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On Regional Borrowing, Default, and Migration

WP 19-04 | Grey Gordon
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On Regional Borrowing, Default, and Migration[*]

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Abstract

Migration plays a key role in city finances with every new entrant reducing debt per person and every exit increasing it. We study the interactions between regional borrowing, migration, and default from empirical, theoretical, and quantitative perspectives. Empirically, we document that in-migration rates are positively correlated with deficits, that many cities appear to be at or near state-imposed borrowing limits, and that defaults can occur after booms or busts in productivity and population. Theoretically, we show that migration creates an externality that results in over-borrowing, and our quantitative model is able to rationalize many features of the data because of it. Counterfactuals reveal (1) Detroit should have deleveraged in the financial crisis to avoid default; (2) a return to the high-interest rate environment prevailing in the 1990s has only small long-run effects on city finances; and (3) anticipated bailouts double default rates.

JEL Codes: E21, F22, F34, R23, R51

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1 Introduction

The local government finances of Detroit and Flint, Michigan, have received significant attention in the media, and for good reason. The two cities have both experienced shrinking populations (with declines exceeding 30% since 1986) and wages (decreasing 10% and 30%, respectively) that have placed significant strain on their finances. Detroit responded to these challenges with burgeoning debt—which grew from $2,000 per person to $12,500 in 2012 dollars—and a default in 2013, while Flint kept debt low but was forced to raise taxes from $3,000 to $4,500 per person. Which of these paths was best? Could anything else have been done? Were these debt levels optimal or too high? Currently, there is scant empirical evidence and no model linking city finances, migration, and default. This paper fills these gaps in the literature in two ways: first, by merging panel datasets on local government finances, labor productivity, and migration to document patterns of cities in general and defaulting cities in particular; and, second, by proposing and analyzing a rich and novel general equilibrium model that successfully captures these patterns.

To guide our investigation of the data, we first consider a comparatively simple two-period Lucas (1972)-type islands model. Each island represents a local economy and has a continuum of households who make migration decisions, an exogenously given per person endowment that is location-specific, and a planner who issues debt in the first period (transferring the proceeds to households) and repays it in the second (using lump-sum taxes). The key assumption is that the local planner maximizes the welfare of *current* residents. The model reveals that, relative to an economy-wide planner, local planners have an incentive to overborrow. The reason is simple: New arrivals in the second period will help repay debt issued in the first period, and the planner does not directly value their utility. There is a potentially offsetting effect, namely, that as borrowing increases, the island's attractiveness to newcomers declines and this increases debt per person. However, we show that in general equilibrium with two heterogeneous island types, the equilibrium is not constrained efficient—i.e., not efficient taking migration decisions as given—and is therefore not Pareto efficient, either.

The theory suggests that cities should accumulate large amounts of debt, that cities with larger in-migration rates all else equal should accumulate more debt, and that states or federal governments should have policies in place that restrict municipal borrowing. (For our purposes, we will refer to cities and municipalities interchangeably.)[1] Using comprehensive datasets on city finances, population, migration, and labor productivity as well as institutional details, we find support for all of these predictions. Results from fixed effects regressions reveal a positive correlation between deficits and in-migration rates. Moreover, we show many states have restrictions on municipal borrowing. While the form of most of the states' borrowing limits precludes us from measuring how close cities are to the limits, for California and Michigan we can tell. And there, it seems most cities are close to, at, or even above the limit.

We further look at the relationship between borrowing, migration, and default by conduct-

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[1]A municipality is a city, town, or village that is incorporated into a local government.

ing a case study of municipal defaults—which are rare overall but have doubled in number since the early 2000s. The study reveals that defaulters have larger debt, expenditures, and deficits than typical cities. However, heterogeneous paths lead to default: Defaults have happened during challenging times characterized by population declines and low productivity (as in Detroit and Flint), but also during productivity or population booms (as in San Bernardino, Stockton, and Vallejo, California).[2] Theoretically, the *bust defaults* are unsurprising: Negative, mean-reverting shocks should result in borrowing for consumption smoothing purposes with persistently negative ones leading to default. However, based on that intuition, the *boom defaults* are very surprising. Yet, our model offers an explanation: In response to rapid population growth, cities over-borrow expecting future entrants to help repay the debt. Then, with a high amount of leverage, default looks attractive when a negative shock eventually occurs.

With these empirical regularities in mind, we extend the two-period model to capture them, allowing for a decentralized economy with an infinite horizon, production, government services, a taste shifter we think of as weather, housing, borrowing limits, and default. After showing the economy can be centralized (at a local level), we demonstrate the calibrated model is capable of matching a host of statistics including the mean and standard deviation of in- and out-migration rates, mean default rates, interest rates on municipal debt, local debt-GDP ratios, the standard deviation of log population, correlations between productivity and migration rates, and population autocorrelations. The model also reproduces the patterns observed in the fixed effects regressions and the proximity of cities to borrowing limits in the cross-section. Importantly, the model generates both boom and bust defaults.

Having established the model's success in matching relevant features of municipal borrowing, migration, and default, we turn to its counterfactual predictions. Feeding in the estimated productivity process for Detroit—which shows a rapid decline beginning in 2006 and continuing through 2012—gives an alternative, and optimal, path for its economy. The results show, perhaps surprisingly, that Detroit's debt levels in the early 2000s (around $7,000 per person in 2010 dollars) were nearly optimal. However, when the financial crisis hit, it should have deleveraged by around $750 per person via large cuts to expenditures and smaller cuts to taxes. In contrast, Detroit raised taxes in 2009 and 2010 and expenditures and debt per person rose almost continually from 2006 to 2011, precipitating default.

We also investigate the consequences of three policy changes. First, we look at increasing the risk-free municipal bond interest rate from 4% (its real value in 2010) to 6.5% (its actual value in the early 1990s). We show raising rates causes migration away from high-debt, low-productivity cities to low-debt, high-productivity ones, providing a slight boost to aggregate output but with few other consequences in the long run. Second, we look at elimination of state-imposed borrowing limits. We show this has small effects precisely because private credit markets constraint city borrowing virtually as much as the state-imposed constraints. Finally, we investigate the conse-

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[2]Technically, Flint did not default. However, in 2002 and 2011, the state appointed emergency financial managers who took over the city finances. Moreover, its 2014 water crisis can be seen as a type of default.

quences of bailouts in our calibrated model. We find ε-bailouts—defined as the smallest transfer sufficient to avoid default—double default rates but otherwise have small effects. The reason is ε-bailouts are not attractive because (1) they give the same utility as default in the bailout period and, (2) to match both large debt positions and low default rates in the data, the calibration makes default costly (equal to 8.4% of annual income). Hence, while bailouts do create a moral hazard problem and lead to some increase in default rates, the moral hazard is mitigated by low ex-post returns from being bailed out.

Since our model features local governments competing (through migration) via taxes and spending, it connects to large literatures on tax competition and fiscal federalism as surveyed by Wilson (1999) and Weingast (2009). In influential work, Tiebout (1956) showed this competition can lead to efficiency, and we show that—in a case where migration reacts to policies in an extreme way—there can be efficiency. Outside of this extreme case, however, we prove the equilibrium is generally inefficient. Interestingly, the inefficiency does not come from either of the two common sources Wilson (1999) highlights.[3]

A few papers in this literature have discussed the potential for local governments to overborrow because of migration. In particular, Bruce (1995) and Schultz and Sjöström (2001) prove that overborrowing generally does occur. However, both of their models are two-period, partial equilibrium models with costless moving. And, in fact, we show that the Bruce (1995) and Schultz and Sjöström (2001) results need not go through in general equilibrium: With symmetry, general equilibrium bond prices undo the incentive to overborrow. (But overborrowing does occur with a modicum of heterogeneity.) More importantly, we contribute to this literature by showing empirically and quantitatively the role of overborrowing in reproducing many of the data's features.

Our paper also builds on the large sovereign default literature begun by Eaton and Gersovitz (1981), which has focused almost exclusively on nation states.[4] Epple and Spatt (1986) is an exception that argues states should restrict local debt because default by one local government makes other local governments appear less creditworthy. Such a force is not at work in our model because we assume full information. We contribute to this literature by showing migration strongly influences debt accumulation and can result in boom defaults.

Our work also connects to a vast literature on intranational migration. The empirical work and to a lesser extent theoretical is surveyed in Greenwood (1997). Two seminal papers in this literature, Rosen (1979) and Roback (1982), employ a static model with perfectly mobile labor. This implies every region provides individuals with the same utility. While this indifference condition allows

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[3]The first source is fiscal externalities induced by tax bases being linked across regions. Gelbach (2004), Akcigit, Baslandze, and Stantcheva (2016), Moretti and Wilson (2017), and Coen-Pirani (2018) provide recent examples of this where migration is affected by tax progressivity. In our model, inefficiency is not driven by changes in tax bases per se since out-migration, by itself, does not lead to inefficiency. The other source of inefficiency comes from pecuniary externalities induced by general equilibrium effects (a recent example of this type is Fajgelbaum, Morales, Serrato, and Zidar, 2015). We show inefficiency results in both closed and open economy versions (the latter having exogenous bond prices) of our model, so this also is not a driving force behind our results.

[4]Some of the key references here are Arellano (2008); Hatchondo and Martinez (2009); Chatterjee and Eyigungor (2012); and Mendoza and Yue (2012). The handbook chapter Aguiar, Chatterjee, Cole, and Stangebye (2016) provides a thorough description of the literature.

for elegant characterizations of equilibrium prices and rents, it also means government policies are completely indeterminate: Every debt, service, or tax choice results in the same utility. Our model breaks this result by assuming labor is imperfectly mobile, which lets it match both the sluggish population adjustments and the small correlations between productivity and migration rates observed in the data.

More recently, Armenter and Ortega (2010), Coen-Pirani (2010), Van Nieuwerburgh and Weill (2010), Kennan and Walker (2011), Davis, Fisher, and Veracierto (2013), Caliendo, Parro, Rossi-Hansberg, and Sarte (2017) have analyzed determinants of migration and its consequences in the U.S. Kennan and Walker (2011) use a structurally estimated model of migration decisions and find expected income differences play a key role, providing external evidence of the model's productivity-driven migration decisions. Outside the U.S., much recent research has been focused on migration in the European Union (Kennan, 2013, 2017; Farhi and Werning, 2014). All these papers abstract from debt. To our knowledge, ours is the only quantitative model of regional borrowing and migration, let alone having default.

The rest of the paper is organized as follows. Section 2 presents a simple model that highlights the relation between borrowing and migration and guides our empirical investigation. Section 3 contains some relevant features about cities and municipalities in the U.S., which will guide us in building the quantitative model described in Section 4. Section 5 gives the calibration and model validation, and Section 6 assesses what leads to default, what (if anything) Detroit should have done differently, and the counterfactuals. Section 7 concludes.

2   On borrowing and migration

Before turning to the data, we highlight how migration influences borrowing decisions and efficiency using a two-period model. To focus purely on the role of borrowing, we assume there is full-commitment to repay debt and, hence, no default.

The economy is comprised of a unit measure of islands and a unit measure of households. Consider an arbitrary island. In the first (second) period, the island has a per person nonstochastic endowment of y_1 (y_2). The local planner / government issues $-b_2$ debt per person ($b_2 > 0$ means assets) at price $\bar{q}$. Total debt issuance is $-b_2 n_1$, where n_1 is the initial measure of households on the island. At the beginning of the second period, households draw an idiosyncratic utility cost of moving $\phi \sim F(\phi)$ with a density f and then decide whether to migrate. If they migrate, they pay ϕ and obtain expected utility J, which is an equilibrium object.

Households value consumption according to $u(c_1) + \beta u(c_2)$, where c_1 (c_2) is consumption in the first (second) period. Household utility in the second period is $u(c_2)$ if they stay and $J - \phi$ if they move, so migration decisions follow a cutoff rule in ϕ with indifference at $J - u(c_2)$. Consequently, the outflow rate is $o_2 = F(J - u(c_2))$. The inflow rate is given by $i_2 = \bar{i}\mathbf{I}(u(c_2))$, where $\mathbf{I}$ is a differentiable, increasing function and $\bar{i}$ is an equilibrium object that ensures aggregate inflows equal aggregate outflows. (Consequently, inflows can depend on the distribution of utility across

islands, but that information must be summarized in $\bar{\imath}$.) The population law of motion is $n_2 = (1 + i_2 - o_2)n_1$. We assume the migration decision is noisy in the sense that $F(0) > 0$, so that some people will move even if $u(c_2) = J$.[5]

After all migration has taken place, the government pays back its total obligation, $-b_2 n_1$, by taxing the n_2 households lump sum. Consequently, per person consumption in the second period is $c_2 = y_2 + b_2 n_1 / n_2$. The government's problem may be written

$$\max_{b_2} u(c_1) + \beta \int \max\{u(c_2), J - \phi\}\, dF(\phi)$$

$$\text{s.t. } c_1 + \bar{q}b_2 = y_1, \quad c_2 = y_2 + b_2 \frac{n_1}{n_2}, \quad n_2 = n_1(1 - o_2 + i_2), \quad c_1, c_2 \geq 0, \tag{1}$$

$$i_2 = \bar{\imath}\mathbf{I}(u(c_2)), \quad o_2 = F(J - u(c_2)).$$

Proposition 1 gives the Euler equation for government bonds (all proofs are in Appendix C).

Proposition 1. *The local government's Euler equation is*

$$u'(c_1)\bar{q} = \beta \frac{1 - o_2}{1 - o_2 + i_2} u'(c_2) \left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}\right). \tag{2}$$

The Euler equation reflects two competing forces. One is an externality seen in the term $\frac{1-o_2}{1-o_2+i_2} \leq 1$. Because the planner does not value the utility of new entrants and because new entrants bear $\frac{i_2}{1-o_2+i_2}$ of the debt burden (which is their share of the second period population), the marginal cost associated with an additional unit of debt—holding fixed migration rates—is $1 - \frac{i_2}{1-o_2+i_2}$ or $\frac{1-o_2}{1-o_2+i_2}$. All else equal, higher in-migration lowers the effective discount factor ($\beta \frac{1-o_2}{1-o_2+i_2}$) and increases borrowing. Clearly, then, the assumption that the planner only values current residents plays a key role. But note that is also the most natural assumption: If households could vote on the planner's policy in the first period, they would unanimously approve it because it maximizes their welfare.

The other force is seen in the term $1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2}$, which is one minus the elasticity of the next period's population with respect to savings. It reflects that for each person attracted to the island through less borrowing, the overall debt burden per person falls. (Conversely, if $b_2 > 0$, each additional entrant reduces assets per person, which discourages savings.) Hence, a rational government, internalizing the effects of city finances on migration decisions, should exercise more financial discipline all else equal to attract individuals to the islands to reduce debt per person.

Consider two equilibrium concepts. A *closed economy equilibrium* is a 3-tuple $\{\bar{\imath}, J, \bar{q}\}$ with optimal migration, consumption, and borrowing decisions such that

1. total inflows equal total outflows, $\bar{\imath}\int \mathbf{I}(u(c_{2,j}))n_{1,j}dj = \int F(J - u(c_{2,j}))n_{1,j}dj$;

2. the expected utility of moving is consistent, $J = \int u(c_{2,j}) \frac{\bar{\imath}\mathbf{I}(u(c_{2,j}))n_{1,j}}{\int \bar{\imath}\mathbf{I}(u(c_{2,i}))n_{1,i}di}dj$; and

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[5]This is relevant because a few times we will assume cross-sectional homogeneity in terms of fundamentals to facilitate the proofs, and we want to ensure migration happens in this case.

3. the bond market is in zero net supply, $\int b_{2,j} n_{1,j} dj = 0$.

An *open economy equilibrium* differs in that $\bar{q}$ is taken parametrically and the bond market clearing is not required.

Is the equilibrium optimal from a societal perspective? To answer this, we need a social planner problem. To this end, let $\hat{c}_{1,i}, \hat{c}_{2,i}$ denote the optimal consumption (in periods 1 and 2, respectively) of household $i \in [0,1]$, and let ϕ_i denote the moving cost shock realization the household receives. Let $y_{1,i}$ denote the endowment agent i will receive in the first period (which depends on their initial island placement). Similarly, let $y_{2,i}^s$ denote the output the household will receive if she decides to stay on her island and $\bar{y}_{2,i}^m$ the output in expectation associated with its moving. Taking migration decisions as given, the planner's objective function is

$$\max_{\hat{c}_{1,i} \geq 0, \hat{c}_{2,i} \geq 0} \int \alpha_i (u(\hat{c}_{1,i}) + \beta(u(\hat{c}_{2,i}) - m_i \phi_i)) di \tag{3}$$

where α_i is the Pareto weight on household i. We will consider two formulations of the resource constraint, an open economy resource constraint given by

$$\int \hat{c}_{1,i} di + \bar{q} \int \hat{c}_{2,i} di = \int y_{1,i} di + \bar{q} \int ((1 - m_i) y_{2,i}^s + m_i \bar{y}_{2,i}^m) di \tag{4}$$

and a closed economy resource constraint given by

$$\int \hat{c}_{1,i} di = \int y_{1,i} di \text{ and } \int \hat{c}_{2,i} di = \int ((1 - m_i) y_{2,i}^s + m_i \bar{y}_{2,i}^m) di. \tag{5}$$

If the planner can choose migration decisions, then $m_i \in [0,1]$ should be added as a choice variable and $\bar{y}_{2,i}^m$ will be identically equal to the maximum second-period endowment value.[6]

Definition 1. *An allocation is* constrained efficient *if it solves the planner problem with migration decisions given for some Pareto weights.*

For either resource constraint, optimality requires that marginal rates of substitution must be equated across individuals, i.e., $\beta u'(\hat{c}_{2,i})/u'(\hat{c}_{1,i}) = \beta u'(\hat{c}_{2,j})/u'(\hat{c}_{1,j})$ for almost all i, j. With the open economy constraint, it is easy to show these must also equal $\bar{q}$, i.e.,

$$u'(\hat{c}_{1,i})\bar{q} = \beta u'(\hat{c}_{2,i}) \tag{6}$$

for almost all i. In comparing (6) with the local government's Euler equation (2), it is clear that overborrowing will occur if the optimal bond choice b_2 is close to zero: In that case, the incentive to attract people—reflected in the term $1 - \frac{b_2}{n_2} \frac{\partial n_2}{\partial b_2}$—is close to zero, while the externality of new entrants shouldering the burden—reflected in $\frac{1-o_2}{1-o_2+i_2}$—is not. (On the other hand, if debt issuance

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[6]There are alternative ways to think of allowing migration here, such as a constrained efficient notion where migration decisions depend on J, but these are beside the point for our purposes.

is large, $b_2 \ll 0$, then attracting new entrants is of primary importance and there could be underborrowing.) Absent cross-sectional heterogeneity and with $\bar{q} = \beta u'(y_2)/u'(y_1)$, implementing the constrained efficient allocation requires $b_2 = 0$. In this case, the externality dominates and the efficient allocation cannot be implemented, which is formalized in Proposition 2:

Proposition 2. *Suppose there is no cross-sectional heterogeneity in y_1 and y_2. Then, if $\bar{q} = \beta u'(y_2)/u'(y_1)$, the open economy equilibrium is not constrained efficient. Moreover, at the constrained efficient allocation, governments would strictly prefer to borrow.*

Tiebout (1956) showed that, under certain assumptions, equilibria are efficient when local governments compete for workers. One of his key assumptions that is not met here is that of costless and fully directed mobility. In fact, the equilibrium can be Pareto efficient if migration is fully directed. To see why, consider trying to implement a Pareto optimal allocation with $b_2 = 0$. For the reasons described above, the Euler equation (2) would typically imply this is impossible. However, if inflow rates "punish" any debt accumulation by falling to zero in a nondifferentiable way, the Euler equation no longer characterizes the optimal choice and the equilibrium can be efficient. We prove this in Proposition 3.

Proposition 3. *Suppose there is no cross-sectional heterogeneity in y_1 and y_2. If migration is completely directed with (1) $\mathbf{I}(u(c_2)) = 0$ for $c_2 < y_2$, (2) the right-hand derivative of $\mathbf{I}(\cdot)$ at $u(y_2)$ infinite, and (3) $\mathbf{I}(\cdot)$ differentiable elsewhere, then a symmetric open economy equilibrium with $\bar{q} = \beta u'(y_2)/u'(y_1)$ and a closed economy equilibrium exist and they are Pareto optimal.*

Note that we do not need any special restrictions on the moving cost distribution as the overborrowing externality reflected in $\frac{1-o_2}{1-o_2+i_2}$ goes away if $i_2 = 0$. In general, the more elastic net migration is to bond holdings, i.e., the larger $\partial n_2/\partial b_2$ is, the less incentive the government has to overborrow.

With a closed economy, there is a different way that the economy can be efficient. In particular, if islands are homogeneous, then the desire for all the islands to overborrow can result in lower equilibrium bond prices / higher interest rates that exactly offset that desire. This result is stated in Proposition 4.

Proposition 4. *If there is no cross-sectional heterogeneity in endowments and initial populations, then any symmetric closed economy equilibrium is Pareto optimal and has $\bar{q} = \beta(1 - F(0))u'(y_2)/u'(y_1)$.*

Note the equilibrium $\bar{q}$ in Proposition 4 includes a $1 - F(0)$ term. Because $i_2 = o_2$ absent heterogeneity, this term decreases the equilibrium price (relative to an equilibrium without migration) in a way that exactly offsets the externality reflected in the Euler equation's $(1 - o_2)/(1 - o_2 + i_2) = 1 - F(0)$ in term. However, with an arbitrarily small amount of heterogeneity, a single price cannot perfectly offset this externality as Proposition 5 shows.[7]

Proposition 5. *Suppose there are two island types. If both types have the same first period endowments and population but different second period endowments, then the closed economy equilibrium is not constrained efficient.*

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[7]The reason two types are required is simply for ease in characterizing asset positions (as in that case $b_2 > 0$ for one island implies $b_2 < 0$ for the other). However, we suspect it holds far more generally.

3 Data and institutions

The theoretic model predicts a dynamic relationship between income, migration, and debt that results in overborrowing. We now investigate these relationships using data collected from a variety of sources described in Appendix A. The goal of this section is not to identify causal effects of, e.g., migration on borrowing or vice versa. Rather, our goal is to provide suggestive evidence of the relationships between migration, debt, and default while also establishing some stylized facts that will be useful for constructing and validating the full model.

3.1 Borrowing and migration

Table 1 reports the results of fixed effects regressions using U.S. county-level data on deficits per person and migration rates (we only have migration data at the county level, so we use county-level deficits to be consistent). The results coincide well with the implications of the two-period Euler equation (2) established in the previous section. Specifically, a regression of deficits on the overborrowing term $\frac{1-o}{1-o+i}$ show a statistically significant negative correlation between discount rates and deficits, consistent with the theory. For in-migration rates—which in large part determine the strength of the overborrowing externality—there is also a statistically significant positive correlation, consistent with the theory. The magnitude is such that going from a 0% to 10% in-migration rate would cause deficits to increase by almost $500 per person (all measures are in 2012 dollars). Moreover, this is overcoming any consumption smoothing effect due to productivity shocks: Positive productivity shocks should increase in-migration while decreasing borrowing, resulting in a negative correlation.

	(1) Deficit	(2) Deficit	(3) Deficit	(4) Deficit	(5) Deficit
Overborrowing term $(1-o)/(1-o+i)$	-4527.0*** (1314.0)				-1964.1 (1706.0)
In-migration rate		4564.4*** (1188.6)			
Out-migration rate			-2220.1* (1155.1)		
Net-migration rate				3284.6*** (818.7)	2503.2** (1063.4)
Observations	7682	7682	7682	7682	7682
R^2	0.018	0.019	0.016	0.019	0.019

Standard errors in parentheses

Constant and year dummies included in estimation

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1: Fixed-effects regressions of deficits per person on migration rates

The out-migration rate is negatively correlated with deficits. If causal, this would work against the theory. However, the quantitative model will also give a negative coefficient, and the reason is twofold. First, a negative productivity shock increases out-migration while simultaneously increasing borrowing (for consumption smoothing purposes), which generates positive comovement between out-migration rates and deficits. Second, the overborrowing externality, as reflected in $\frac{1-o}{1-o+i}$, is far less influenced by out-migration than in-migration, which should make it harder to detect. For example, since i is around 6% in the data, a 10 percentage point increase in the out-migration rate (say from 0 to 0.1) makes the overborrowing term go from around 0.943 to 0.938. In contrast, if o is 6% and i increases from 0 to 10%, the overborrowing term should go from 1 to 0.904. Higher net-migration rates also should cause increased borrowing since the effective discount factor can be written $(1-o)/(1+net)$ where net is the net-migration rate, and this is borne out in the fixed-effects regressions.

In the final specification, we include net-migration rates alongside the overborrowing term. This specification is designed to control for effects that may increase deficits not due to overborrowing but rather due to financing of large capital projects, for example. In this specification, the overborrowing term is still negatively correlated with deficits but not at standard confidence levels. While these regressions should not be interpreted causally, they are consistent with the model's theoretical predictions and indicate a connection between borrowing and migration. Later, we will use these regression results in validating our calibrated model.

3.2 Borrowing limits

One of the model's predictions is that a *supra-local* planner would generally like to restrict local government borrowing, and that this constraint would generally be binding.[8] In fact, many states do have statutory limits on how much cities can borrow. To show the variety both in sizes and types of limits, we report in Table 2 borrowing limits for nine states. The table reveals states have implemented a variety of rules. E.g., California (CA) limits are tied to spending or revenue that year. In contrast, most of the states restrict debt based on a percentage of property valuations, but the percentages can differ substantially from as little as one-third of 0.5% (IL) to 10% (NY). Almost all the states have known exceptions, and these usually include debt related to education and or water supplies or voter-approved debt. Qualitatively, the rules in CA and the spending-based exceptions in other states could produce an incentive to have big budgets in order to borrow more, and the quantitative model will allow for this feature of the law.

Are these constraints binding? Here we are limited because our main dataset does not have data on taxable property valuations, and many states use this information in their borrowing limit formulation. However, California uses expenditures to constrain debt, and so we can tell how close cities are to their limit as displayed in the top panel of Figure 1. The graph reveals that many cities—

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[8]While we did not state this as a separate proposition, this is the case in the environment of Proposition 2. There, the constrained efficient allocation cannot be supported because, at $b_2 = 0$, the local government would strictly prefer to borrow. If borrowing were forbidden, then the allocation would be supported as an equilibrium.

State	Limit	Known exceptions
CA	Indebtedness less than revenue that year	Authorization by referendum, special projects, and public school spending.
IL	Limits range from 0.5%-3% of assessed value (1/3 of market value)	Schools have debt limits of 13.8% value of taxable property.
IN	2% of assessed value (usually 100% of market value)	Some revenue bonds.
MA	5% of taxable property valuation last year	
MI	10% (5% for townships and school districts) of assessed value (usually 50% of market)	Approval of voters for more school district debt; revenue bonds (see definition in notes) generally excepted.
MN	"Net debt" less than 3% of market value of taxable property	Charter can authorize higher rates up to 3.67%, "first class" cities have a 2% limit.
NY	Roughly 10% of the property valuation over the previous five years	Debt related to water supplies and sewers.
OH	Net indebtedness less than 5.5% (or 10.5% with vote) of tax valuation	Self-supporting projects for water facilities, airports, public attractions, et al.
WI	5% of taxable property value	School's have a 10% debt limit, may issue $1 million without voter approval.

Sources are as follows: CA's is Harris (2002); MA's is MCTA (2009); MN's is Bubul (2017); IL, IN, MI, and WI's is Faulk and Killian (2017); NY's is ONYSC (2018); OH's is (OMAC, 2013, p.50). Revenue bonds are municipal bonds that are paid by revenue from a particular project (they are non-GO bonds).

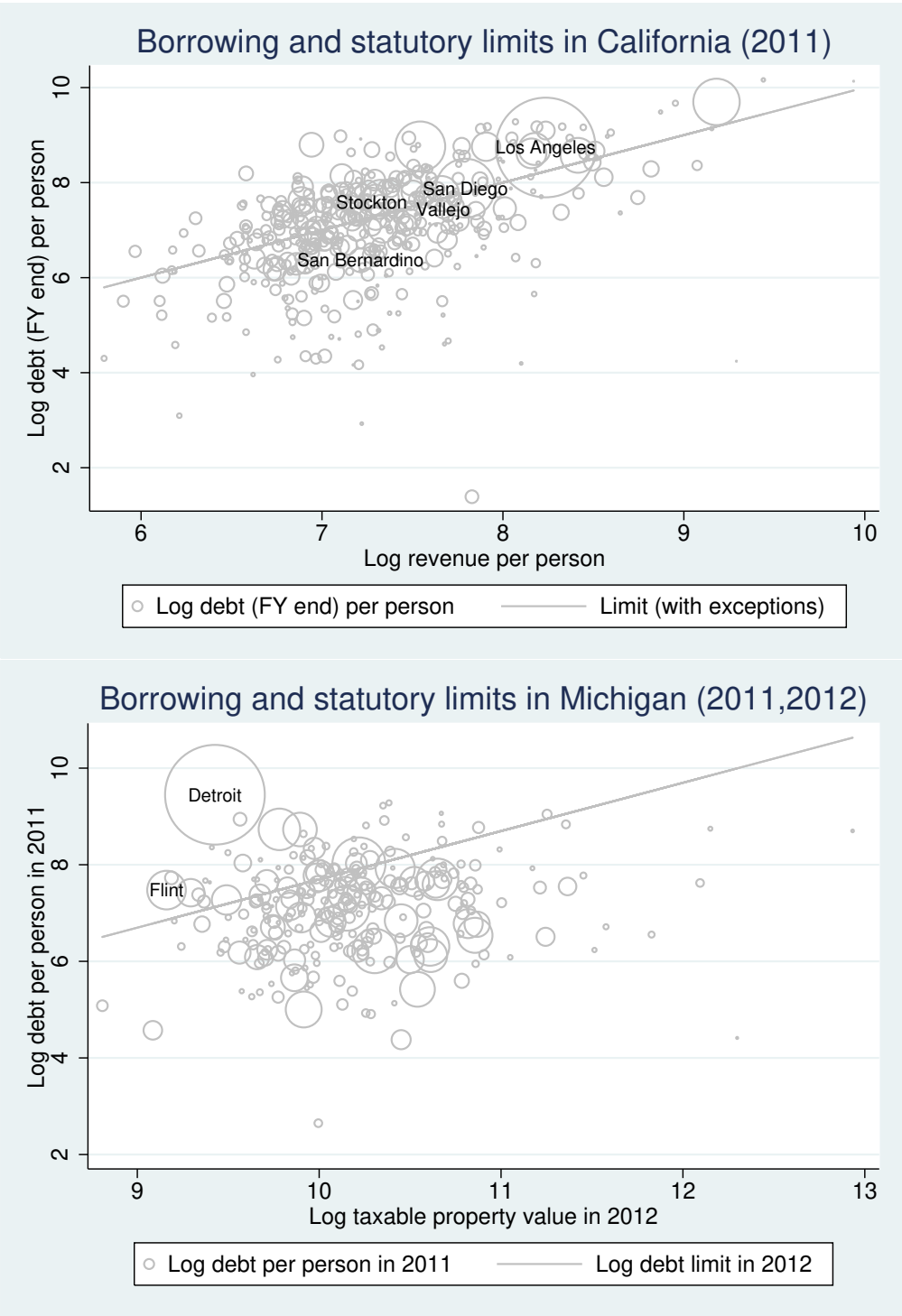Table 2: Sample of statutory borrowing limits by state

Figure 1: Statutory borrowing limits and closeness to the limits

including very large ones—are borrowing beyond the revenue per person limit (which could reflect spending on special projects or borrowing authorized by referendum). Additionally, we were able to find taxable valuation data for Michigan (from Kleine and Schulz, 2017), and the bottom panel of Figure 1 displays how close MI cities are to their limits. Again, many cities are at, near, or above the limit, including Detroit and Flint. Even the wealthiest cities (as measured by property values) are borrowing. In summary, it seems cities regularly borrow, and many of them borrow as much as they legally can. This evidence suggests the borrowing limits are binding, consistent with the model prediction that cities are overborrowing and that states are optimally restricting them. Additionally, the quantitative model—where we know overborrowing does occur and can tell explicitly whether the limits are binding—will produce similar patterns and generate a positive (though small) welfare gain from having borrowing limits.
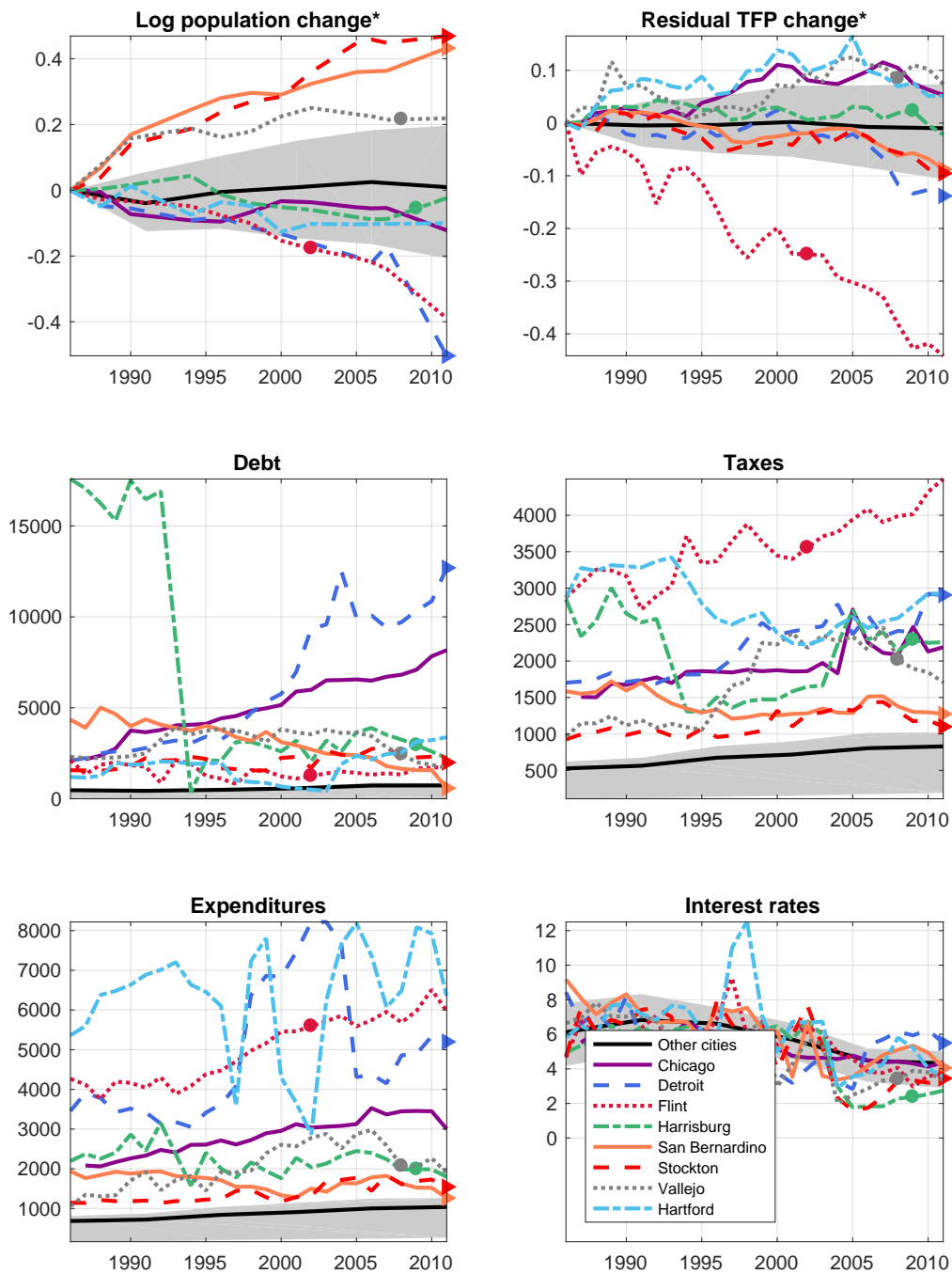
3.3   A case study of default and financial distress

To uncover some stylized facts on municipal defaults and financial distress, we turn to a case study of local governments. Our sample is Detroit (MI), Flint (MI), Harrisburg (PA), San Bernardino (CA), Stockton (CA), Vallejo (CA), Chicago (IL), and Hartford (CT), cities that have defaulted or been reported as having financial difficulties in the last few years. (News coverage on these and other cities is listed in Appendix A.)

As emphasized in the introduction, the population growth in the top left panel of Figure 2 data reveal heterogeneous paths to default or, more generally, fiscal stress. San Bernardino, Stockton, and Vallejo all experience unusually large population *growth* leading up to default. Detroit and Flint, and to a lesser extent Chicago, Hartford, and Harrisburg, experience the opposite. Similarly, the top right panel reveals that cities encounter fiscal stress during periods of adverse productivity shocks (Detroit, Flint, Stockton, San Bernardino) or after unusually large productivity gains (Chicago, Vallejo, Hartford). From the lens of standard default models, the latter observation is surprising; but we will show our model can generate defaults after productivity and population booms (as well as busts).

The middle left panel in Figure 2 displays the dynamics of debt in 2012 dollars per person since the mid-1980s. One can see that financially struggling cities tend to have debt far above average. For example, while the average city owed less than $1,000 in 2011, Chicago and Detroit owed about $8,000 and $12,000, respectively. In some cases, financial maneuvering has been used to underplay the amount of debt: Harrisburg's massive debt in the early 1990s plummeted due to a sale of its incinerator project to a separate government entity (Murphy, 2013).[9] While the average debt position of all cities looks flat because it is so much smaller than the case study cities' average, it has

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[9]In particular, it was sold to the Harrisburg Authority, a municipal authority with the power to issue debt (Murphy, 2013, p.4). The sale occurred in 1993, but Harrisburg "continued to operate the facility" and has guaranteed debt issuance of the authority totaling at least $299 million (Murphy, 2013, p.5). These guarantees do not show up as debt in our data at the city level. Faulk and Killian (2017) show empirically that having more special districts (and the Harrisburg Authority is classified as one of these) is positively correlated with increased local government debt in four of the five states they consider, which suggests this type of behavior is not unique.

Log population change*

Residual TFP change*

Debt

Taxes

Expenditures

Interest rates

Other cities
Chicago
Detroit
Flint
Harrisburg
San Bernardino
Stockton
Vallejo
Hartford

Note: Changes are log differences relative to 1986 except for Chicago, which is relative to 1987; circles denote periods of acute fiscal stress such as defaults, bankruptcies, or emergency manager takeovers (the last only for Flint); triangles denote acute fiscal stress periods that occur after 2011; "other cities" is not the universe of cities but covers 64% to 74% of the U.S. population over the time range; fiscal variables are in 2010 dollars per person; the interquartile range is given by the shaded area.

Figure 2: Case Study – Cities under Financial Stress

increased by 56% since the 1980s, going from $469 per person to $732.

Regarding inlays and outlays (the middle right panel and lower left panel, respectively), we observe that defaulters' expenditures tend to outstrip their tax revenues, but not always. Furthermore, defaulters have large expenditures per person. In contrast, typical cities seem to run close to balanced budgets, which is consistent with the comparatively low average debt per person. Hartford's large tax revenue shortfall (with expenditures often around $6,000 and with tax revenue closer to $2,500) has been offset with large cash infusions by the state (Rojas and Walsh, 2017). While Connecticut's support for Hartford has been exceptional, we will analyze the consequences of this bailout-like behavior in Section 6.[10]

The data reveal a secular decline in interest rates over the past decades and show the financial crisis pushed up the borrowing costs of defaulters in our sample.[11] (The data for "other cities" is plotted only in years ending in two or seven, which is when the coverage is almost universal. Consequently, the high frequency variation is missed; see Appendix A.2 for details.) This run up in interest rates likely contributed to the wave of defaults at the end of our sample. Furthermore, the recent increase in the federal fund rates raises the question of whether increased borrowing costs will have significant deleterious effects on city finances. We will also investigate this in Section 6.

In summary, our case studies uncover the existence of multiple paths to default. These paths are quite heterogeneous with defaults happening during booms and busts. As we will show, the model we will propose is rich enough to capture these heterogeneous default episodes.

3.4 Default rates, trends, and institutional details

We close this section by discussing overall default rates and trends, as well as few relevant institutional details. According to Moody's (2012), there have been 73 municipal bond defaults (of bonds rated by Moody's) since 1970. This figure translates into a default rate of roughly one municipal default every 4.3 years. These very low default rates imply that municipal bonds generally carry low interest rates, as was seen in Figure 2.

However, this low default rate belies the precarious nature of city finances. For instance, as happened in Flint, cities can come under state management and thereby lose control of their finances. In fact, Kleine and Schulz (2017) report that Michigan in and just before 2017 had 11 cities (4%), one township, and one county under state oversight due to a financial emergency (p. 9).

Additionally, the last decade has seen a substantial increase in default rates. Figure 3 shows the default rates for speculative-grade municipal bonds over the past 40 years, and these rates have doubled since the onset of the Great Recession (going from 1% during 1991-2007 to 2% during 2008-2012). These results show that while overall default rates are low, fiscal stress is substantial and potentially growing.

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[10]State support of local governments is generally on the order of $100-$300 per person, an order of magnitude smaller than Connecticut's support for Hartford. E.g., median and mean non-tax revenue was $129 and $256, respectively, in 2011.

[11]Instrumental-variable-based evidence for the link between municipal default risk and interest rates can be found in Capeci (1994).
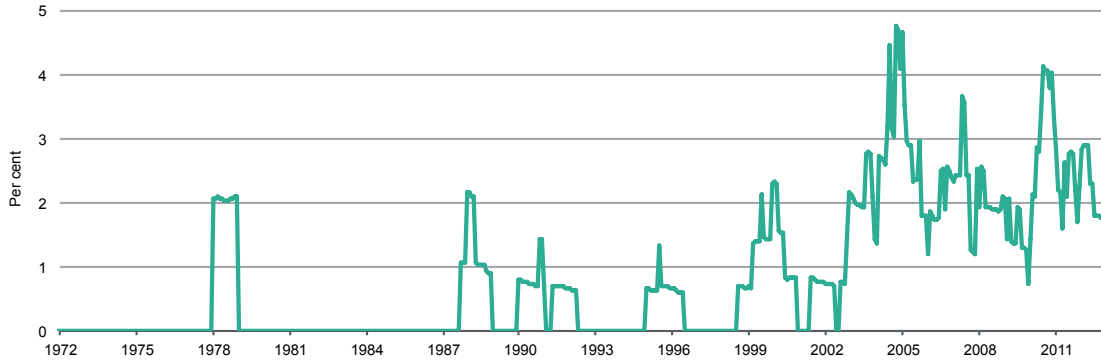
Figure 3: Default Rates on Speculative Grade Bonds (Source: Moody's)

Before moving to the quantitative model, we would comment on one institutional aspect that will impact modeling decisions and the calibration. Specifically, municipal bankruptcy (i.e., Chapter 9 bankruptcy) allows municipalities to discharge their debt while keeping essentially all their assets (in contrast to consumer and firm bankruptcy under Chapter 7).[12] Because of this, we will focus on gross debt (not debt net of assets) in the calibration and treat bankruptcy as a complete discharge of debt.

4 The quantitative model

We first provide an overview of the model and its timing. Then, we describe the household, firm, and government problems. Finally, we define equilibrium.

4.1 Overview and timing

We model municipalities in the U.S. as a unit measure of islands. Each island consists of a continuum of households (whose measure in the aggregate is one), a local government, and a neoclassical firm. The government is a sovereign entity that issues debt, taxes its residents, and provides government services. Households consume, work, and crucially decide whether to stay on the island or migrate to another one. Finally, there is a financial intermediary who buys portfolios of municipal debt as well as a risk-free bond.

The timing of the model is as follows. At the beginning of the period, all shocks are realized. Upon observing them, households make migration decisions. After migration occurs, the govern-

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[12]The legal reason for this difference is state sovereignty over local affairs: Chapter 9 " is significantly different in that there is no provision in the law for liquidation of the assets of the municipality and distribution of the proceeds to creditors. Such a liquidation or dissolution would undoubtedly violate the Tenth Amendment to the Constitution and the reservation to the states of sovereignty over their internal affairs" (United States Courts, 2018). This was seen in the courts' rejection of creditor demands that Detroit sell part of its art museum collection (US Bankruptcy Court, City of Detroit Case No. 13-53846).

ment chooses its policies, including debt issuance.[13] Finally, households make consumption and labor decisions simultaneously with firms while taking prices and government policies as given.

4.2 Households

Define the state vector of a generic island as $\mathbf{x} := (b, n, z, f, \omega)$, where b is assets per person measured before migration, n is the population before migration, z is the island's productivity, $f \in \{0, 1\}$ indicates whether the government is in a state of default, and ω is a fixed island type we will think of as weather. Including "weather" allows the model to match the variance of population across cities without producing a counterfactually large correlation between productivity and in-migration. We assume z follows a finite-state Markov chain.

Households, knowing $\mathbf{x}$, decide whether to stay $m = 0$ or move $m = 1$. If they stay, they expect to receive lifetime utility $S(\mathbf{x})$ (specified below). If they move, they are assigned to another island, receive J in expected lifetime utility, and pay an idiosyncratic utility cost $\phi \sim F(\phi|z)$. The dependence of ϕ on z allows us to capture, in a reduced form way, the notion that high-income workers are more mobile than low-income ones; however, the estimated dependence of F on z turns out to be very small. Their problem is

$$V(\phi, \mathbf{x}) = \max_{m \in \{0,1\}} (1 - m)S(\mathbf{x}) + m(J - \phi). \tag{7}$$

The moving decision follows a reservation strategy $R(\mathbf{x})$ with $m = 1$ when $\phi < R(\mathbf{x})$.

The utility conditional on staying is

$$S(\mathbf{x}) = \max_{c \geq 0, h \geq 0} u(c, g(\mathbf{x}), h, \omega) + \beta \mathbb{E}_{\phi', \mathbf{x}'|\mathbf{x}} V(\phi', \mathbf{x}')$$

$$\text{s.t. } c + r(\mathbf{x})h = w(\mathbf{x}) + \pi(\mathbf{x}) - T(\mathbf{x}) \tag{8}$$

where $w(\mathbf{x})$ is the island's wage; $\pi(\mathbf{x})$ is the per person profit from the island's firm; $g(\mathbf{x})$ is government services; $T(\mathbf{x})$ are lump-sum taxes (which we will show is virtually equivalent to using property taxes); and h is a housing good, owned by the firm and rented to households at price $r(\mathbf{x})$. The expectation term $\mathbb{E}_{\phi', \mathbf{x}'|\mathbf{x}}$ embeds household beliefs about the local government's policies. We assume u is continuously differentiable, strictly concave, strictly increasing, and satisfies the Inada conditions in its first three arguments.

If a household decides to move, they migrate to island $\mathbf{x}$ at rate $i(\mathbf{x})$ and must stay there for at least one period. We assume that this rate is given by

$$i(\mathbf{x}) = \left(\int n F(R(\mathbf{x})|z) d\mu(\mathbf{x}) \right) \frac{\exp(\lambda S(\mathbf{x}))}{\int \exp(\lambda S(\mathbf{x})) d\mu(\mathbf{x})} \tag{9}$$

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[13]This timing means that unanticipated changes in government policies do not immediately alter the population. We view this "sticky population" assumption as reasonable in that migration is a time-consuming process that often involves searching for a new job, finishing a school year, selling an existing home, and finishing rental agreements.

where μ is the invariant distribution of islands.[14] By construction, the measure of households leaving equals the measure entering in aggregate, $\int i(\mathbf{x})d\mu(x) = \int nF(R(\mathbf{x}))d\mu(\mathbf{x})$. If $\lambda = 0$, households are uniformly assigned to each island ("random search"). As $\lambda \to \infty$, the city with the largest $S(\mathbf{x})$ receives all the inflows ("directed search"). Note that these inflows are what would arise from using Type 1 extreme value shocks (as in Kennan and Walker, 2011, and many others) were there a finite number of islands.[15] Given these inflows, the expected value of moving in equilibrium is

$$J = \int S(\mathbf{x})\frac{i(\mathbf{x})}{\int i(\mathbf{x})d\mu(\mathbf{x})}d\mu(\mathbf{x}) \tag{10}$$

and the law of motion for population is

$$\dot{n}(\mathbf{x}) = n(1 - F(R(\mathbf{x})|z)) + i(\mathbf{x}), \tag{11}$$

where $\dot{n}$ denotes the population after migration has taken place.

4.3   Firms

Each island has a firm that operates a linear production technology zL and owns the island's housing stock $\bar{H}$. Alternatively, $\bar{H}$ may be thought of as the island's land. We assume $\bar{H}$ is in fixed supply and homogeneous across islands to prevent adding an extra state variable, but our inclusion of weather ω will capture some of this fixed heterogeneity across islands. Firms solve

$$\dot{n}(\mathbf{x})\pi(\mathbf{x}) = \max_{L,H \leq \bar{H}}(1 - \kappa(\mathbf{x}))zL - w(\mathbf{x})L + r(\mathbf{x})H. \tag{12}$$

taking w and r competitively, and the solution of this problem gives labor demand, L^d (and the housing supply, $H = \bar{H}$). The term $\kappa(\mathbf{x}) = \bar{\kappa}\max\{d(\mathbf{x}), f\}$ is a pecuniary cost of default. Since $\dot{n}(\mathbf{x})$ denotes the number of households remaining after migration and each household inelastically supplies one unit of labor, labor market clearing requires

$$L^d(\mathbf{x}) = \dot{n}(\mathbf{x}). \tag{13}$$

It is worth making a few observations about the firm problem. First, in equilibrium, per person profits π equal $r\bar{H}/\dot{n}$. Consequently, by making local residents the firm shareholders, we are effectively assuming each gets the rent associated with owning an equal share of the housing/land stock. Second, if there were property taxes, say via $\tau r(\mathbf{x})\bar{H}$ for $\tau \in [0, 1]$, the taxes would reduce these rents by $\tau r\bar{H}/\dot{n}$ in the same way that the lump-sum tax T in (8) does. For this reason, we loosely interpret the lump-sum tax T as a property tax. Last, we have assumed there are no agglomeration or congestion effects in the production function (or that they are both present and

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[14]This rule has the same form as in the two-period model. In particular, one can take $\mathbf{I}(S(\mathbf{x})) = \exp(\lambda S(\mathbf{x}))$.

[15]The usual specification would be written $\max_{\mathbf{x}} S(\mathbf{x}) + \varepsilon_{\mathbf{x}}/\lambda$ where each $\varepsilon_{\mathbf{x}}$ is i.i.d. with a Type 1 extreme value distribution. Then the probability of choosing $\mathbf{x}$ is proportional to $\exp(\lambda S(\mathbf{x}))$, as it is in our formulation. However, a continuum of $\mathbf{x}$ choices makes $\mathbb{E}[\max_{\mathbf{x}} S(\mathbf{x}) + \varepsilon_{\mathbf{x}}/\lambda]$ infinite, and so it is difficult to micro-found our inflow assumption.

cancel).[16] Their absence could result in the model under- or over-predicting the relationship between population and productivity. However, the model generates a signficant positive correlation between city density and productivity like that found in the data (Glaeser, 2010). Also, the model has congestion externalities in the form of reduced housing per person and agglomeration effects in that local governments provide a partly non-rival service, as will be discussed shortly.

4.4 Local governments

Each local government decides the level of services $g \geq 0$ it wishes to provide. These services are potentially non-rival in that, to provide g services to each of the $\dot{n}$ households, the government must only invest $\dot{n}^{1-\eta}g$ units of the consumption good where $\eta \in [0, 1]$ is a parameter. The government pays for these services using tax revenue $T\dot{n}$ or, potentially, debt issuance. The default flag f (which is a component of x) indicates the city's standing with creditors. If $f = 0$, then the city is in good standing with its creditors and can borrow and default. If $f = 1$, it is in bad standing and cannot borrow (and has no debt). In the case of default or bad standing, firm output drops by $\bar{\kappa}$ and the government returns to good standing $f' = 0$ and no debt $b' = 0$ with probability χ. A government with $f = 0$ that repays its debt $-bn$ chooses a new level of debt per person $-b'$, implying a total obligation next period of $-b'\dot{n}$. The discount price it receives on this pledge is $q(b', \dot{n}, z, \omega)$, which depends on the debt level, population after migration $\dot{n}$ (which equals the next period's population before migration n'), productivity, and weather, as all of these potentially influence repayment rates.[17]

In keeping with the statutory borrowing limits discussed in Section 3, we impose a borrowing limit $b' \geq \underline{b}(z, \dot{n}, g)$. For the quantitative work, we further assume that

$$- b' \leq g\dot{n}^{-\eta}\delta \tag{14}$$

where $\delta \in \mathbb{R}^+$ controls how tight the limit is. Hence, we require total debt issued in a period $-b'\dot{n}$ to be less than a fraction δ of total expenditures $g\dot{n}^{1-\eta}$. Note that this limit is qualitatively closer to the standard limits in CA than the other states. However, the exemptions in many states allow for spending on projects, which this form permits. Given the large variation in laws across states, we will choose δ to match observed debt levels rather than trying to choose it based on statutory law. However, the estimated value will be not far from California's statutory limit of $\delta = 1$.

To define the government's problem, we need to specify how the economy will respond to deviations in government policies. To this end, we assume r and w adjust dynamically in response to the government policies (d, g, b', and T) clearing the labor and housing markets and that households and firms optimize given those prices and implied profits. Formally, we assume that c, h, r, w, π, L^d

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[16]A simple way to introduce agglomeration is with the modified production function $zL\dot{n}^{\varpi}$, where N is population and $\varpi > 1$. Duranton and Puga (2004) provide micro-foundations for this type of agglomeration.

[17]Capeci (1994) provides empirical evidence on the link between municipal default risk and interest rates. This bond pricing framework was first introduced in Eaton and Gersovitz (1981). Our use of short-term debt significantly simplifies the computation as long-term debt models suffer from convergence problems (Chatterjee and Eyigungor, 2012).

always solve the following equations

$$\underbrace{\begin{array}{c} u_c r = u_h \\ c + rh = w + \pi - T \end{array}}_{\text{Household optimization}}, \quad \underbrace{\begin{array}{c} w = (1 - \bar{\kappa}\hat{d})z \\ \dot{n}\pi = (1 - \bar{\kappa}\hat{d})zL^d - wL^d + r\overline{H} \end{array}}_{\text{Firm optimization}}, \quad \text{and} \quad \underbrace{\begin{array}{c} \dot{n} = L^d, \\ \dot{n}h = \overline{H} \end{array}}_{\text{Market clearing}}, \quad (15)$$

where $\hat{d} := \max\{d, f\}$. Letting U denote the indirect flow utility associated with g, T, and d, it is easy to show

$$U(g, T, \hat{d}, \dot{n}, z, \omega) = u((1 - \bar{\kappa}\hat{d})z - T, g, \overline{H}/\dot{n}, \omega). \quad (16)$$

Now we can state the government's problem as

$$\tilde{S}(\mathbf{x}) = \begin{cases} \max_{d \in \{0,1\}} (1 - d)S^N(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) + dS^D(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) & \text{if } f = 0 \\ S^D(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) & \text{if } f = 1 \end{cases} \quad (17)$$

where $\dot{b}(\mathbf{x}) := bn/\dot{n}(\mathbf{x})$ gives assets per person after migration and

$$\tilde{S}^N(\dot{b}, \dot{n}, z, \omega) = \max_{g \geq 0, T \leq z, b' \in \mathbf{B}} U(g, T, 0, \dot{n}, z, \omega) + \beta \mathbb{E}_{\phi', z'|z} \tilde{V}(\phi', b', \dot{n}, z', 0, \omega)$$

$$\text{s.t. } g\dot{n}^{1-\eta} + q(b', \dot{n}, z, \omega)b'\dot{n} = T\dot{n} + \dot{b}\dot{n} \quad (18)$$

$$b' \geq \underline{b}(z, \dot{n}, g)$$

and

$$\tilde{S}^D(\dot{b}, \dot{n}, z, \omega) = \max_{g \geq 0, T \leq (1-\bar{\kappa})z} U(g, T, 1, \dot{n}, z, \omega) + \beta \mathbb{E}_{\phi', z'|z} \left(\begin{array}{c} \chi \tilde{V}(\phi', 0, \dot{n}, z', 0, \omega) \\ +(1 - \chi)\tilde{V}(\phi', \dot{b}, \dot{n}, z', 1, \omega) \end{array} \right)$$

$$\text{s.t. } g\dot{n}^{1-\eta} = T\dot{n} \quad (19)$$

with $\tilde{V}(\phi, \mathbf{x}) = \max\{\tilde{S}(\mathbf{x}), J - \phi\}$. (Note that in (18) we used the identity $bn = \dot{b}\dot{n}$.) For the quantitative work and most of the theoretical results, we restrict the bond choice b' to be in a finite set $\mathbf{B}$ that includes 0. For the derivation of the Euler equation, however, we treat $\mathbf{B}$ as an interval $[\underline{b}, \infty)$ with $\underline{b} < 0$.

4.5 Financial intermediaries

Following Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), we posit a competitive financial intermediary who purchases measures of contracts. To write the intermediary's problem, we think of governments and the intermediary as choosing contracts indexed by (b', n', z, ω)—note $n' = \dot{n}$. From the government's perspective, a contract costs $Q(b', n', z, \omega) := q(b', n', z, \omega)b'n'$ in the current period, yields $b'n'$ if the city does not default, and yields zero otherwise. Because the intermediary purchases a measure of contracts, a law of large numbers gives a certain yield $\sum_{z'} \mathbb{P}(z'|z)(1 - d(b', n', z', \omega))b'n'$. The intermediary behaves competitively, taking Q and d as given.

The intermediary chooses a measure M' over contracts to maximize the net present value of dividends D discounted at rate $\bar{q}$. His problem is

$$W(B, M) = \max_{D, B', M'} D + \bar{q} W(B', M')$$

$$\text{s.t. } D + \bar{q} B' + \int Q(b', n', z, \omega) dM'(b', n', z, \omega) = B + \int \sum_z \mathbb{P}(z|z_{-1})(1 - d(b, n, z, \omega))(bn) dM(b, n, z_{-1}, \omega),$$
(20)

with a no-Ponzi condition.

In equilibrium, we require that (1) the intermediary issues zero dividends each period $D = 0$; (2) the intermediary has zero wealth $W = 0$; (3) contract markets clear; and (4) the risk-free bond market clears. We assume the risk-free bond is in net supply $\overline{B}$, and so bond market clearing requires $B' = B = \overline{B}$. We further assume each contract is in zero net supply, which means the intermediary must be the counterparty to every contract purchased by cities. Formally, contract markets clear if

$$M'(\mathcal{B}, \mathcal{N}, z, \omega) = -\int \mathbf{1}_{[b'(b, n, z, 0, \omega) \in \mathcal{B}, \dot{n}(b, n, z, 0, \omega) \in \mathcal{N}]}(1 - d(b, n, z, \omega)) \mu(db, dn, z, 0, \omega)$$
(21)

for all z, ω and all $\mathcal{B} \times \mathcal{N}$ in a product Borel σ-algebra. Our assumption that the economy is closed is particularly appropriate for municipal bonds, as approximately 97% of them are held domestically (Cohen and Eappen, 2015). An attractive implication of the zero-dividend condition is that we do not need to specify who owns the intermediary.

4.6 Equilibrium

A steady-state recursive competitive equilibrium is value functions $S, V, \tilde{S}, \tilde{V}$; an expected value of moving J; household policies c, h, m; government policies g, T, b', d; prices and profit $\bar{q}, q, w, r, \pi$; labor demand L^d; the intermediary policies and value function D, M', B', W; an intermediary state M; a law of motion for population $\dot{n}$; and a distribution of islands μ, such that (1) household policies c, h and migration decisions are optimal taking V, S, J, prices and government policies as given; (2) government policies g, T, b', d are optimal taking $\tilde{V}, \tilde{S}, J$, the population law of motion $\dot{n}(\mathbf{x})$, and prices q as given; (3) firms optimally choose $L^d(\mathbf{x})$ taking $w(\mathbf{x}), r(\mathbf{x})$ as given and optimal per person profits are $\pi(\mathbf{x})$; (4) the intermediary policies D, M', B' are optimal given $W, \bar{q}, q$, and d; (5) beliefs are consistent: $S(\mathbf{x}) = \tilde{S}(\mathbf{x})$ and $V(\phi, \mathbf{x}) = \tilde{V}(\phi, \mathbf{x})$; (6) the distribution of islands μ is invariant; (7) the intermediary's portfolio is time-invariant, $M = M'(\overline{B}, M)$; (8) J and $\dot{n}$ are consistent with μ and household and government policies; (9) the intermediary makes zero profits, $D(\overline{B}, M) = 0$ and $W(\overline{B}, M) = 0$; and (10) markets clear.

4.7 Centralization and the Euler equation

To characterize equilibrium, we first simplify the equilibrium conditions by providing sufficient conditions for the intermediary's problem, bond-market clearing, and contract market-clearing to

be satisfied. We then show how the government and household problems may be centralized into a single problem. Last, we derive the Euler equation.

Proposition 6. *If prices satisfy*

$$q(b', n', z, \omega) = \bar{q}\mathbb{E}_{z'|z}(1 - d(b', n', z', \omega)). \tag{22}$$

and if

$$\overline{B} = \int (1 - d(b, n, z, \omega))bn\mu(db, dn, dz, 0, d\omega) \tag{23}$$

then there exist prices and an optimal policy M', with M' invariant, such that contract markets and the risk-free bond market clear and zero profits obtain (provided the other equilibrium conditions are met).

Note that if there is no default, this simply says $\overline{B} = \int bn d\mu$, which means in equilibrium the intermediary holds a portfolio of city assets / debt ($\int bn d\mu$) that corresponds to aggregate debt demand ($-\overline{B}$).

Proposition 7 shows the government, household, and firm problem may be centralized into a single problem, which we use as the basis for computation.

Proposition 7. *Suppose $\hat{S}$ satisfies*

$$\hat{S}(\mathbf{x}) = \begin{cases} \max_{d \in \{0,1\}}(1-d)\hat{S}^N(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) + d\hat{S}^D(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) & \text{if } f = 0 \\ \hat{S}^D(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) & \text{if } f = 1 \end{cases} \tag{24}$$

where

$$\hat{S}^N(\dot{b}, \dot{n}, z, \omega) = \max_{c>0, g \geq 0, b' \in \mathbf{B}} u(c, g, \overline{H}/\dot{n}, \omega) + \beta \mathbb{E}_{\phi', z'|z} \max\{\hat{S}(b', \dot{n}, z', 0, \omega), J - \phi'\}$$
$$\text{s.t. } \dot{n}c + \dot{n}^{1-\eta}g + q(b', \dot{n}, z, \omega)b'\dot{n} = z\dot{n} + \dot{b}\dot{n} \tag{25}$$
$$b' \geq \underline{b}(z, \dot{n}, g)$$

and

$$\hat{S}^D(\dot{b}, \dot{n}, z, \omega) = \max_{c>0, g \geq 0} u(c, g, \overline{H}/\dot{n}, \omega) + \beta \mathbb{E}_{\phi', z'|z} \left(\begin{array}{c} \chi \max\{\hat{S}(0, \dot{n}, z', 0, \omega), J - \phi'\} \\ +(1-\chi)\max\{\hat{S}(\dot{b}, \dot{n}, z', 1, \omega), J - \phi'\} \end{array} \right) \tag{26}$$
$$\text{s.t. } \dot{n}c + \dot{n}^{1-\eta}g = (1 - \overline{\kappa})z\dot{n}$$

with associated optimal policies $c(\mathbf{x}), g(\mathbf{x}), d(\mathbf{x}), b'(\mathbf{x})$ (with d and b' arbitrary for $f = 1$). Then (1) $\hat{S}$ is a solution to the household problem and c, l are optimal policies; (2) $\hat{S}$ is a solution to the government problem and g, d, b' are optimal policies; and (3) there exists prices r, w such that labor and housing markets clear and firms optimize.

In what follows, we will use S, S^N, S^D in place of $\hat{S}, \hat{S}^N, \hat{S}^D$, respectively.

Proposition 8 characterizes the government Euler equation:

Proposition 8. *Consider the S^N problem in (25) at some state $(\dot{b}, n', z)$. Suppose that at the optimal choices, the borrowing constraint is not binding and that locally about b' repaying is strictly preferred. If in addition $S^N_{\dot{b}}(b' \frac{n'}{n''}, n'', z', \omega)$, $S^N_{\dot{n}}(b' \frac{n'}{n''}, n'', z', \omega)$, and $\dot{n}_b(b', n', z', 0, \omega)$ exist locally for $n'' := \dot{n}(b', n', z', 0, \omega)$ about b', then the Euler equation satisfies*

$$u_c \bar{q} = \beta \mathbb{E}_{z'|z} \left[u_c \left(\frac{1-o'}{1+i'-o'} \right) \left(1 - \frac{b'}{n''} \dot{n}_b \right) + (1-o') S^N_{\dot{n}} \left(b' \frac{n'}{n''}, n'', z', \omega \right) \dot{n}_b \right] \qquad (27)$$

where o' is the outflow rate $F(R(b', n', z', 0)|z')$ and $i' = i(b', n', z', 0)/n'$ is the inflow rate next period.

Relative to the two-period model Euler equation in (2), there is an additional term connected to $S^N_{\dot{n}}$. In the two-period model, the *level* of population only effects utility through its effect on debt per person. Here, there are additional effects through reduced housing per person ($\overline{H}/\dot{n}$) and partially non-rival government services (if $\eta > 0$).

One result we would like to have is a proof of equilibrium uniqueness. Unfortunately, we have not been able to show uniqueness even in the general version of the two-period model. This is potentially worrisome in that, given the link between migration and borrowing we demonstrated, expectations of how many people are moving might feed into debt accumulation decisions and justify those expectations. However, we investigate uniqueness quantitatively by using 100 randomly drawn initial guesses for the equilibrium objects. Each guess converged to the same equilibrium values, and so at least computationally there is no evidence of indeterminacy at the calibrated values. See Appendix C.6 for more details.

5 Calibration and estimation

In this section, we show that the model can reproduce a large number of key empirical moments, targeted and untargeted. The data, including definitions, construction, and cleaning of key variables, are described in Appendix A. We take a model period to be a year.

5.1 Productivity

As productivity (TFP) plays a vital role in the model, it is necessary to have a process that accurately captures location-specific productivity dynamics. To this end, we begin by constructing a TFP series in the data using the County Business Patterns (CBP), which is an annual panel dataset published by the Census covering the universe of counties (not cities, unfortunately) dating back to 1986.[18] For our TFP measure, we use real annual payrolls per employee.

Let the log of TFP for a county-year pair be denoted z_{it}. We specify

$$z_{it} = \varsigma_i + \varpi_t + \tilde{z}_{it} \qquad (28)$$

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[18]In fact, it goes back as far as 1946 but the data are not easily accessible. The sample is available at https://www.census.gov/programs-surveys/cbp/about.html.

and obtain the residual $\tilde{z}_{it}$ using a fixed-effects regression. To discretize the fixed effects ς_i, we non-parametrically break the estimates into bins corresponding to to 0-10%, 10-50%, 50-90%, 90-99%, and 99-100%. The estimated fixed effects averaged within these bins are -0.34, -0.13, 0.09, 0.37, and 0.65, respectively. We discard the time effects ϖ_t as we will only consider steady states.

For the residual TFP $\tilde{z}_{it}$, we use an AR(2) specification, which allows more persistent movements in TFP that better capture decade-long persistent movements in productivity such as what occurred in Detroit and Flint (see Figure 2). Restricting the sample to cities of at least one million residents, the estimated first and second AR coefficients are 0.73 (0.02) and 0.23 (0.03), respectively, with an innovation variance 0.001 (2×10^{-5}).[19] We describe our discretization process in the appendix.

5.2 Preferences and moving costs

We set $\beta = 0.96$ and assume the flow utility exhibits constant relative risk aversion over a Cobb-Douglas aggregate of consumption, government services, and housing plus a taste shifter for weather:

$$u(c, g, h, \omega) = \frac{(c^{1-\zeta_g-\zeta_h} g^{\zeta_g} h^{\zeta_h})^{1-\sigma}}{1-\sigma} + \omega. \tag{29}$$

As ζ_g and ζ_h are relatively small, the constant relative risk aversion over consumption is approximately σ, which we take to be 2. The free parameters ζ_g and ζ_h are estimated jointly, strongly controlling the mean level of government expenditures and housing expenditures, respectively. We take the weather term ω to be normally distributed with mean zero (a normalization) and standard deviation σ_ω. We discipline σ_ω by matching the standard deviation of log population across cities.

We assume the moving cost is distributed

$$\phi|z \sim \begin{cases} \underline{\phi} & \text{w.p. } p_{\underline{\phi}}/2 \\ Logistic(\mu_\phi - \beta_\phi \log z, \varsigma_\phi) & \text{w.p. } 1 - p_{\underline{\phi}} \\ -\underline{\phi} & \text{w.p. } p_{\underline{\phi}}/2 \end{cases} . \tag{30}$$

Having the $\pm\underline{\phi}$ shock means that, for a sufficiently large $\underline{\phi}$, every island's departure rate is in $[p_{\underline{\phi}}/2, 1 - p_{\underline{\phi}}/2]$, which ensures some minimal stability in calibrating the model. Having the mean of the logistic distribution be contingent on z is meant to capture the idea that high-productivity individuals have lower moving costs, but the estimated value of β_ϕ is approximately zero. We set $p_{\underline{\phi}} = 10^{-4}$ and take $\underline{\phi}$ arbitrarily large giving $\int V(\phi, \mathbf{x}) dF(\phi|z)$, the expected utility of being in an island with state $\mathbf{x}$, equal to

$$\frac{p_{\underline{\phi}}}{2}(J + S(\mathbf{x})) + (1 - p_{\underline{\phi}})(S(\mathbf{x}) + \varsigma_\phi \log(1 + e^{(S(\mathbf{x}) - J - \mu_\phi + \beta_\phi \log(z))/\varsigma_\phi})) \tag{31}$$

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[19] We use large cities to reduce the role of measurement error. Weighting by population gives essentially the same estimates (.68,.18, and .001, respectively), while unweighted estimates have smaller persistence parameters (the estimates are .60,.19, and .004, respectively), consistent with attenuation bias.

plus a constant that we offset via a normalization.[20]

The parameters controlling moving costs μ_ϕ, β_ϕ, ς_ϕ and the parameter λ controlling how directed moving is (see (9)) are jointly estimated. We identify the cost parameters using mean departure and arrival rates as well as coefficients from the regressions in Table 3 of productivity and population on outflow rates in the data. β_ϕ controls how the outflow rate varies with productivity, so we use it to target the regression coefficient for log productivity on outflow rates. ς_ϕ controls how much the migration decisions vary with fundamentals, so we use the standard deviation of out-migration rates to discipline it. μ_ϕ controls the overall level of outflow rates, so we use the mean departure rate of 6.4% to discipline it. We identify λ by matching the regression coefficient for log TFP fixed effect on log population.

	(1) In mig. rate	(2) Out mig. rate	(3) In mig. rate	(4) Out mig. rate
Log TFP	-0.00456	0.00288		
Log TFP residual			0.0261***	0.00481
Log TFP fixed effect			-0.0155***	0.00220
Log population	-0.0000821	-0.00385***	0.00134***	-0.00376***
Constant	0.0821***	0.0933***	0.0515***	0.102***
Observations	2736	2736	2736	2736
R^2	0.001	0.058	0.017	0.058

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: In- and out-migration rate determinants

The borrowing limit δ and risk-free bond net supply $\overline{B}$ jointly determine the overall level of debt in the economy and the equilibrium interest rate associated with $\bar{q}$. Consequently, we use them to target a mean risk-free interest rate of 4% (the recent average, see Figure 2) and the data's total debt to GDP ratio of 0.089.[21] Since η controls the relative price of government services as population grows, we use the coefficient of a regression of log population on log expenditures (1.113) to discipline it. Finally, we calibrate the default cost $\overline{\kappa}$ to match default rates in the data. Since it is difficult to separately identify $\overline{\kappa}$ and χ (the probability of returning from autarky), we set $\chi = 1$, which concentrates all default costs in the period of default.

5.3 Fit of targeted and untargeted moments

Table 4 reports the targeted and untargeted statistics alongside the jointly calibrated parameter values. The model closely matches all of the targeted statistics. The estimated debt limit, δ, allows cities to borrow up to 128% of their expenditures, which is fairly close to California's statutory limit

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[20]The constant is $\underline{\phi}p_\phi/2$. We subtract β times it from flow utility in (29) each period.

[21]As discussed in Section 3.4, we use a gross debt measure since assets are not seized in a Chapter 9 bankruptcy. Additionally, we do not include debt from the underfunding of pensions because they are only underfunded at a local level if one assumes risk-free discount rates when in fact local governments have generally earned much higher returns. We discuss this more with appropriate references in Appendix A.2.

of 100% (plus exceptions). The consumption-equivalent flow cost of default, $\overline{\kappa}$, is estimated to be around 3%.[22] The utility shares ζ_g and ζ_h are close to the shares observed in the data. Migration is partially directed with $\lambda > 0$. Weather plays a large role, with a rough calculation giving the lifetime consumption equivalent variation of permanently moving from the median weather $\omega = 0$ to $2\sigma_\omega$ at 67%.[23] The importance of weather for utility helps the model match the very low (in fact, negative) correlation between in-migration and productivity and, simultaneously, the large dispersion in population. The large value for μ_ϕ with a correspondingly large ς_ϕ makes out-migration largely dependent on moving cost shocks rather than fundamentals like productivity. The value for β_ϕ is nearly zero. Finally, the provision of public goods displays some rivalry with $\eta = 0.33$.

The model gets most of the untargeted predictions qualitatively correct while missing on a few statistics. The model recreates the very slow population adjustments seen in the data with log population autocorrelation exceeding the data's 0.999. It also matches the small correlations between migration rates and productivity and migration rates and population. Last, it reproduces the dispersion in net migration rates and the large dispersion in government expenditures.

As an additional validation step, we rerun the fixed effects regressions from Table 1 on simulated data from the model. To convert the numeraire to dollars for this example and in the remainder of the paper, we multiply by 50,000, giving average output per person in our benchmark as $68,200. This is between the median and mean household income in 2010 but closer to the median.[24] The results are presented in Table 5, which reveals the model has similar patterns with the overborrowing term $(1-o)/(1-o+i)$ and in-migration significantly moving borrowing. One difference is that out-migration, while having the same negative sign as in the data, is much more negative. This reflects an optimal response present in the model that is less dramatic in the data. In particular, when bad productivity causes populations to shrink, the model suggests cities should cut deficits sharply to prevent exploding debt per person, but in the data cities seem not to do this (at least to such an extent). Overall, the magnitudes (except for out-migration) are fairly similar to the data. This feature will reappear in comparing Detroit's optimal and actual path. The R^2 is naturally larger than in the data as there are fewer determinants of spending in the model, but still the regressors explain at most 15% of the variation.

A final validation step is to compare the pattern of cities in relation to the borrowing constraint, which is done in Figure 4. The pattern is remarkably similar to that for California seen in Figure 1. (Note we use a CA-type borrowing limit that ties debt to spending rather than debt to property

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[22]We targeted a default rate of 0.0003. Since there are around 35,000 cities/townships/villages in our data, this implies 10.5 defaults a year, which is exaggerated. In large part, we do this for computational reasons since the simulations, which already use 16 million cities, must scale even larger to generate reasonable sample sizes conditional on default.

[23]Approximating lifetime utility as $\omega/(1-\beta) + \mathbb{E}_0 \sum \beta^t c_t^{1-\sigma}/(1-\sigma)$ gives the consumption equivalent variation γ as $\gamma = \frac{\mathbb{E}_0 \sum \beta^t (c_t^0)^{1-\sigma}/(1-\sigma)}{\frac{2\sigma_\omega}{1-\beta} + \mathbb{E}_0 \sum \beta^t (c_t^1)^{1-\sigma}/(1-\sigma)} - 1$ where c_t^0 and c_t^1 are the optimal plans in the original and new environment, respectively. Assuming these consumption plans equal 1 (the median unconditional value of output per person in our normalization), plugging in the parameters gives 2/3 or 67%.

[24]In 2010, median household income was $49,276 while GDP per household was $117,538. There is not a single appropriate measure to look at because the model has far less inequality than in the data. But any desired conversion can be done via dividing by 50,000 and multiplying by the desired number.

Targeted Statistics	Data	Model	Parameter	Value
Default rate ($\times 100$) $\int dd\mu$	0.030	0.049	$\overline{\kappa}$	0.084
Interest rate $1/\bar{q} - 1$	0.040	0.040	δ	1.286
Debt / GDP $\int -bnd\mu / \int z\dot{n}d\mu$	0.089	0.089	$\overline{B}$	-0.120
Gov. expenditures / GDP $\int g\dot{n}^{1-\eta}d\mu / \int z\dot{n}d\mu$	0.082	0.074	ζ_g	0.066
Housing expenditures / GDP $\int r\bar{H}d\mu / \int z\dot{n}d\mu$	0.125	0.125	ζ_h	0.112
*Out rate mean $\int F(R)/nd\mu$	0.064	0.063	μ_ϕ	22.513
*Out rate st. dev.	0.022	0.022	ς_ϕ	6.504
Std. deviation of $\log n$	1.846	1.797	σ_ω	0.326
*Population reg. coef., $\log z$ FE	4.224	4.152	λ	0.582
*Out rate reg. coef., $\log z$	0.003	-0.005	β_ϕ	0.000
Regression coef., log expenditures on $\log n$	1.113	1.049	η	0.325

Untargeted Statistics	Data	Model
Autocorrelation of $\log n$	0.999	1.000
Std. deviation of net migration rates	0.021	0.022
Correlation of log expenditures and $\log n$	0.858	0.994
Std. deviation of log expenditures	2.388	1.896
*Out rate reg. coef., $\log z$ res	0.005	-0.019
*In rate reg. coef., $\log z$ res	0.026	0.070
*In rate mean	0.066	0.066
*In rate st. dev.	0.028	0.029
*In rate reg. coef., $\log n$	0.001	-0.013

Note: Parameters are listed beside statistics they strongly influence; * means the underlying data is county-level; in all cases, income measures are at the county level; debt measures are gross (i.e., excluding any assets); and the debt and government expenditures are the sum of measures at county, city, and special district levels (see the appendix for more details).

Table 4: Calibration targets and parameter values

	(1) Deficit	(2) Deficit	(3) Deficit	(4) Deficit	(5) Deficit
Overborrowing term	-10802				-16276
In-migration rate		9665			
Out-migration rate			-38402		
Net-migration rate				8406	-4340
R^2	0.153	0.152	0.079	0.150	0.154

Note: A constant is included in each regression; the largest standard error is 58.8, and all the values are significant at a 1% level; each regression has 15,200,000 observations.

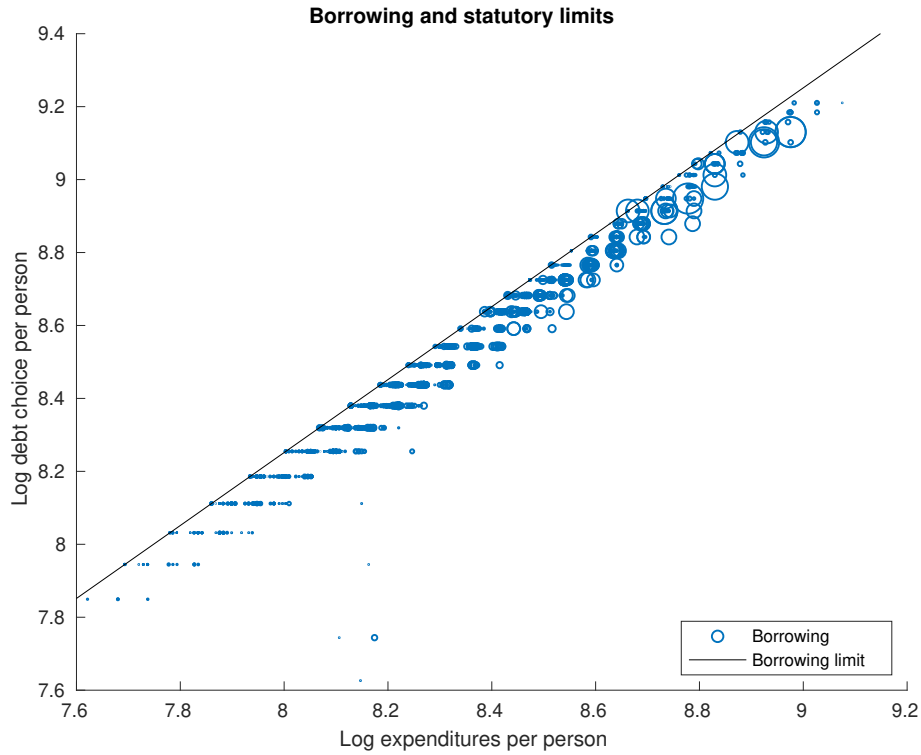Table 5: Fixed-effects regressions using model data

Figure 4: Model distribution of cities relative to their borrowing limit

values like in MI.) Specifically, almost all the cities are close to the statutory borrowing limit, though only a few are exactly at it. As this pattern is nontargeted, it lends additional credence to the model's other predictions and the counterfactuals that we now analyze.

6   Model predictions and counterfactuals

With our calibrated model in hand, we first ask what leads to and triggers default. We then turn to counterfactuals, looking at the elimination of state-imposed borrowing limits and the effects of a return to higher interest rates. Finally, we use the model to assess what, if anything, Detroit should have done differently, and the consequences of bailing out it and other cities.

6.1   What leads to and triggers default?

Default is a rare event in both the data and model, but the model-generated data, via an unrestricted sample size, let us determine what leads to and triggers default. We begin answering these questions by comparing the unconditional marginal distributions with the distributions conditional on a default in three years, both plotted in Figure 5.

At first glance, cities more or less likely to default look the same, as the overall means and standard deviations are fairly close to each other. However, on further inspection, a few important

differences emerge, especially with respect to migration flows. In particular, default is predicted by higher out-migration rates and higher in-migration. The combined effect is a smaller overborrowing term $(1-o)/(1-o+i)$, giving greater borrowing incentive for cities prone to default, and because of that one might expect the debt-expenditure ratio to be larger for cities likely to default. However, this is not the case because *all* cities are close to their borrowing limits.

Importantly, default in the model can happen in all types of cities except the largest and least productive. This means the model is capable of producing default in big, average productivity cities (like Detroit); medium-sized, high-productivity cities (like Stockton); and small, low-productivity cities (like Harrisburg). Default also can happen when a city is shedding population at 5% a year (like Flint or Detroit); growing at 5% a year (like San Bernardino or Stockton); or with a stable population (like Harrisburg).

An alternative way to assess what leads to default is to consider default events, i.e., windows before and after default. These are displayed in Figure 6. We break default episodes into three cases: an average default event (blue line), a default during a technology boom (red dashed line), and a default during a technology bust (green circled line).[25] On average, default episodes coincide with slightly growing productivity followed by a sharp decline in productivity (a drop close to 12%). Although on average population increases slightly pre-default, cities see their population decline post-default, losing about 10% of their inhabitants within five years. This is driven by an uptick in the out-migration rate and a sharper contraction in the in-migration rate.

Because the mean default episodes average over boom and bust defaults, they hide a large amount of heterogeneity. Looking specifically at bust defaults, one finds a prolonged decline in population with a 10% population loss over the 10 years leading up to default, qualitatively similar to the experience of Flint and Detroit. Facing this shrinking population, the sovereign holds debt per person constant, which requires running a significant primary *surplus* of almost $200 per year.[26] This surplus is financed with cuts to expenditures *and* taxes. In the few years before default, interest rates do increase, reflecting the increased default risk, but the largest interest rates are still low, not unlike in the data (as can be seen in Figure 2).

Looking at boom defaults (red dashed lines), the first thing to notice is the model can generate them (which was not clear a priori). The population and productivity growth is strong until just a year before default, like in Vallejo, CA. With the boom, the cities have virtually no primary deficit, which results in growing debt per person as interest payments pile up, and this is despite substantial population growth that all else equal reduces debt per person. The debt growth is paired with a noticeable increase in expenditures and taxes. Interest rates trend upwards, showing that the city is taking on increasing (albeit small) amounts of risk. When an unusually large decline in productivity occurs, in-migration plummets and debt per person increases, triggering default. While boom defaults are triggered by a decline in productivity, a *necessary* ingredient is that the city must be leveraged enough to make default worthwhile. This is where overborrowing plays

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[25]A boom (bust) is formally defined as having log productivity growth in the 10 years preceding default above the 75th (below the 25th) percentile.

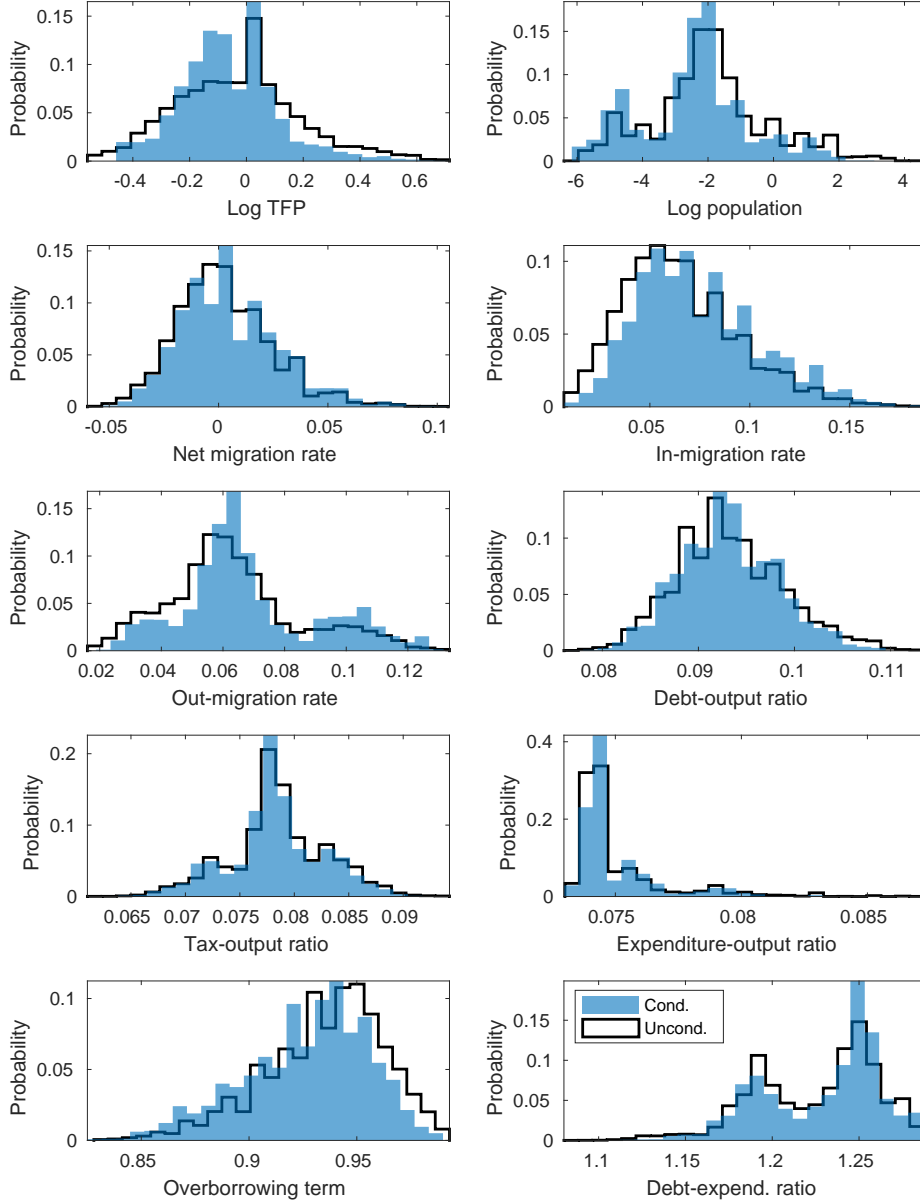[26]The primary deficit is $g\dot{n}^{1-\eta} - T\dot{n}$.

Figure 5: Distributions, unconditional and conditional on a default in three years

a crucial role in generating boom defaults: By keeping cities in debt even after long periods of growing productivity, even high-productivity cities are at risk of default.

A striking difference between the boom and bust defaults appears in the incentive to borrow as reflected in the "overborrowing term" $(1 - o)/(1 - o + i)$. In boom defaults, increases in in-migration drive down the overborrowing term, which increases borrowing incentives. The converse is true for bust defaults. Additionally, the magnitudes of the changes are large: Recalling that $1 - (1 - o)/(1 - o + i)$ is the share of debt subsidized (i.e., paid for) by new entrants, in boom (bust) defaults this subsidy increases (decreases) by 2 percentage points. While the debt paths show trends qualitatively consistent with these changes, the dynamics are muted in part because cities are close
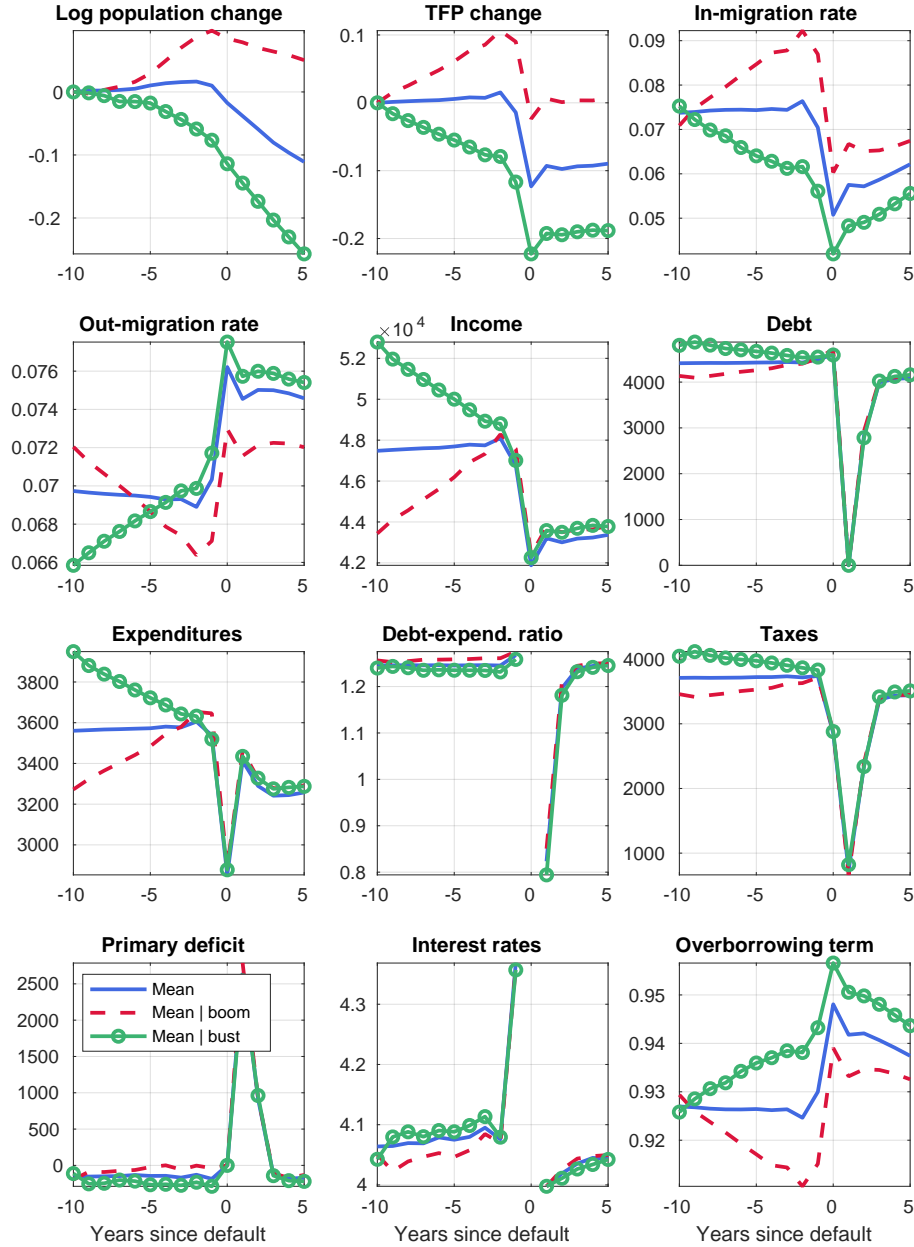
Note: all financial variables are per person.

Figure 6: Default episodes

to their debt limits: The debt-expenditure ratio is close to its limit of δ (1.286) in every year leading up to default.

In answer to the questions of what leads to and triggers default, the model reveals much the same answer as the data: The causes are myriad. Bust defaults lead to shrinking populations and primary surpluses financed by cuts to taxes and larger cuts to expenditures. In contrast, boom defaults are characterized by growing populations, growing debt per person, and growing taxes and expenditures. Boom defaults are possible in the model because of large overborrowing incentives

that keep cities leveraged even after a decade of productivity and income growth.

6.2 Disentangling the roles of migration, borrowing limits, credit markets, and general equilibrium effects

The analysis of what drives defaults showed that while defaulters tend to have large debt-expenditure ratios and greater overborrowing incentives than typical cities, the dynamics of debt per person are surprisingly muted. The reason given above was that debt is constrained by the exogenous borrowing limit, which keeps it from from moving much. However, the model has three external forces that restrict borrowing: Exogenous borrowing limits, a Laffer curve for borrowing in private credit markets, and the endogenously-determined risk-free interest rate. Additionally, the level of in-migration and its elasticity with respect to debt act as internal restraints on borrowing. We now try to assess the role of each of these forces before turning to the counterfactuals.

6.2.1 External restraints on borrowing

To start, we first eliminate the borrowing constraint in partial equilibrium, i.e., holding $\bar{q}$ fixed at the benchmark. The percent change relative to the benchmark is displayed in the third column (labeled "$(\infty, \cdot)$") of Table 6. One can see there is very little change from the benchmark except that the default rate increases somewhat. This is because private credit markets are restraining cities. To show this, we make default infeasible by setting the default cost $\bar{\kappa} = 1$ in addition to having $\delta = \infty$. This eliminates the Laffer curve for debt by turning the borrowing schedule $-q(b', \dot{n}, z, \omega)b'$, which has a peak, into just $-\bar{q}b'$, which does not. Those results are displayed in the fifth column, labeled "$(\infty, 1)$". Now debt per person expands by 65.9%, reaching the maximum debt amount allowed by the computational method ($10,000 per person).[27] One interesting feature of these results is that private credit markets restrain borrowing significantly despite small equilibrium default rates. Hence, in our calibration the threat of default, rather than outright losses from default, constrains borrowing.

In fact, our calibration suggests that private credit markets restrain municipal borrowing more than the exogenous borrowing limit. This can be seen in the fourth column, labeled $(\cdot, 1)$, where we have made default costs infinite but kept the borrowing limit at the benchmark value. In this case, debt per person increases by 12.5%. Note that in this case aggregate debt over aggregate expenditures rises from 1.201 to 1.285. Since the benchmark value of $\delta = 1.286$ constrains the debt-expenditure ratio in individual cities to 1.286, this shows that without the Laffer curve from borrowing in private credit markets, virtually *every* city goes to the borrowing limit in the model.

A final force in the model that restrains borrowing is the general equilibrium response of the risk-free interest rate. In particular, it is sufficient to discuss the most extreme case of $(\delta, \bar{\kappa}) = (\infty, 1)$ in the rightmost column of Table 6. In it, we see that while the overborrowing incentives are very large, they can be undone by a strong general equilibrium response that increases the risk-free rate

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[27]The equilibrium is solved for using discrete state space techniques, and as part of this the grid for debt per person specifies both a minimum ($0) and maximum ($10,000).

	Bench.	$(\delta, \bar{\kappa})$			
		$(\infty, \cdot)$	$(\cdot, 1)$	$(\infty, 1)$	$(\infty, 1)$
General equilibrium effects	—	No	No	No	Yes
Aggregate measures					
Output per person	68.2	-0.2	0.1	0.2	0.9
Consumption per person	62.9	-0.2	-0.4	0.1	0.6
Consumption per person s.d.	16.0	1.0	0.7	0.8	2.5
Expenditures per person	5.06	-0.3	5.4	-0.0	0.4
Taxes per person	5.30	-0.3	5.6	1.7	4.5
Debt per person	6.08	-0.2	12.5	65.9	-0.6
Weather	0.562	-0.2	-0.0	-0.0	0.2
Interest rate (%)	4.00	-0.0	-0.0	-0.0	121.4
Average across cities					
Default rate $\times 100$	0.049	12.2	-100.0	-100.0	-100.0

Note: the benchmark financial variables are in thousands of dollars; non-benchmark are values are percent deviation from the benchmark; a · means the value is the same as in the benchmark.

Table 6: Changes due to exogenous and endogenous borrowing constraints

from 4% to 8.8%, thereby keeping debt levels approximately at the benchmark level.[28] From these experiments, we conclude that the strong overborrowing incentives seen in the two-period model are present in the quantitative model, but that they are significantly restrained by both exogenous and endogenous (i.e., Laffer curve) credit limits.

6.2.2 Internal restraints on borrowing

In the two-period model, there were two forces that determined how much the city wanted to overborrow. First was the magnitude of the overborrowing term $(1-o)/(1-o+i)$. Second was the elasticity of population with respect to debt accumulation. We change the magnitudes of these forces by increasing and decreasing the mean moving costs μ_ϕ (which primarily affects the first force) and the search directedness parameter λ (which substantially affects both) holding the risk-free price $\bar{q}$ fixed. The results are presented in Table 7.

The first thing to notice is that—in each case—changes in debt per person have the same sign and roughly the same magnitude as changes in expenditures per person and output per person. To a large extent this is caused by all cities having a large incentive to overborrow that pushes them close to their borrowing limits. Because the exogenous borrowing limit restricts the debt-expenditure ratio, debt and expenditures positively comove. Additionally, credit markets keep the debt-output ratio from getting too large, and so debt and output also tend to comove. Last, output and expenditures comove because government services are a normal good. Consequently, debt, expenditures, and output move together irrespective of whether the city is constrained by the exogenous borrowing limit or endogenous ones. The lesson here is that modest changes in either the overborrowing term or the population elasticity do not restrain borrowing because there is a large incentive to borrow that modest changes do not overcome.

Keeping this in mind, consider specifically the impacts of a decrease in moving costs. This increases in- and out-migration, which also increases the overborrowing term and, in the two-period model, leads to more debt accumulation. However, debt per person decreases because output decreases, and as just discussed, debt and output comove. Smaller moving costs reduce output because, in our calibration, migration is very noisy: Many people leave high-productivity islands for no reason other than that they wanted to, i.e., their moving cost draw ϕ was substantially negative. (Recall that the data dictate this feature because of the near-zero correlation between out-migration and log productivity.) If the moving costs are even lower, that means even more people are leaving cities for purely idiosyncratic reasons rather than fundamentals.[29]

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[28]If the market perfectly cleared, the debt per person average change would literally be zero, but here aggregate debt declines by 0.6%. This particular experiment has more numerical error than the others because, without exogenous or endogenous borrowing limits, borrowing becomes very elastic to guesses on the risk-free rate.

[29]These comparative statics hinge on the calibrated values. For instance, consider the setup in the two-period model without borrowing and with random search ($\lambda = 0$)—so that J is fixed—and no dispersion in ϕ with $\phi = \mu_\phi$. Then efficiency—in terms of total output—dictates that people should move whenever $S < J$ and stay whenever $S > J$. Decreasing μ_ϕ from a positive value to zero improves sorting and increases output: People who previously stayed at islands with $S < J$ because moving was too costly now leave; but, people at islands with $S > J$ stay because in expectation they would be worse off from a move. On the other hand, decreasing μ_ϕ further to a negative number would worsen output: Now even people with $S > J$ move simply to obtain utility $J - \mu_\phi > J$.

| | | Pct. change from bench. | | | |
| | | μ_ϕ | | λ | |
	Bench.	↓	↑	↓	↑
General equilibrium effects	—	No	No	No	No
Aggregate measures					
Output per person	68.2	-2.1	1.7	-1.2	1.2
Consumption per person	62.9	-2.1	1.7	-1.2	1.2
Consumption per person s.d.	16.0	0.0	1.3	-1.4	0.5
Expenditures per person	5.06	-1.9	1.6	-1.2	1.2
Taxes per person	5.30	-1.9	1.6	-1.2	1.2
Debt per person	6.08	-0.8	0.3	-1.3	1.2
Weather	0.562	-4.2	3.3	-1.9	2.0
Average across cities					
Default rate $\times 100$	0.049	29.6	-20.5	1.9	16.2
In-migration rate (%)	6.56	22.8	-20.9	-0.9	0.6
Out-migration rate (%)	6.31	23.3	-21.2	-0.8	0.5
Log population s.d.	1.80	-7.7	8.3	-4.3	4.7

Note: the benchmark financial variables are in thousands of dollars; non-benchmark are values are percent deviation from the benchmark.

Table 7: Changes due to moving costs and search directedness

Now consider an increase in search directedness (i.e., $\lambda\uparrow$), which has three effects. First, it makes moving more attractive, which tends to increase in-migration and, consequently, increase debt. Second, it increases output, which, based on arguments above, also tends to increase debt. Third, it makes in-migration more elastic to city debt positions, which reduces borrowing by "punishing"—in the sense of lower in-migration—cities that accumulate debt. Here, quantitatively, the first two forces dominate locally around the calibrated levels because λ is small, as required to match the data's small correlations between in-migration and productivity. For very large values of λ (which would imply counterfactually large correlations), the third force would likely dominate (as suggested by the efficiency result obtained assuming perfect search directedness in Proposition 3).

6.3 Optimal borrowing limits?

With a better understanding of the model mechanisms, we now turn to the model's counterfactuals. One prediction of the theoretical model is that a supralocal government may wish to constrain local government borrowing. And, as we showed, CA, MI, and several other states have imposed municipal borrowing limits. Using the model, we now explore what would be the impact of lifting these limits by setting δ to a large value. In contrast to the earlier exercise (in Section 6.2), we allow for general equilibrium effects. The results are displayed in column $\delta = \infty$ of Table 8.

To understand why output goes down, one must remember there are essentially two types of cities in the model: Cities constrained by the exogenous borrowing limit and cities constrained by the private credit markets. The first tend to be *high*-productivity cities (because their bond prices are better) while the latter tend to have low productivity. By eliminating the exogenous limit, high-productivity cities accumulate more debt. This reduces household incentives to relocate to high-productivity cities, which results in lower output. Everything that comoves with output, such as consumption, expenditures, and taxes, also falls. Moreover, welfare, as measured by J (see equation 10), also falls (because J is negative in the benchmark, a positive percent change means a lower J). At face value, this measure—which is potentially problematic as it ignores transitions costs—indicates that borrowing limits are in fact optimal. However, the magnitudes of the changes are small because, as previously discussed, private credit markets significantly constrain city borrowing.

6.4 A return to high interest rates

In light of the recent increases in the federal funds rate, we assess the impact of a higher risk-free rate on municipalities. In particular, we increase the risk-free interest rate in the model from 4% to 6.5%—the latter corresponding to the interest rates in the early 1990s as can be seen in Figure 2—by reverse-engineering a risk-free bond supply $\overline{B}$ that delivers a 6.5% interest rate. The results are displayed in Table 8 in the column labeled $\overline{q}^{-1} \uparrow$.

More expensive borrowing reduces the incentives to issue debt, and debt per person declines by 16%. The higher debt service costs encourage migration away from high-debt, low-productivity

	Bench.	Pct. change from bench.		
		$\bar{q}^{-1}\uparrow$	$\delta=\infty$	$\pi_b=\frac{1}{2}$
General equilibrium effects	—	Yes	Yes	Yes
Aggregate measures				
Output per person	68.2	0.2	-0.2	-0.1
Consumption per person	62.9	0.1	-0.2	-0.2
Consumption per person s.d.	16.0	-0.0	0.9	0.2
Expenditures per person	5.06	-0.0	-0.3	-0.1
Taxes per person	5.30	1.4	-0.3	0.1
Debt per person	6.08	-16.0	-0.2	-0.1
Utility conditional on moving (J)	-12.9	0.2	0.0	0.0
Weather	0.562	0.1	-0.2	-0.1
Interest rate (%)	4.00	62.6	0.1	8.4
Tax for financing bailouts (%)	0.000	0.000*	0.000*	0.057*
Average across cities				
Default rate ×100	0.049	-13.0	12.2	157.9
Bailout rate ×100	0.000	0.000*	0.000*	0.064*

Note: The benchmark financial variables are in thousands of dollars; non-benchmark are values are percent deviation from the benchmark unless marked by *. The experiments are as follows: $\bar{q}^{-1}\uparrow$ increases the risk-free interest rate by 2.5 percentage points; $\delta=\infty$ removes the borrowing limit; $\pi_b=\frac{1}{2}$ is anticipated bailouts where bailouts occur 50% of the time.

Table 8: Counterfactual experiment results

cities, which increases aggregate output and "weather" slightly (both of which only reflect sorting). Naturally, then, consumption also increases.[30] Default rates fall slightly in the long run, while still remaining low overall. From these results, we conclude that, while there will be some effects from a return to high interest rates, the overall impact is likely to be small, at least in the long run.

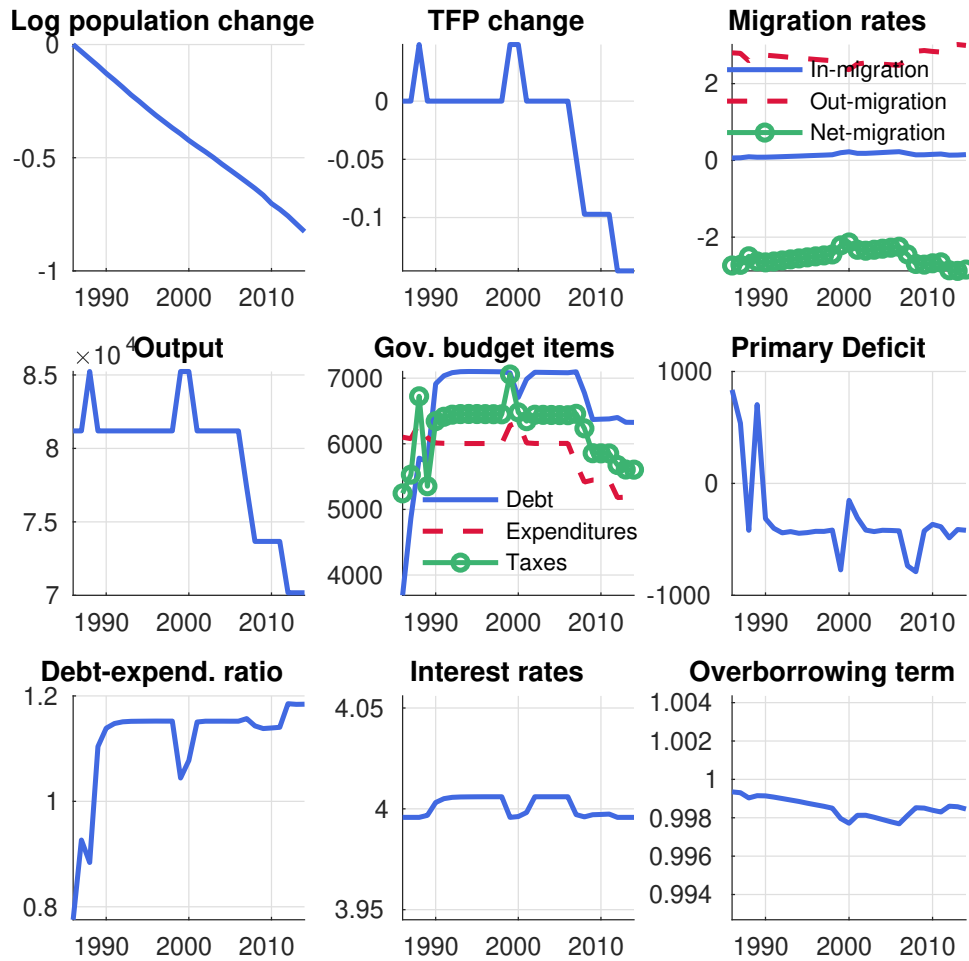## 6.5   What should Detroit have done differently?

Since our model has city planners who maximize the welfare of current residents, the planner's response to shocks gives an optimal strategy for dealing with adverse (or beneficial) shocks. Hence, by feeding in Detroit's observed shock path in our simulation, our model provides a counterfactually optimal path for the city including policy prescriptions.[31] We conduct this simulation by feeding in the estimated fixed and residual productivity series when starting the economy with the initial debt and population observed in the data. (For robustness, Appendix C.8 gives the paths where the simulation starts after a long burn-in.)

The simulation results for Detroit are displayed in Figure 7, and they show that Detroit's model-implied long-run level of population is much lower than its 1986 level. From 1986 to 2011, the model predicts a 0.75 log decrease or a 53% decline in population. While the decline in the data over this period was 0.45 log point or 36% (see Figure 2), we note that Detroit's population has continued in its downward march in the data. In the model, the levels of debt, expenditures, and taxes stabilize after a few years of debt accumulation, and the city then optimally runs a primary surplus of around $500 per person. The optimal model-implied debt level in the early 2000s of around $7,000 per person is not far from the value Detroit actually held, which was around $7,000 in 2001. However, Detroit's debt continued to grow, and at the start of the financial crisis was around $10,000 per person. More importantly, when productivity sharply fell in the financial crisis, precipitating the General Motors bankruptcy, the optimal response was to cut expenditures and—to a lesser extent—taxes, reducing debt per person by $750. In contrast, Detroit raised taxes and expenditures simultaneously, ultimately increasing debt per person by $2,500.

While Detroit is a special case in terms of the magnitude of changes it underwent, its suboptimal response to adverse productivity shocks is not. In particular, one signficant disagreement between the model and data is the correlation between deficits and out-migration. In the fixed-effects regressions of Table 1, the negative correlation between between deficits and out-migration is only weakly negative. Emblematic of this weak correlation is that Detroit's out-migration in the financial crisis was paired with larger deficits (as seen in Figure 2). In contrast, the same fixed-effects regressions on model data (in Table 5) reveal a strong negative correlation between deficits and out-migration: When adverse shocks hit and increase out-migration, the optimal response is

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[30]Unlike in the counterfactuals seen thus far, output and debt do not comove. The reason is the interest rate increase endogenously pushes the largest and most productive cities away from their borrowing constraint, which breaks the tight link between expenditures, output, and debt. We show this in Figure 10 in Appendix C.7.

[31]Since we have productivity data only at the county level, we use Wayne county's measure in place of Detroit's. We also assume that Detroit has the best discretized "weather" state—in this case, clearly one should understand we are not talking about weather per se, but rather some fixed characteristic that resulted in Detroit's very large population.

Figure 7: The optimal response to Detroit's productivity shocks

to significantly deleverage. While Detroit's residents in the model unanimously favor the record primary surpluses of $800 per person, one can see that in practice these policies would not be particularly popular, especially at a time when output has fallen by more than $10,000 per person.

6.6 Bailouts (or, what should the U.S. have done differently?)

The dire effects of the financial crisis not only precipitated Detroit's bankruptcy, but also led to bailouts for General Motors and Chrysler. This led to discussion on whether Detroit should be bailed out, to which the governor of Michigan replied "bankruptcy is there to deal with the debt question" (Condon, 2013). Moreover, bailouts are not just a Detroit issue. For instance, Hartford's finances exhibit massive swings in expenditures while taxes and debt are steady (as can be seen in Figure 2) because of repeated, but uncertain, cash infusions from the Connecticut state government. We now examine a general bailout policy that applies to any city at risk of default. Specifically, we will focus on ε-bailouts as made precise by the following definition:

Definition 2. *An ε-bailout is an unconditional transfer of resources $\mathfrak{b}$ (measured per person and after migration) that makes the government indifferent between repaying and default (and we assume they repay in such a case). Formally, $\mathfrak{b}(\mathbf{x})$ is given by $S^N(\dot{b}(\mathbf{x}) + \mathfrak{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega) = S^D(\dot{n}(\mathbf{x}), z, \omega)$.*

We assume bailouts occur with probability π_b (i.i.d.) known to the local government. We also assume that households make migration decisions before seeing whether the city is bailed out or not. Hence, $\dot{n}$ and $\dot{b}$ are functions of $\mathbf{x}$ (as before) and do not depend on whether a bailout is realized or not. To pay for the bailouts, we suppose a federal government taxes purchases of the risk-free bond at rate τ. These assumptions change debt pricing and the intermediary's problem and require a federal government budget constraint, and we describe these modifications in Appendix C.5.

As can be seen in column $\pi_b = \frac{1}{2}$ of Table 8, anticipated bailouts that occur with 50% probability more than double default rates. However, apart from this change, the effects on the other statistics are quite weak with output, consumption, and expenditures all falling. Again, the exogenous borrowing limits play a significant role: Cities would like to borrow significantly, but loosening the borrowing constraint by increasing expenditures (at the expense of consumption) is distortionary. Additionally, with ε-bailouts, utility in the bailout period is the same as utility in default, which is very low. This also limits any utility gain from being bailed out to excessive spending in the periods leading up to the bailout.

To examine the moral hazard effects, we again turn to default episodes. These are displayed in Figure 8, which plots the benchmark's default (blue lines) and anticipated bailout (green circled lines) episodes, as well as surprise bailouts (red dashed lines) that are measure zero events. Anticipated bailouts do not lead to excessive borrowing in anticipation of a bailout as cities at risk of default are already at or very near to their borrowing limit. The surprise bailouts, which show just the cash-injection effects without any moral hazard or general equilibrium effects, reveal effects on debt, taxes, expenditures, and deficits, but all those changes do little to change the population dynamics. Overall, we find the effects of ε-bailouts to be quite small.
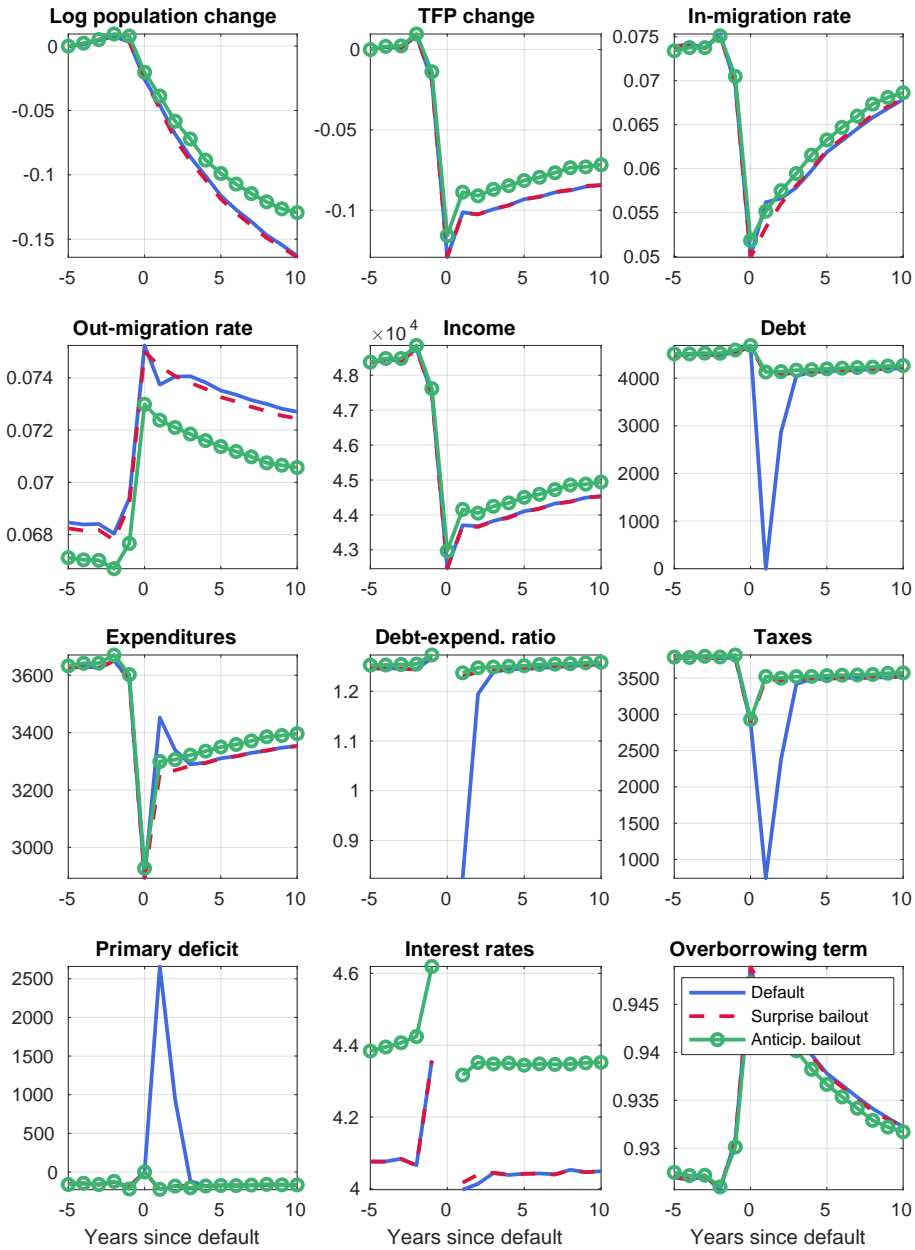
Figure 8: Comparison of Default and Bailout Episodes

7 Conclusion

Borrowing, migration, and default are intimately connected. Theoretically, we demonstrated that migration tends to result in over borrowing. Empirically, we documented that defaults can occur after booms or busts in labor productivity and population, that in-migration rates are negatively correlated with deficits, and that many cities appear to be at or near state-imposed borrowing limits. We interpreted these empirical findings loosely in light of the theoretical results, but our quantitative model was able to formalize these explanations by replicating these and many other

features of the data.

Our counterfactuals revealed a number of interesting results. One novel result was that Detroit should have cut spending, taxes, and debt in response to its large negative TFP shocks in 2007, 2008, and 2009, thereby avoiding default. We also found that bailing out Detroit and cities like it does not generate large moral hazard costs as long as the bailouts are not generous. With regards to monetary policy, we found a return to a high interest rate environment, while slightly improving GDP via migration incentives, has fairly small effects. Last, we found that eliminating state-imposed borrowing limits should have small effects because private credit markets restrain cities nearly as much as the state-imposed limits.

While our paper has focused on municipalities, it should also prove useful for analyzing states and nations. The drastic population flight from Puerto Rico, a less well-known population decline in Greece, and large in-migration to Spain are three such applications where our framework could prove useful.

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A Additional data details [Not for Publication]

This appendix describes our data sources, definitions of key variables, and cleaning procedures in Sections A.1, A.2, and A.3. Section A.4 records newspaper headlines on local government finances.

A.1 Census County Business Patterns data

To construct TFP measures, we use data from the Census' County Business Patterns (CBP) database from 1986 to 2014. The main measures we use are the payroll variable `ap` (converted to constant dollars using the standard CPI series obtained from FRED) and the mid-March employment variable `emp`, along with the FIPS codes. In the CBP database, missing or bad values are assigned a value of zero, so we treat `ap` and `emp` as missing whenever they are 0. Our overall productivity measure z_{it} is `ap`/`emp`. The data includes disaggregated employment levels by sectors (NAICS and SIC), so we keep only the observations corresponding to aggregates. The panel includes 91,800 year-county non-missing observations for z_{it}.

A.2 Annual Survey of State & Local Government Finances data

For our data on government finances and population, we use the Annual Survey of State & Local Government Finances (IndFin) compiled by the Census Bureau. Every five years (in years ending in two or seven), the aim is to construct a comprehensive record of state and local finances. (In practice, surveys are sent out for most cities and not all are returned, but the coverage is good enough to cover 64-74% of the U.S. population depending on the year.) In intervening years, a non-representative sample is selected from the population. Some of the larger cities are "jacket units," and instead of surveys the Census sends its own workers to record the data. The data are aggregated at different levels, with "cities"—i.e., municipalities and townships—counties, and states. We consider two samples, one corresponding to cities (`typecode` equal to 1 or 2) and the other to data aggregated at a county level (the aggregation of `typecode` values 1 through 5). Some of the data go back to 1967. However, the first population records begin in 1986 (survey year 1987), so we restrict ourselves to the 1987-2012 survey years.

   The `population` is not recorded in each year (the data for it does not necessarily correspond to the survey year but are given by `yearpop`), and so we construct estimates. We restrict the sample so that each city/county has at least two population measures. We fill in missing observations using linear interpolation of the log population. We also allow for some extrapolation, but do not allow extrapolation beyond five years.

   The raw sample consists of 390,557 year-county or year-city observations. We then use the sample restrictions as described in Table 9. We compute annual trust returns as `totinstrustinvrev` divided by `insurtrustcashsec` less the CPI-measured inflation rate. We compute implied interest rates via the interest paid during the year over the total debt, short and long term: 100* `totalinterestondebt` / (`stdebtendofyear` + `totallongtermdebtout`). All financial variables are converted to real 2012 dollars using the CPI.

Sample selection condition	Obs.
Raw	390557
Require non-missing `name`, require no id changes (`idchanged=0`), drop if "no data" (`zerodata>0`)	387963
Require `yearpop` not missing, drop if `datayearcode`="N", require at least two population observations	387219
Drop observations with non-missing investment annual returns on trust funds exceeding 30%	386723
Dropping missing population estimates	386515
Dropping observations where population growth rates could not be estimated	386511
Dropping Louisville, KY observations before 2003	386494
Require annual population growth rates of less than 25%	386494
Require revenue per person of less than $25,000	386090
Require debt per person of less than $30,000	376778
Require accounting identity for the evolution of long-term debt to nearly hold	363365
Require estimated interest rates be less than 40% annually	362122

Table 9: Sample selection in IndFin

By following the IndFin definitions, our debt measure excludes any implicit debt in the form of underfunded pensions. In attempting to construct an alternate measure that included this "debt," we found that whether there is any underfunded pension liabilities or not hinges on the discount rate assumed in accounting. If one assumes Treasury bill rates, then there is underfunding. However, many pension assets (75% according to Biggs, 2016) are invested in stocks, private equity, and hedge funds, and the average return on pension assets has historically been much larger. If one uses the expected returns, the liabilities turn into an asset for local pensions; see Table 1 of Rauh (2017). Since the model has no aggregate risk, the impact of assuming a risk-free or risky rate only affects the target for the debt-output ratio, and we suspect changes the model little. Because of this, we ignore any underfunding of pensions, effectively treating them as fully funded. Additionally, a municipality's assets generally *cannot* be seized in default. Because of this, we look at gross debt, not liabilities minus assets.

A.3   Migration data

We retrieve information on migration flows from the Census database on county-level migration. The database contains data on migration in five-year intersecting intervals starting from 2006-2010 originating from the American Community Survey (ACS) and the Puerto Rico Community Survey (PRCS). The respondents are asked where they lived one year prior to the survey.

The datasets include population moving within the same county, from a different county but same state, from a different state, from abroad, and non-movers. As a measure of migration inflows, we are using the sum of population moving from a different county within the same state, from a different state, and from abroad. Thus, we do not count as migrants those who move within the

same county. Outflows are calculated analogously.

The data have limitations, especially for small counties. Geographies with populations of 65,000 or more have annual estimates produced, three years of data are needed for geographies of 20,000-65,000, and five years of data for smaller geographies. We address this issue by, e.g., taking the city-county-migration database and creating, using 2010-2014 migration data that corresponds to the year 2012 estimate, five variable types, assigning the 2010-2014 data as *_F2 for 2010, *_F1 for 2011, * for 2012, *_L1 for 2013, and *_L2 for 2014 where * is one of inflow, outflow, or population. We then compute a weighted average for each year via $\frac{1}{4}$(*_F2+*_L2) + $\frac{1}{2}$(*_F1+*_L1) + * divided by 2.5. (When one or more of the variables were missing, we computed the weighted average using only the nonmissing data.)

A.4 Cities making headlines

Here, we document some cities/municipalities experiencing financial difficulties as reported by different media outlets. In quotations, we include excerpts of these news. To retrieve the source, the interested reader should click on the city's name.

U.S. Virgin Islands: *"With just over 100,000 inhabitants, the protectorate now owes north of $2 billion to bondholders and creditors. That is the biggest per capita debt load of any U.S. territory or state - more than $19,000 for every man, woman and child scattered across the island chain of St. Croix, St. Thomas and St. John. The territory is on the hook for billions more in unfunded pension and healthcare obligations."*

Chicago: *"Chicago's finances are already sagging under an unfunded pension liability Moody's has pegged at $32 billion and that is equal to eight times the city's operating revenue. The city has a $300 million structural deficit in its $3.53 billion operating budget and is required by an Illinois law to boost the 2016 contribution to its police and fire pension funds by $550 million.*
Cost-saving reforms for the city's other two pension funds, which face insolvency in a matter of years, are being challenged in court by labor unions and retirees.
State funding due Chicago would drop by $210 million between July 1 and the end of 2016 under a plan proposed by Illinois Governor Bruce Rauner."

Detroit: *"'It is indeed a momentous day,' U.S. Bankruptcy Judge Steven Rhodes said at the end of a 90-minute summary of his ruling. 'We have here a judicial finding that this once-proud city cannot pay its debts. At the same time, it has an opportunity for a fresh start. I hope that everybody associated with the city will recognize that opportunity.'*
In a surprise decision Tuesday morning, Rhodes also said he will allow pension cuts in Detroit's bankruptcy. He emphasized that he won't necessarily agree to pension cuts in the city's final reorganization plan unless the entire plan is fair and equitable. 'Resolving this issue now will likely expedite the resolution of this bankruptcy case,' he said."

Flint: *"Flint once thrived as the home of the nation's largest General Motors plant. The city's economic decline began during the 1980s, when GM downsized. In 2011, the state of Michigan took over Flint's finances after an audit projected a $25 million deficit. In order to reduce the water fund shortfall, the city announced that a new pipeline would be built to deliver water from Lake Huron to Flint. In 2014, while it was under construction, the city turned to the Flint River as a water source. Soon after the switch, residents said the water started to look, smell and taste funny. Tests in 2015 by the Environmental Protection Agency (EPA) and Virginia Tech indicated dangerous levels of lead in the water at residents' homes."*

Hartford: *"Hartford's biggest bond insurer said it had offered to help the city postpone payments on as much as $300 million in outstanding debt, in a move designed to help prevent a bankruptcy filing for Connecticut's capital. Under Assured Guaranty's proposal, debt payments due in the next 15 years would instead be spread out over the next 30 years without bankruptcy or default. The city would issue new longer-dated bonds and use the proceeds to make the near-term debt payments."*

Puerto Rico: *"The Puerto Rican government failed to pay almost half of $2 billion in bond payments due Friday, marking the commonwealth's first-ever default on its constitutionally guaranteed debt."*

New Jersey and other states: *"The particular factors are as diverse as the states. But one thing is clear: More states are facing financial trouble than at any time since the economy began to emerge from the Great Recession, according to experts who say the situation will grow more dire as the Trump administration and GOP leaders on Capitol Hill try to cut spending and rely on states to pick up a greater share of expensive services like education and health care."*

On the State Crisis: *"States and cities around the country will soon book similar losses because of new, widely followed accounting guidelines that apply to most governments starting in fiscal 2018 – a shift that could potentially lead to cuts to retiree heath benefits."*

Illinois: *"After decades of historic mismanagement, Illinois is now grappling with $15 billion of unpaid bills and an unthinkable quarter-trillion dollars owed to public employees when they retire."*

B   Computation [Not for Publication]

This appendix describes the computational algorithms used.

B.1   Discretization of the AR(2) process

To discretize the AR(2), we follow Gordon (2019), which exploits information about the autocorrelation function to discretize efficiently. We use a "coverage" (i.e., the support in Tauchen's 1986 method) of two unconditional standard deviations, treating any discretized state $(\tilde{z}_{it}, \tilde{z}_{it-1})$ occur-

ring with probability less than 10^{-6} in the invariant distribution as irrelevant and dropping it from the set of discretized states (with appropriate reweighting of transition probabilities). The algorithm delivers 58 discrete $(\tilde{z}_{it}, \tilde{z}_{it-1})$ states. (These, combined with the five permanent productivity states and three weather states, make 870 exogenous states.)

B.2 Equilibrium computation

To compute the equilibrium, we guess on three objects: the expected utility conditional on moving J, the risk-free price $\bar{q}$,[32] and the average inflows over a "normalization" term for the logit probabilites,

$$\bar{i} := \frac{\int n F(R(\mathbf{x})|z) d\mu(\mathbf{x})}{\exp(\lambda(S(\mathbf{x}) - \max_{\mathbf{x}} S(\mathbf{x})) d\mu(\mathbf{x})}, \tag{32}$$

Subtracting off $\max_{\mathbf{x}} S(\mathbf{x})$ prevents overflows in the computation. Note that knowing $\bar{i}$, $i(\mathbf{x})$ can be obtained via

$$i(\mathbf{x}) = \bar{i} \exp(\lambda(S(\mathbf{x}) - \max_{\mathbf{x}} S(\mathbf{x}))). \tag{33}$$

B.2.1 Solving for the law of motion and value and price functions

With the tuple $(J, \bar{i}, \bar{q})$, we solve for the value function $S(\mathbf{x})$, the law of motion $\dot{n}(\mathbf{x})$, and the price schedule $q(b', \dot{n}, z)$ as follows:

1. Construct discrete grids of of debt per person $\mathcal{B}$, population $\mathcal{N}$, productivity $\mathcal{Z}$, and weather $\mathcal{W}$.

   For $\mathcal{B}$, we use 20 linearly spaced points from -0.2 to 0. Since average income across cities is normalized to 1 and the debt-output ratio is around .02, this allows for a given city to hold roughly 4 times as much debt as the average and it is not binding in the benchmark. (This grid is coarse relative to those used in Bewley-Huggett-Aiyagari type models, but the dispersion in debt holdings is much more concentrated for cities.) For $\mathcal{N}$, we use 64 *log-linearly* spaced grid points over $\pm 5 * 1.8$ since the standard deviation of the log population is roughly 1.8. For $\mathcal{Z}$, we discretize the process as described in Section B.1 and tensor product it with the nonparametrically discretized permanent shocks. For $\mathcal{W}$, we use a three-point discretization $\{-2\sigma_\omega, 0, 2\sigma_\omega\}$ with Tauchen's method.

2. Fix tolerances (tol_q, tol_n, tol_S).

   We use $(tol_q, tol_n, tol_S) = (10^{-3}, 10^{-5}, 10^{-7})$.

3. Guess on $S(\mathbf{x})$, $\dot{n}(\mathbf{x})$, $q(b', \dot{n}, z, \omega)$.

   The initial guess we use is $S(\mathbf{x}) = 0$, $\dot{n}(\mathbf{x}) = n$, and $q(b', \dot{n}, z, \omega) = \bar{q}$.

---

[32]For the bailout case, we guess on $\bar{q}(1-\tau)$ rather than $\bar{q}$ as this is the relevant object for households and cities. Having computed the equilibrium value of $\bar{q}(1 - \tau)$, we then compute τ using the invariant distribution and bailout amounts and use it to recover $\bar{q}$.

4. Solve for $S^N(\dot{b}, \dot{n}, z, \omega)$ via grid search and update $S^D(\dot{b}, \dot{n}, z, \omega)$.

   For this, we use the analytic solution—conditional on b'—of the intratemporal problem. Whenever we interpolate, we use linear interpolation.

5. Solve for $d(\mathbf{x})$ and an update $S^*(\mathbf{x})$ by comparing S^N and S^D evaluated at $(\dot{b}(\mathbf{x}), \dot{n}(\mathbf{x}), z, \omega)$.

6. Compute an update $q^*(b', \dot{n}, z, \omega)$ using $d(\mathbf{x})$.

7. Compute an update $\dot{n}^*(\mathbf{x})$ using $S^*(\mathbf{x})$ and J.

8. Determine whether the convergence criteria $||q^* - q||_\infty < tol_q$, $||\dot{n}^* - \dot{n}||_\infty < tol_n$, and $||S^* - S||_\infty < tol_S \cdot ||S||_\infty$ are satisfied. If so, stop. Otherwise, update the guesses as $S := S^*$, $\dot{n} := \frac{1}{2}\dot{n}^* + \frac{1}{2}\dot{n}$, and $q := q^*$ and go to Step 4.

B.2.2 Solving for the invariant distribution and key equilibrium object updates

Given the converged values for $\dot{n}(\mathbf{x})$, the bond policy $b'(\dot{b}, \dot{n}, z, \omega)$ (from the S^N problem), and the default decision $d(\mathbf{x})$, we compute the invariant distribution $\mu(\mathbf{x})$ and updates $J^*, \vec{i}^*, \vec{q}^*$ as follows:

1. Fix a tolerance tol_μ.

   We use $tol_\mu = 10^{-10}$.

2. Guess on μ.

   Our initial guess is $\mu(0, 1, z, 1) = \mathbb{P}(z)$ with $\mu = 0$ elsewhere. (Consequently, the mass of households is 1 initially.) On subsequent invariant distribution computations, we use the previously computed μ.

3. For all $\mathbf{x}$, compute the out-migration rate $o(\mathbf{x}) := F(R(\mathbf{x})|z)$ where $R(\mathbf{x}) = S(\mathbf{x}) - J$.

4. Using $o(\mathbf{x})$, $\bar{i}$, $\mu(\mathbf{x})$, compute updates $\dot{n}^*(\mathbf{x})$, J^*, $\vec{i}^*$.

5. Using $\dot{n}^*(\mathbf{x})$, and $\mu(\mathbf{x})$, the bond and default policies, compute an update on the invariant distribution $\mu^*(\mathbf{x})$.

   Again, we use linear interpolation to distribute the mass from μ to μ^*. (An important advantage of linear interpolation is that it keeps the number of *households* the same on each iteration, i.e., $\int \mu(\mathbf{x}) n dx = \int \mu^*(\mathbf{x}) n dx$.) We do not linearly interpolate the bond policies, which are discontinuous because they are computed via grid search. Rather, we compute the four sets of linear interpolation weights and knots for $\dot{b}$ and $\dot{n}$ then evaluate $b'(\dot{b}, \dot{n}, z)$ four distinct times at the knots.

6. Determine whether the convergence criteria $||\mu^* - \mu||_\infty < tol_\mu$ is satisfied. If so, continue to the next step. Otherwise, update the guess $\mu := \mu^*$ and go to Step 3.

7. For the updates J^* and $\bar{i}^*$, use the values associated with the computed invariant distribution μ. For an "update" on $\bar{q}$, there is no natural fixed point update. However, construct a "pseudo-update" by defining $\bar{q}^* := \bar{q} - 10(\overline{B} - \int(1 - d(\mathbf{x}))(bn)d\mu(\mathbf{x}))$.

   Note that if cities are borrowing too little in that $\int(1 - d(\mathbf{x}))(bn)d\mu(\mathbf{x}) > \overline{B}$, this makes $\bar{q}^* > \bar{q}$, lowering interest rates and making borrowing more attractive. The reason we multiply by 10 is because $\overline{B}$ is on the order of .001-.004, so if $\int(1 - d(\mathbf{x}))(bn)d\mu(\mathbf{x}) = 0$ this implies $\bar{q}^*$ would be increased by .01-.04 (i.e., a few percentage point increase in the interest rate).

B.2.3 Solving for the key equilibrium objects

With the initial guesses $J, \bar{i}, \bar{q}$ and the updates $J^*, \bar{i}^*, \bar{q}^*$, produce new initial guesses as follows:

1. Fix tolerances $(tol_J, tol_{\bar{i}}, tol_{\bar{q}})$. Fix an initial step size $\delta_{\bar{q}} > 0$. Fix $\xi_J > 0$ and $\xi_{\bar{i}} > 0$.

   We use $(tol_J, tol_{\bar{i}}, tol_{\bar{q}}) = (10^{-7}, 10^{-5}, 10^{-3})$. Our $\delta_{\bar{q}}$ varies based on the experiment (with larger values when the equilibrium value is suspected to be far away), but in the benchmark we use $\delta_{\bar{q}} = .01$. Our initial values for ξ_J and $\xi_{\bar{i}}$ are 1.5 and 1, respectively.

2. Check whether $|J^* - J| < tol_J \cdot |J|, |\bar{i}^* - \bar{i}| < tol_{\bar{i}} \cdot \max\{|\bar{i}|, .01\}$, and $|\bar{q}^* - \bar{q}| < tol_{\bar{q}}$. If so, STOP: an equilibrium has been computed. Otherwise, go on.

3. Save the $d_J^* := J^* - J$, $d_{\bar{i}}^* := \bar{i}^* - \bar{i}$, $d_{\bar{q}}^* := \bar{q}^* - \bar{q}$ and—before doing so—store the previous changes (except on the first iteration) as d_J, $d_{\bar{i}}$, and $d_{\bar{q}}$.

4. Update the equilibrium values:

   (a) J according to $J := \xi_J J^* + (1 - \xi_J)J$

   (b) $\bar{i}$ according to $\bar{i} := \xi_{\bar{i}}\bar{i}^* + (1 - \xi_{\bar{i}})\bar{i}$

   (c) $\bar{q}$ according to $\bar{q} := \bar{q} + sign(\bar{q}^* - \bar{q})\min\{\delta_{\bar{q}}, |\bar{q}^* - \bar{q}|\}$.

5. If not the first iteration and if $d_{\bar{q}}d_{\bar{q}}^* < 0$, replace $\delta_{\bar{q}}$ with $\delta_{\bar{q}} := 0.5 \cdot \delta_{\bar{q}}$.

   The update on $\delta_{\bar{q}}$ is necessary. To speed the computation, we also increase (decrease) ξ_J when $d_{\bar{q}}d_{\bar{q}}^* \geq 0$ $(d_{\bar{q}}d_{\bar{q}}^* < 0)$ and similarly for $\xi_{\bar{i}}$, but this is not necessary.

If convergence has not been obtained, the new guesses on $J, \bar{i}, \bar{q}$ are used to solve for the value functions, price functions, law of motion, invariant distribution, and key equilibrium objects as described in Sections B.2.1 and B.2.2.

C Omitted proofs and results [Not for Publication]

This section contains omitted proofs from the two-period model and additional theoretic and quantitative results.

C.1 Two-period model proofs

We first give the two-period model proofs.

C.1.1 The Euler equation

Proof of Proposition 1. The objective function may be written

$$u(c_1) + \beta \left((1 - o_2)u(c_2) + \int_{-\infty}^{J-u(c_2)} (J - \phi)f(\phi)d\phi \right) \tag{34}$$

Using Leibniz's rule,

$$0 = u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} + u(c_2)\frac{-\partial o_2}{\partial b_2} + \frac{\partial(J - u(c_2))}{\partial b_2}(J - \phi)f(\phi)\Big|_{\phi = J - u(c_2)} \right)$$

$$= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} + u(c_2)\frac{-\partial o_2}{\partial b_2} + \frac{\partial(J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right)$$

$$= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} - u(c_2)\frac{\partial F(J - u(c_2))}{\partial b_2} + \frac{\partial(J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right)$$

$$= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta \left((1 - o_2)\frac{\partial u(c_2)}{\partial b_2} - u(c_2)f(J - u(c_2))\frac{\partial(J - u(c_2))}{\partial b_2} + \frac{\partial(J - u(c_2))}{\partial b_2}u(c_2)f(J - u(c_2)) \right)$$

$$= u'(c_1)\frac{\partial c_1}{\partial b_2} + \beta(1 - o_2)u'(c_2)\frac{\partial c_2}{\partial b_2}$$

$$= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\frac{\partial b_2 \frac{n_1}{n_2}}{\partial b_2}$$

$$= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} + b_2 n_1 \frac{\partial n_2^{-1}}{\partial b_2} \right)$$

$$= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} + b_2 n_1 (-1)n_2^{-2}\frac{\partial n_2}{\partial b_2} \right)$$

$$= -\bar{q}u'(c_1) + \beta(1 - o_2)u'(c_2)\left(\frac{n_1}{n_2} - b_2 \frac{n_1}{n_2}\frac{1}{n_2}\frac{\partial n_2}{\partial b_2} \right)$$

$$= -\bar{q}u'(c_1) + \beta\frac{n_1}{n_2}(1 - o_2)u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} \right)$$

$$= -\bar{q}u'(c_1) + \beta\frac{n_1}{n_1(1 - o_2 + i_2)}(1 - o_2)u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} \right)$$

$$= -\bar{q}u'(c_1) + \beta\frac{1 - o_2}{1 - o_2 + i_2}u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} \right)$$

Consequently, the Euler equation reads

$$\bar{q}u'(c_1) = \beta\frac{1 - o_2}{1 - o_2 + i_2}u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} \right). \tag{35}$$

□

C.1.2 Constrained efficiency

Proof of Proposition 2. With no cross-sectional heterogeneity, constrained efficiency with the open economy constraint requires (6). If $\bar{q} = \beta u'(y_2)/u'(y_1)$, then this requires

$$\frac{u'(y_2)}{u'(y_1)} = \frac{u'(c_2)}{u'(c_1)} = \frac{u'(y_2 + b_2 \frac{n_1}{n_2})}{u'(y_1 - \bar{q}b_2)} \tag{*}$$

Evidently, this requires $b_2 = 0$. However, $b_2 = 0$ is not compatible with the government's Euler equation. In particular, at $b_2 = 0$ and at $\bar{q}$, the government Euler equation can be written

$$\frac{u'(y_2)}{u'(y_1)} = \frac{1 - o_2}{1 - o_2 + i_2} \frac{u'(c_2)}{u'(c_1)}. \tag{**}$$

Hence, if $i_2 > 0$, then (*) and (**) cannot simultaneously hold. And in fact, some people will enter (i.e., i_2 is greater than 0) because in the constrained efficient allocation $c_2 = y_2$ for every island and so $J = u(c_2)$ and—given this—some people will move since $F(0) > 0$ (i.e., migration is noisy). Hence, the constrained efficient allocation cannot be supported as an equilibrium.

For the claim that at the constrained efficient allocation governments would strictly prefer to borrow, note the Euler equation at the constrained efficient allocation is not satisfied with

$$u'(y_1)\bar{q} > \beta u'(y_2)\frac{1 - o_2}{1 - o_2 + i_2} \Leftrightarrow 1 > \frac{1 - o_2}{1 - o_2 + i_2}.$$

The way to equate marginal utilities would then be to increase c_1 by borrowing. $\qquad\square$

Proof of Proposition 3. Under these assumptions, the Pareto optimal allocation is $c_1 = y_1$, $c_2 = y_2$, with households moving whenever $\phi < 0$ and staying whenever $\phi > 0$ (with indifference elsewhere)—these conditions are the same for both the open economy (with the specified $\bar{q}$) and closed economy constraints. Note that in contrast to the hypothesis of Proposition 2, inflow rates are assumed to be not differentiable at $b_2 = 0$, which means the Euler equation is not valid at that point.

We will prove the existence of a symmetric open economy equilibrium with $\bar{q} = \beta u'(y_2)/u'(y_1)$ and a closed economy equilibrium with the same price, both of which have $b_2 = 0$ as optimal. We will do so by establishing that at this price $b_2 < 0$ is not optimal, that $b_2 > 0$ is not optimal, and that an optimal choice exists (in which case it must be $b_2 = 0$). This will then support the allocation $(c_1, c_2) = (y_1, y_2)$ (and the migration decisions).

For use below, we note that whenever the derivative $\partial n_2/\partial b_2$ exists, one has

$$\begin{aligned}
\frac{b_2}{n_2}\frac{\partial n_2}{\partial b_2} &= \frac{b_2}{n_2}n_1\left(\bar{i}\mathbf{I}'(u(c_2)) + f(J - u(c_2))\right)u'(c_2)\frac{\partial c_2}{\partial b_2} \\
&= b_2 u'(c_2)\left(\frac{n_1}{n_2}\right)^2\left(\bar{i}\mathbf{I}'(u(c_2)) + f(J - u(c_2))\right)
\end{aligned} \tag{36}$$

Because $\mathbf{I}$ is increasing and f is positive, this has the same sign as b_2.

First we will show that $b_2 < 0$ is not optimal by showing the Euler equation does not hold there. Given no inflows for $b_2 < 0$, borrowing is not optimal because the Euler equation (which is valid everywhere except at $b_2 = 0$) requires

$$\beta \frac{u'(y_2)}{u'(y_1)} = \bar{q} = \beta \frac{u'(c_2)}{u'(c_1)} \underbrace{\frac{1 - o_2}{1 - o_2 + i_2}}_{=1} \underbrace{\left(1 - \frac{b_2}{n_2} \frac{\partial n_2}{\partial b_2}\right)}_{\geq 1 \text{ since } b_2 < 0} \geq \beta \frac{u'(c_2)}{u'(c_1)}. \tag{37}$$

However, with $b_2 < 0$, $c_1 > y_1$ and $c_2 < y_2$, which gives a contradiction.

Now we will show that $b_2 > 0$ is not optimal. The Euler equation in this case reads

$$\beta \frac{u'(y_2)}{u'(y_1)} = \bar{q} = \beta \frac{u'(c_2)}{u'(c_1)} \underbrace{\frac{1 - o_2}{1 - o_2 + i_2}}_{\leq 1} \underbrace{\left(1 - \frac{b_2}{n_2} \frac{\partial n_2}{\partial b_2}\right)}_{\leq 1 \text{ since } b_2 > 0} \leq \beta \frac{u'(c_2)}{u'(c_1)}. \tag{38}$$

However, with $b_2 > 0$, $c_1 < y_1$ and $c_2 > y_2$, which gives a contradiction.

Since $b_2 < 0$ and $b_2 > 0$ are not optimal, all that remains to show is that an optimal choice exists. Without loss of generality, we can restrict the choice set to $b_2 \in [-\delta, \delta]$ for δ arbitrarily small such that every choice is feasible. Then, with a continuous objective function being maximized over a compact set, a maximum exists, which must be $b_2 = 0$. $\qquad\square$

Proof of Proposition 4. Absent cross-sectional heterogeneity, Pareto efficiency in the closed economy dictates that moving should occur if $\phi < 0$ and staying if $\phi > 0$ (with indifference for $\phi = 0$) and that $c_1 = y_1$ and $c_2 = y_2$.

In a symmetric equilibrium, bond market clearing dictates $b_2 = 0$ for all islands. Consequently, $c_2 = y_2$ and $c_1 = y_1$ for all islands. Since there is no cross-sectional heterogeneity in endowments, this means in equilibrium $J = u(c_2)$. Therefore, the reservation cutoff for moving is $\phi = 0$, and each island experiences outflows $o_2 = F(0)$. Because there is no heterogeneity in initial populations, outflows and inflows are the same, so $i_2 = F(0)$ as well. Consequently, if $\bar{q}$ is given by

$$\bar{q} = \beta \frac{1 - o_2}{1 - o_2 + i_2} \frac{u'(c_2)}{u'(c_1)} \left(1 - \frac{b_2}{n_2} \frac{\partial n_2}{\partial b_2}\right) = \beta (1 - F(0)) \frac{u'(y_2)}{u'(y_1)},$$

then $b_2 = 0, c_2 = y_2, c_1 = y_1$ is optimal and constitutes an equilibrium. Moreover, the equilibrium is Pareto optimal.

$\qquad\square$

To prove Proposition 5, we first establish the following lemma:

Lemma 1. *If there are two island types with homogeneous first-period endowments and heterogeneous second-period endowments, then in equilibrium the island with a larger second period endowment has strictly greater second period consumption and strictly borrows.*

Proof. For use below, we first note that

$$\frac{\partial n_2}{\partial b_2} = \frac{n_1 \partial [(1 - F(J - u(c_2)) + \bar{i}\mathbf{I}(u(c_2))]}{\partial b_2}$$

$$= n_1 [-f(J - u(c_2))(-u'(c_2)) + \bar{i}\mathbf{I}'(u(c_2))u'(c_2)]\frac{\partial c_2}{\partial b_2}$$

$$= n_1 u'(c_2)(f(J - u(c_2)) + \bar{i}\mathbf{I}'(u(c_2)))\frac{n_1}{n_2}.$$

Defining

$$g(c_2) := \frac{n_2}{n_1}\frac{\partial n_2}{\partial b_2}$$

$$= \frac{n_2}{n_1}\left(n_1 u'(c_2)(f(J - u(c_2)) + \bar{i}\mathbf{I}'(u(c_2)))\frac{n_1}{n_2}\right)$$

$$= n_1 u'(c_2)(f(J - u(c_2)) + \bar{i}\mathbf{I}'(u(c_2))),$$

we have the Euler equation as

$$\bar{q}u'(c_1) = \beta\frac{1 - o_2}{1 - o_2 + i_2}u'(c_2)\left(1 - \frac{b_2}{n_2}\frac{n_1}{n_2}g(c_2)\right)$$

and note that $g(c_2) \geq 0$.

Now, use tildes to denote variables associated with the island having higher endowments in the second period.

For contradiction, suppose that $\tilde{b}_2 \geq 0$. Then $b_2 \leq 0$ and $\tilde{c}_2 > c_2$ and $\tilde{c}_1 < c_1$. Because $\tilde{c}_2 > c_2$, $\tilde{i}_2$ is larger and $\tilde{o}_2$ is smaller than i_2 and o_2, respectively. Consequently,

$$\frac{\beta}{\bar{q}}\frac{1 - \tilde{o}_2}{1 - \tilde{o}_2 + \tilde{i}_2} = \frac{\beta}{\bar{q}}\frac{1}{1 + \frac{\tilde{i}_2}{1 - \tilde{o}_2}} < \frac{\beta}{\bar{q}}\frac{1}{1 + \frac{i_2}{1 - o_2}} = \frac{\beta}{\bar{q}}\frac{1 - o_2}{1 - o_2 + i_2}$$

From the Euler equations, this implies

$$\frac{u'(\tilde{c}_2)}{u'(\tilde{c}_1)}\left(1 - \frac{n_1}{\tilde{n}_2}\frac{\tilde{b}_2}{\tilde{n}_2}g(\tilde{c}_2)\right) > \frac{u'(c_2)}{u'(c_1)}\left(1 - \frac{n_1}{n_2}\frac{b_2}{n_2}g(c_2)\right).$$

Because $\tilde{b}_2 \geq 0$ and $b_2 \leq 0$ and $g \geq 0$, one necessarily has

$$\frac{u'(\tilde{c}_2)}{u'(\tilde{c}_1)} > \frac{u'(c_2)}{u'(c_1)}.$$

or

$$\frac{u'(c_1)}{u'(\tilde{c}_1)} > \frac{u'(c_2)}{u'(\tilde{c}_2)}.$$

Since the islands have the same initial endowment and $\tilde{b}_2 \geq 0$, $\tilde{c}_1 < c_1$ and so the lefthand side is less than 1. That means the righthand side must also be less than 1, which implies $c_2 > \tilde{c}_2$. But $\tilde{y}_2 > y_2$ and $\tilde{b}_2 \geq b_2$, so clearly $\tilde{c}_2 \geq c_2$. This contradicts $\tilde{b}_2 \geq 0$. Therefore, $\tilde{b}_2 < 0$.

Now, with $\tilde{b}_2 < 0$, one has $b_2 > 0$. From the Euler equation,

$$\frac{\beta}{\bar{q}} \frac{u'(c_2)}{u'(c_1)} = \frac{1 - o_2 + i_2}{1 - o_2} \left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)^{-1}$$

This implies

$$\frac{u'(\tilde{c}_2)}{u'(\tilde{c}_1)} \frac{1 - \tilde{o}_2}{1 - \tilde{o}_2 + \tilde{i}_2} \left(1 - \frac{n_1}{\tilde{n}_2} \frac{\tilde{b}_2}{\tilde{n}_2} g(\tilde{c}_2)\right) = \frac{u'(c_2)}{u'(c_1)} \frac{1 - o_2}{1 - o_2 + i_2} \left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)$$

$$\frac{\frac{u'(\tilde{c}_2)}{u'(\tilde{c}_1)}}{\frac{u'(c_2)}{u'(c_1)}} = \frac{\frac{1-o_2}{1-o_2+i_2} \left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)}{\frac{1-\tilde{o}_2}{1-\tilde{o}_2+\tilde{i}_2} \left(1 - \frac{n_1}{\tilde{n}_2} \frac{\tilde{b}_2}{\tilde{n}_2} g(\tilde{c}_2)\right)}$$

$$\frac{\frac{u'(\tilde{c}_2)}{u'(c_2)}}{\frac{u'(\tilde{c}_1)}{u'(c_1)}} = \frac{\frac{1}{1+\frac{i_2}{1-o_2}} \left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)}{\frac{1}{1+\frac{\tilde{i}_2}{1-\tilde{o}_2}} \left(1 - \frac{n_1}{\tilde{n}_2} \frac{\tilde{b}_2}{\tilde{n}_2} g(\tilde{c}_2)\right)}$$

Suppose for contradiction that $c_2 \geq \tilde{c}_2$. Then the numerator of the lefthand side is weakly greater than 1. Since $\tilde{b}_2 < 0$, $\tilde{c}_1 > c_1$ so the denominator of the lefthand side is strictly less than 1. Therefore, the lefthand side is strictly greater than 1. Additionally, since $c_2 \geq \tilde{c}_2$, $i_2/(1 - o_2) \geq \tilde{i}_2/(1 - \tilde{o}_2)$. Consequently, $1/(1 + i_2/(1 - o_2)) \leq 1/(1 + \tilde{i}_2/(1 - \tilde{o}_2))$. So, the only way the equality could hold is if

$$\frac{\left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)}{\left(1 - \frac{n_1}{\tilde{n}_2} \frac{\tilde{b}_2}{\tilde{n}_2} g(\tilde{c}_2)\right)} > 1$$

However, since $g \geq 0$ and $\tilde{b}_2 < 0$, this is impossible. Therefore, it must be the case that $c_2 < \tilde{c}_2$.
□

Proof of Proposition 5. Consider two islands of different types. Let the allocations etc. associated with the one having higher endowments in the second period be denoted with tildes.

By Lemma 1, $\tilde{c}_2 > c_2$, $\tilde{b}_2 < 0$, and consequently $b_2 > 0$. So, $\tilde{i}_2$ is larger and $\tilde{o}_2$ is smaller than i_2 and o_2, respectively. Consequently,

$$\frac{1 - \tilde{o}_2}{1 - \tilde{o}_2 + \tilde{i}_2} = \frac{1}{1 + \frac{\tilde{i}_2}{1-\tilde{o}_2}} < \frac{1}{1 + \frac{i_2}{1-o_2}} = \frac{1 - o_2}{1 - o_2 + i_2}$$

Now, with $\tilde{b}_2 < 0$, one has $b_2 > 0$. From the Euler equation,

$$\frac{\bar{q}}{\beta} \frac{u'(c_1)}{u'(c_2)} = \frac{1 - o_2}{1 - o_2 + i_2} \left(1 - \frac{n_1}{n_2} \frac{b_2}{n_2} g(c_2)\right)$$

So, the lefthand side, and hence the marginal rate of substitution, will differ across islands if

$$\frac{1-\tilde{o}_2}{1-\tilde{o}_2+\tilde{i}_2}\left(1-\frac{n_1}{\tilde{n}_2}\frac{\tilde{b}_2}{\tilde{n}_2}g(\tilde{c}_2)\right) \neq \frac{1-o_2}{1-o_2+i_2}\left(1-\frac{n_1}{n_2}\frac{b_2}{n_2}g(c_2)\right)$$

With $\tilde{b}_2 < 0$ and $b_2 > 0$, this must be the case, and, in particular,

$$\frac{u'(\tilde{c}_1)}{u'(\tilde{c}_2)}\frac{\bar{q}}{\beta} = \frac{1-\tilde{o}_2}{1-\tilde{o}_2+\tilde{i}_2}\left(1-\frac{n_1}{\tilde{n}_2}\frac{\tilde{b}_2}{\tilde{n}_2}g(\tilde{c}_2)\right) > \frac{1-o_2}{1-o_2+i_2}\left(1-\frac{n_1}{n_2}\frac{b_2}{n_2}g(c_2)\right) = \frac{u'(c_1)}{u'(c_2)}\frac{\bar{q}}{\beta}$$

From the Euler equations,

$$\frac{u'(\tilde{c}_1)}{u'(\tilde{c}_2)} > \frac{u'(c_1)}{u'(c_2)}.$$

Hence, the equilibrium is not constrained efficient because consumption growth is not equated across individuals (even excluding individuals who migrate).

$\square$

C.2 The intermediary's problem

We now give the results used to simplify the intermediary's problem.

For the intermediary problem, we consider a slightly more general formulation from that in the main text, allowing for a proportional tax on risk-free bond holdings τ. Consequently, the intermediary's problem is

$$W(B, M) = \max_{D, B', M'} D + \bar{q}(1-\tau)W(B', M')$$

$$\text{s.t. } D + \bar{q}(1+\tau)B' + \int Q(b', n', z, \omega)dM'(b', n', z, \omega) = \tag{39}$$

$$B + \int \sum_z \mathbb{P}(z|z_{-1})(1 - d(b, n, z))(bn)dM(b, n, z_{-1}, \omega)$$

We will first prove the following, with Proposition 6 following as an immediate corollary:

Proposition 9. *If prices satisfy*

$$q(b', n', z, \omega) = \bar{q}(1+\tau)\mathbb{E}_{z'|z}(1 - d(b', n', z', \omega)) \tag{40}$$

and if

$$\overline{B} = (1 - d(b, n, z, \omega))(bn)\mu(db, dn, dz, 0, d\omega) \tag{41}$$

then there exist prices and an optimal policy M', with M' invariant, such that contract markets and the risk-free bond market clear and zero profits obtain (provided the other equilibrium conditions are met).

Proof of Proposition 9. To characterize the intermediary's problem, first consider its first-order conditions (FOCs). The FOC for B' is trivially satisfied for any B'.

Suppress dependence on ω—it enters the same way as z everywhere, but with $\omega' = \omega$ always. Equivalently, expand the definition of z to a pair giving TFP and ω. From the FOC for $M(b', n', z)$ one must have

$$Q(b', n', z) = \bar{q}(1 + \tau) \sum_{z'} \mathbb{P}(z'|z)(1 - d(b', n', z'))(b'n'). \tag{42}$$

Replacing $Q(b', n', z)$ with $q(b', n', z)b'n'$ and simplifying for $b' \neq 0$, this becomes

$$q(b', n', z) = \bar{q}(1 + \tau)\mathbb{E}_{z'|z}(1 - d(b', n', z')). \tag{43}$$

$$q(b', n', z) = \bar{q}(1 + \tau)\mathbb{E}_{z'|z}(1 - d(b', n', z')). \tag{44}$$

Hence, in equilibrium, if prices satisfy the above equations, then the indifferent is intermediary over all feasible contract, bond holding, and dividend distribution schemes.

Contract market clearing as stated in (21) dictates what M' must be as a function of the distribution μ, household policies b', d, and law of motion $\dot{n}$. Additionally, $B'(\overline{B}, M)$ must equal $\overline{B}$ for risk-free bond market clearing. The other equilibrium conditions pertaining to intermediary are that (1) M must be invariant ($M' = M$) and (2) that the intermediary makes zero profits. To satisfy the first, we take $M = M'$.

To satisfy zero profits, first, note that for any $\mathcal{B} \cap \{0\} = \emptyset$ and any integrable function $g(b', n', z')$,

$$\sum_z \int \mathbb{P}(z'|z)g(b', n', z')\mathbf{1}_{[(b',n')\in\mathcal{B}\times\mathcal{N}]}M'(db', dn', z)$$

$$= -\int g(b', n', z')\mathbb{P}(z'|z)\mathbf{1}_{[(b'(b,n,z,0),\dot{n}(b,n,z,0))\in\mathcal{B}\times\mathcal{N}]}(1 - d(b, n, z))d\mu(b, n, z, 0) \tag{45}$$

$$= -\int_{\mathcal{B}\times\mathcal{N}} g(b', n', z')\mu(db', dn', z', 0)$$

which follows from μ being invariant and $\mathbb{P}(z'|z)\mathbf{1}_{[(b'(b,n,z,0),\dot{n}(b,n,z,0))\in\mathcal{B}\times\mathcal{N}]}(1 - d(b, n, z))$ being the transition probability from b, n, z to b', n', z' as long as $b' \neq 0$ (for $b' = 0$, there is an additional arrival coming from cities that transition from $f = 1$ to $f = 0$).

Now we need to show the intermediary makes zero profits supposing that the other conditions hold. Consider the intermediary's budget constraint assuming zero dividends and bond market clearing:

$$(1 - \bar{q}(1 + \tau))\overline{B}$$
$$= \int Q(b', n', z)dM'(b', n', z) - \int \sum_z \mathbb{P}(z|z_{-1})(1 - d(b, n, z))(bn)dM(b, n, z_{-1}). \tag{46}$$

If this holds, then zero dividends is feasible and hence optimal. Then one has

$$
\begin{aligned}
& \int Q(b', n', z) dM'(b', n', z) \\
&= \bar{q}(1+\tau) \sum_{z'} \int (1 - d(b', n', z'))(b'n') \mathbb{P}(z'|z) dM'(b', n', z) \\
&= -\bar{q}(1+\tau) \sum_{z'} \int (1 - d(b', n', z'))(b'n') \mu(db', dn', z', 0) \\
&= -\bar{q}(1+\tau) \int (1 - d(b', n', z'))(b'n') \mu(db', dn', dz', 0)
\end{aligned}
\tag{47}
$$

where the first equality follows from equilibrium pricing of $Q(b', n', z)$, the second from (45), and the third is just different notation.

From the hypothesis that $\overline{B} = \int (1-d(b, n, z))(bn)\mu(db, dn, dz, 0)$ and (47), we have $\int Q(b', n', z) dM'(b', n', z) = -\bar{q}(1+\tau)\overline{B}$. So, (46) will hold as long as

$$
\overline{B} = -\int \sum_{z} \mathbb{P}(z|z_{-1}) \left((1 - d(b, n, z))bn \right) dM(b, n, z_{-1})
\tag{48}
$$

From M being invariant, we can replace M with M' so this condition is equivalent to

$$
\overline{B} = -\int \sum_{z'} \mathbb{P}(z'|z) \left((1 - d(b', n', z'))b'n' \right) dM'(b', n', z)
\tag{49}
$$

Rewriting and using (45), this is

$$
\begin{aligned}
\overline{B} &= -\sum_{z'} \int \mathbb{P}(z'|z) \left((1 - d(b', n', z'))b'n' \right) dM'(b', n', z) \\
&= -\sum_{z'} \left(-\int \left((1 - d(b', n', z'))b'n' \right) \mu(db', dn', z', 0) \right) \\
&= \int \left((1 - d(b', n', z'))b'n' \right) \mu(db', dn', dz', 0)
\end{aligned}
\tag{50}
$$

which holds from the hypothesis.

$\square$

Because the $q(b', n', z)$ prices are pinned down by $\bar{q}(1+\tau)$, exactly what $\bar{q}$ and τ are is irrelevant for them. However, given the equilibrium value of $\bar{q}(1+\tau)$ and a τ, one immediately has $\bar{q}$. Note that τ is only used to finance bailouts, and in this case it is determined by how many resources are needed in equilibrium to bail out cities (as implied by the government budget constraint).

Proof of Proposition 6. The result follows from Proposition 9 taking $\tau = 0$. $\square$

C.3 The centralized problem

We now give the proof that the model can be centralized at a local level.

Proof of Proposition 7. Consider an arbitrary choice (c, g, d, b') in the centralized problem. At this choice, household and firm optimization and market clearing will be satisfied if we take $r = u_h/u_c$, $w = (1 - \bar{\kappa}\hat{d})z$, $L^d = \dot{n}$, $h = \bar{H}/\dot{n}$, π solving $\dot{n}\pi = (1 - \bar{\kappa}\hat{d})z\dot{n} - w\dot{n} + r\overline{H}$, and T solving $\dot{n}c + rh\dot{n} = w\dot{n} + \pi\dot{n} - T\dot{n}$ (see equation 15). Eliminating profit from the consumption equation, one has $c = (1 - \bar{\kappa}\hat{d})z - T$ (and clearly $h = \bar{H}/\dot{n}$). Hence, the flow utility associated with this allocation is $u((1 - \bar{\kappa}\hat{d})z - T, g, \bar{H}/\dot{n}, \omega)$, which according to (16) is the same as $U(g, T, \hat{d}, \dot{n}, z, \omega)$. Hence, an arbitrary choice delivers $u(c, g, \bar{H}/\dot{n}, \omega)$ flow utility in the centralized problem, which—when supported using the above prices and allocations—is the same as $U(g, T, \hat{d}, \dot{n}, z, \omega)$. Moreover, at these prices and allocations, b', d, T is feasible for the government as guaranteed by Walras's law.[33] Then, since the centralized planner is maximizing the same flow utility, discounting, and expectations as the government, optimal choices for the centralized planner must simultaneously solve the government's problem. Hence, the optimal choices from the centralized problem can be supported as a decentralized equilibrium using the prices r, w, firm allocation L^d, household housing consumption allocation h, firm profits π, and taxes T. □

C.4 The full-model Euler equation

We now give the derivation of the full-model's Euler equation.

Proof of Proposition 8. Fix an ω (weather) and suppress dependence on it.

With repayment strictly preferred, default rates are zero the next period, $q(b', \dot{n}, z) = \bar{q}$. Also, note that for the same reason, S^N inherits the differentiability of S. Additionally, the continuation utility of the decentralized problem $S^N(\dot{b}, n', z)$—to avoid confusion, we are using n' as the state rather than $\dot{n}$—may be written

$$\beta\mathbb{E}_{\phi', z'|z} \max \left\{ S^N \left(\frac{b'n'}{\dot{n}(b', n', z', 0)}, \dot{n}(b', n', z', 0), z' \right), J - \phi' \right\} dF(\phi'), \tag{51}$$

which is valid for any b' locally. Equivalently,

$$\beta\mathbb{E}_{z'|z} \left(\int_{R(b', n', z', 0)}^{\infty} S^N \left(\frac{b'n'}{\dot{n}(b', n', z', 0)}, \dot{n}(b', n', z', 0), z' \right) dF(\phi'|z') + \int_{-\infty}^{R(b', n', z', 0)} (J - \phi') dF(\phi'|z') \right). \tag{52}$$

---

[33]One can verify this easily. For instance, if $f = d = 0$, then the centralized budget constraint reads $\dot{n}c + \dot{n}^{1-\eta}g + qb'\dot{n} = z\dot{n} + \dot{b}\dot{n}$. Using $c = (1 - \bar{\kappa}\hat{d})z - T$ as was found above with $\hat{d} = 0$ to eliminate c, one finds $\dot{n}^{1-\eta}g + qb'\dot{n} = T\dot{n} + \dot{b}\dot{n}$, which is the government's budget constraint.

Using Leibniz's rule and noting $S^N = J - R$, the derivative with respect to b' is

$$\beta\mathbb{E}_{z'|z}(1 - F(R(b', n', z', 0)|z'))\left(S_b^N\left(\frac{n'}{\dot{n}} + b'n'\frac{\partial\dot{n}^{-1}}{\partial b'}\right) + S_{\dot{n}}^N\frac{\partial\dot{n}}{\partial b'}\right). \tag{53}$$

where the arguments for S_b^N and $S_{\dot{n}}^N$ are $(\frac{b'n'}{\dot{n}(b',n',z',0)}, \dot{n}(b', n', z', 0), z')$ and the argument for $\dot{n}$ is $(b', n', z', 0)$. The envelope condition gives $S_b^N(b', n', z) = u_c$. Also, $\partial\dot{n}^{-1}/\partial b' = -\dot{n}^{-2}\dot{n}_{b'}$. Plugging these in,

$$\beta\mathbb{E}_{z'|z}(1 - F(R(b', n', z', 0)|z'))\left(u_c\left(\frac{n'}{\dot{n}} - b'n'\frac{1}{\dot{n}^2}\frac{\partial\dot{n}}{\partial b'}\right) + S_{\dot{n}}^N\frac{\partial\dot{n}}{\partial b'}\right)$$

$$\beta\mathbb{E}_{z'|z}(1 - F(R(b', n', z', 0)|z'))\left(u_c\frac{n'}{\dot{n}}\left(1 - \frac{b'}{\dot{n}}\frac{\partial\dot{n}}{\partial b'}\right) + S_{\dot{n}}^N\frac{\partial\dot{n}}{\partial b'}\right) \tag{54}$$

So, the FOC for b' is

$$u_c\bar{q} = \beta\mathbb{E}_{z'|z}(1 - F(R(b', n', z', 0)|z'))\left(u_c\frac{n'}{\dot{n}}\left(1 - \frac{b'}{\dot{n}}\frac{\partial\dot{n}}{\partial b'}\right) + S_{\dot{n}}^N\frac{\partial\dot{n}}{\partial b'}\right) \tag{55}$$

Using the i' and o' notation from the hypothesis, note that $\dot{n}(b', n', z', 0)/n'$ is $1+i'-o'$ and $F(R(b', n', z', 0)|z') = o'$. Hence,

$$u_c\bar{q} = \beta\mathbb{E}_{z'|z}\left[\left(\frac{1 - o'}{1 + i' - o'}\right)\left(1 - \frac{b'}{\dot{n}}\frac{\partial\dot{n}}{\partial b'}\right)u_c + (1 - o')S_{\dot{n}}^N\frac{\partial\dot{n}}{\partial b'}\right]. \tag{56}$$

$\square$

C.5 Modifications necessary for bailouts

In having bailouts financed by a bailout tax, the intermediary (who is the only purchaser of the risk-free bond) has a new budget constraint:

$$D + \bar{q}(1+\tau)B' + \int Q(b', n', z, \omega)dM'(b', n', z, \omega) = B + \int\sum_z\mathbb{P}(z|z_{-1})(1 - d(b, n, z))(bn)dM(b, n, z_{-1}, \omega). \tag{57}$$

This changes the equilibrium bond prices in Proposition 6 to

$$q(b', n', z, \omega) = \bar{q}(1 + \tau)\mathbb{E}_{z'|z}(1 - d(b', n', z', \omega)), \tag{58}$$

but the condition $\overline{B} = \int(1 - d(b, n, z, \omega))bn\mu(db, dn, dz, 0, \omega)$ is unchanged. The federal government's budget constraint holds provided

$$\tau\bar{q}(-\overline{B}) = \int\mathfrak{b}(\mathbf{x})\dot{n}(\mathbf{x})\pi_b\mathbf{1}[S^N(\dot{b}, \dot{n}, z, \omega) < S^D(\dot{b}, \dot{n}, z, \omega)]d\mu(\mathbf{x}). \tag{59}$$

C.6 Quantitative testing of indeterminacy

To test for indeterminacy, we proceed by drawing 100 random starting guesses for J, $\bar{i}$, and $\bar{q}$ uniformly distributed about $\pm 50\%$ of the benchmark's computed equilibrium values. (For the defini-

tion of $\bar{i}$, see Appendix B.2.) We then compute the implied equilibrium solution. Figure 9 shows a scatter plot of the guesses in three-dimensional space, and also reveals that they all converge to the same solution (up to small numerical differences). This suggests that the equilibrium is unique in a wide-range about the computed benchmark equilibrium.
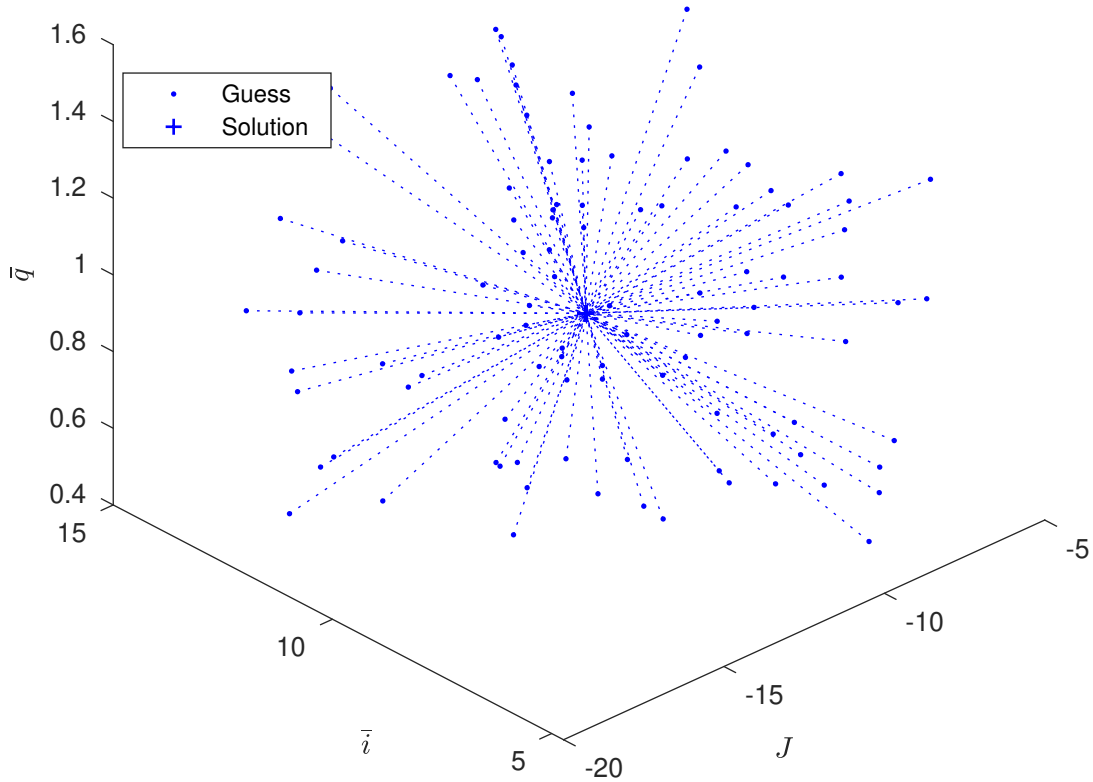

Figure 9: Quantitative testing of indeterminacy

C.7   Proximity to borrowing limits for a higher risk-free rate

Analogously to Figure 4, Figure 10 shows the proximity of cities relative to their borrowing limits after a 2.5% increase in the risk-free interest rate. The largest cities are the ones that deleverage first in response to the interest rate increase.

C.8   Detroit's optimal path after its population stabilizes

Figure 11 presents Detroit's optimal response to shocks after allowing the population to stabilize (which takes almost 200 years and entails the population shrinking by 85% relative to 1987). Overall, the response is similar to that seen in Figure 7 with a large deleveraging occurring in 2008 with cuts to expenditures that outweigh the tax cuts.
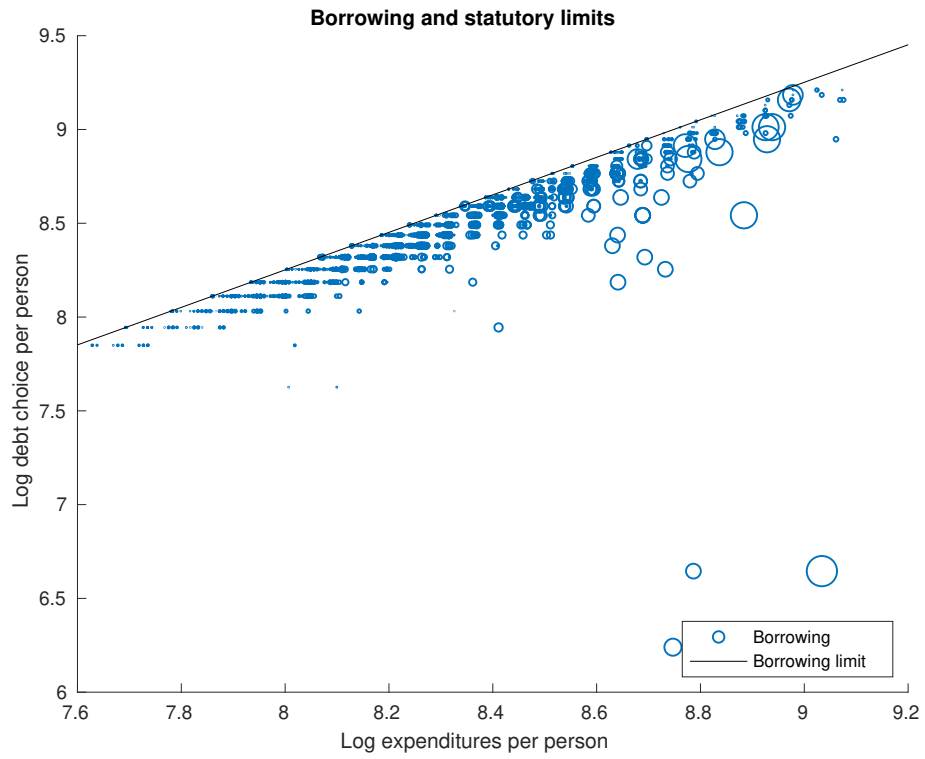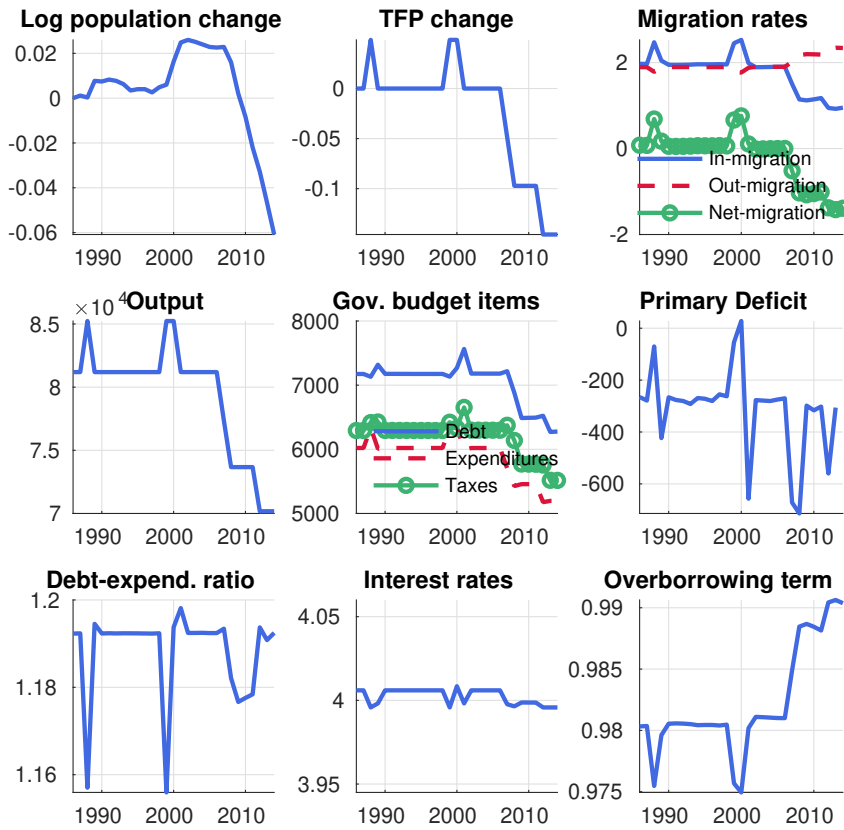
Borrowing and statutory limits

Note: A 1% subsample is used (to limit file size).

Figure 10: Model distribution of cities relative to their borrowing limit after an interest rate increase

C.9 Partial equilibrium results

Table 10 gives the partial equilibrium results (i.e., holding $\bar{q}$ fixed) for the counterfactuals.

Figure 11: The optimal response to Detroit's productivity shocks

	Bench.	Pct. change from bench.		
		$\bar{q}^{-1}\uparrow$	$\delta=\infty$	$\pi_b=\frac{1}{2}$
General equilibrium effects	—	No	No	No
Aggregate measures				
Output per person	68.2	0.2	-0.2	0.2
Consumption per person	62.9	0.1	-0.2	0.2
Consumption per person s.d.	16.0	-0.0	1.0	0.7
Expenditures per person	5.06	-0.0	-0.3	0.2
Taxes per person	5.30	1.4	-0.3	0.2
Debt per person	6.08	-16.0	-0.2	0.7
Utility conditional on moving (J)	-12.9	0.2	-0.0	0.0
Weather	0.562	0.1	-0.2	0.1
Interest rate (%)	4.00	62.6	-0.0	1.4
Tax for financing bailouts (%)	0.000	0.000*	0.000*	0.057*
Average across cities				
Default rate $\times 100$	0.049	-13.0	12.2	197.7
Bailout rate $\times 100$	0.000	0.000*	0.000*	0.074*

Note: The benchmark financial variables are in thousands of dollars; non-benchmark are values are percent deviation from the benchmark unless marked by *. The experiments are as follows: $\bar{q}^{-1}\uparrow$ increases the risk-free interest rate by 2.5 percentage points; $\delta=\infty$ removes the borrowing limit; $\pi_b=\frac{1}{2}$ is anticipated bailouts where bailouts occur 50% of the time.

Table 10: Counterfactuals in partial equilibrium