving equation (7), where V_t solves equation (8), for given $\boldsymbol{p}$, and $\boldsymbol{\sigma}^{c,n}$ is truth-telling for all c and n. Note that by construction condition (ii) is satisfied. And condition (iii), the market clearing condition, is also satisfied since the excess demand in the original economy is $\alpha/\hat{\alpha}$ the excess demand in the alternative economy when we are comparing the same asset policy and assuming truth-telling strategies. Since the latter is zero, the former must also be zero.

Now let's show condition (i). Let us start showing that, given $\boldsymbol{p}$, the asset allocation $\boldsymbol{a}$ solves the financial institution problem (10). We can rewrite equation (25) to show that for all period t and h^t with $\iota_t = 1$, the sequence $\boldsymbol{p}$ satisfies

$$(26) \quad \lambda_t = p_t - \sum_{d=1}^{\infty} \beta^d (1-\hat{\alpha})^{d-1} \alpha p_{t+d} = p_t - \hat{\mathbb{E}}\{\beta^{d_k} p_{t+d_k} \mid h^t\}.$$

Equations (23) and (26) combined imply equations (11) and (12) of the first order conditions of the financial institution problem. Since $\lim_{t\to\infty} \beta^t \lambda_t = 0$, equation (13) of the first order conditions is also satisfied. Since these conditions are necessary and sufficient, we can assert that $\boldsymbol{a}$ solves maximizes the asset policy problem of the financial institution. Note then that, when $\eta = 0$, the balanced team mechanism, $\boldsymbol{\mu}$, associated with the asset allocation, $\boldsymbol{a}$, and the transfer, $\boldsymbol{\tau}$, implied by equation (17) satisfies all the conditions of corollary 3 in Athey and Segal (2013). To see that it is also a PBE in a neighborhood of $\eta = 0$, first note that, when $\eta = 0$, truth-telling is a strict best response since a non-truthful announcement would imply a discrete jump on the allocation. Moreover, all the payoffs of the financial institution game are continuous in η. As a result, truth-telling will be a PBE in an open neighborhood of $\eta = 0$. Therefore, there exists $\bar{\eta} \in (0,1)$ such that for all $\eta \in [0, \bar{\eta})$, the family $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}$ constitutes an equilibrium.

For the last, we need to show that an equilibrium outcome is constrained Pareto efficient if, and only if, $\eta = 0$. First note that, if $\eta = 0$, the planner's problem of the alternative economy and the original economy coincide and the equilibrium outcome will solve the planner's problem. This is the sufficiency part. To see the necessity, assume by the way of contradiction that for some $\alpha \in (0,1)$ and $\eta \in (0,1]$ the associated economy has an equilibrium $\{\{\boldsymbol{\sigma}^{c,n}\}_{c,n}, \boldsymbol{\phi}, \boldsymbol{p}\}$ whose outcome is Pareto efficient. Now we can use equation (26) to construct the Lagrange multipliers for the alternative economy and show that the asset allocation associated with this equilibrium solves the planner's problem of the alternative economy. But the solution of the alternative economy and the original economy cannot be the same since $\boldsymbol{\psi}$ and $\hat{\boldsymbol{\psi}}$ do not coincide when $\eta \neq 0$. Therefore, the implied outcome is not constrained Pareto efficient, which concludes the proof. $\qquad \square$

APPENDIX B—PROOF OF PROPOSITION 2

PROOF: Let us first consider the case where $\alpha = 1$ and $\eta = 0$. Because of proposition 1, we

know this economy has a truth-telling equilibrium. We just need to show it is unique. Note that in this case the financial institution finds a dealer at every period and, since the dealers have no bargaining power, the financial institutions trade in the competitive inter-dealer market with no frictions. In other words, the asset market is equivalent to a Walrasian market. Now let us show that equilibrium strategies must be truth-telling. Consider the first order conditions of the financial institution problem, (11)-(12). Since $\alpha = 1$ and $\eta = 0$, we can rewrite these equations as

$$p_t - \beta p_{t+1} = u'(a_t^n, \theta_t^n) = u'(a_t^m, \theta_t^m) \text{ for all } n \neq m.$$

This implies that the asset allocation of agents n and m are independent at any given period. Therefore, the investor n incentive term is

$$\gamma_t^n(h^{t-1}, x_t, \theta_t^n) = \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} \psi_s^n(h^s) \mid h^{t-1}, x_t, \theta_t^n\right] - \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} \psi_s^n(h^s) \mid h^{t-1}, x_t\right] = 0$$

and the transfer to investor n is

$$\tau_t^n = p_t\left[a_{t-1}^n - a_t^n\right] - \frac{\phi_t}{N} + \gamma_t^n - \frac{\sum_{i \neq n} \gamma_t^i}{N-1} = p_t\left[a_{t-1}^n - a_t^n\right].$$

Therefore, the payoff of an investor n under truth-telling is exactly

(27) $\mathbb{E}\{\sum_t \beta^t[u(a_t^n, \theta_t^n) + p_t(a_{t-1}^n - a_t^n)]\}$

But an asset allocation for investor n maximizes (27) if, and only if, it satisfies the first order conditions

$$p_t - \beta p_{t+1} = u'(a_t^n, \theta_t^n).$$

This is exactly the same condition implied by the first order condition of the coalition problem. Therefore, truth-telling gives the agent the maximum payoff he could achieve, and any other announcement would lead to strictly worse payoff. As a result, truth-telling is a dominant strategy and the equilibrium is unique and in truth-telling.

Note then that given any price, the asset allocation given to an investor by his financial institution is exactly the equivalent Walrasian allocation. Hence, we can apply the regular first and second welfare theorems that hold for this environment. Therefore, any equilibrium is Pareto efficient and, since there is a unique Pareto efficient allocation, the equilibrium is unique.

For the last, note that all payoffs are continuous in α and η, which implies that truth-telling will also be the unique equilibrium of the economy in a neighborhood of $\alpha = 1$ and $\eta = 0$. □