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Idea Diffusion and Property Rights

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Idea Diffusion and Property Rights

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Abstract

We study innovation and diffusion of technology at the industry level. We derive an industry's evolution, from birth to its maturity, and we characterize how diffusion affects the incentive to innovate. The model implies that protection of innovators should be only partial due to the matching externality in the meetings in which idea transfers take place. The model also shows that enhancing idea diffusion is socially beneficial and can generate industry overtaking patterns endogenously. We fit the model to the early experiences of the U.S. automobile and personal computer industries and quantify the theoretical predictions.

1   Introduction

Innovation and diffusion are fundamental drivers of technological progress and long-run growth. An innovation cannot fulfill its potential without being widely adopted, but rapid diffusion and imitation may reduce the incentive to innovate. In this paper, we study the interplay between innovation and diffusion in a competitive industry setting, and discuss welfare and policy implications.

The model features an industry with a fixed demand curve for a homogeneous product and a group of zero measure potential producers. An innovation or "idea" enables an agent to produce the good at zero cost subject to a capacity constraint. At the outset, agents decide whether to pay a sunk cost to innovate. Some will do so immediately; others may consider innovating later, or wait to imitate an innovation.

Imitation occurs in random pairwise meetings between those who have the idea and those who do not. Imitation is costless, but the imitator may have to pay a fee to the idea seller and the fee is determined by the latter's bargaining share.

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We study two regimes regarding the payment for ideas. In Regime 1, imitators *cannot* resell ideas to other imitators. A potential adopter can copy an idea from an imitator but the fee goes to the idea's original innovator and not to the imitator – this scenario is typically seen in patent licensing or franchising. In Regime 2, by contrast, imitators *can* resell ideas to other imitators and keep the proceeds, a scenario which is often relevant for non-patented know-how.

Our model leads to the following findings. First, under either regime, innovators enter the industry only at the beginning and the number of imitators then follows an S-shaped logistic diffusion curve over time. More innovators enter in Regime 1 or when idea sellers' bargaining share is larger, resulting in faster industry growth.

Second, socially optimal compensation for innovators should be only partial. Innovators do generate positive knowledge spillovers, but they also generate a matching externality, and from the welfare viewpoint they should be compensated, but not fully. Moreover, the socially optimal bargaining share of idea sellers is larger in Regime 2 where innovators collect the payoff of ideas partly indirectly.

Third, differences in idea diffusion speed can explain industry overtaking patterns. Specifically, our model shows that a lower diffusion speed may encourage entry of innovators and raise initial industry capacity, but that it lowers imitation and leads to slower growth of capacity. In reality, diffusion speed may vary across locations due to differences in laws and regulations restricting entry of imitators. In the U.S., for example, some states enforce non-compete contracts that restrict labor turnover and idea diffusion, while other states do not; Saxenian (1994), Gilson (1999) and Franco and Mitchell (2008) point out that the enforcement of non-compete contracts in Massachusetts but not in California accounted for the overtaking of venture activity on Route 128 by that in Silicon Valley. Our model generates such overtaking endogenously and shows that restricting diffusion reduces welfare.

We fit the model to the early experiences of the U.S. automobile and personal computer industries, both of which show S-shaped growth in the number of producers in the period before the shakeout, a pattern shared by many industries. We thus add to the literature on industry life cycles – e.g., Gort and Klepper (1982), Jovanovic and MacDonald (1994), Klepper (1996), Filson (2001), and Hayashi et al. (2017).

Those studies focus on explaining the shakeout of firms, while our study explains the expansion of firm numbers prior to the shakeout. We find that the auto and the PC industries both face highly elastic demands, under which entry of imitators can drive prices down only slowly. Because this encourages innovation and exacerbates the matching externality, the socially optimal bargaining share of idea sellers should be low for both industries, and lower for the more price elastic PC. We also find that in both industries, restricting diffusion would forgo too much free imitation and reduce social welfare.

In our model, random meetings between agents who have ideas and those who do not give rise to a logistic diffusion process. This is consistent with prior work that features logistic diffusion curves in the technology diffusion literature (e.g., Griliches

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1957, Mansfield 1961, Bass 1969, 2004, Young 2009),[1] as well as in the epidemics studies (e.g., Atkeson 2020, Garibaldi, Moen, and Pissarides 2020, among many applications of the SIR model to the spread of the COVID-19 disease). And the quadratic matching function underlying the logistic diffusion was recently studied by Lauermann, Nöldeke, and Tröger (2020).

We add to several other strands of the literature. First, the work on competitive innovation. Boldrin and Levine (2008) among others provide evidence that competitive innovation is pervasive in history and in modern day markets. They point out that in both theory and practice, limited supply due to capacity constraints would keep the price of a new good above the marginal cost of production over a sustained period of time. This generates competitive rents and hence provides incentives for innovation. They consider a single innovator's entry decision in a market where the number of imitators grows at a constant rate. By contrast, our model endogenizes the entry number of innovators and generates S-shaped growth in the number of imitators. We also consider compensation from imitators to innovators for transferring intangible ideas, and our policy implications are different.

Second, our finding that protection of innovators should be only partial agrees with findings in some recent papers on aggregate growth. For example, Hopenhayn and Shi (2020) show that due to matching congestion, the growth-maximizing bargaining share of innovators is sensitive to the parameters in the meeting function. Benhabib, Perla, and Tonetti (2021) show that innovators' licensing income becomes highly elastic with regard to the license price when innovators' bargaining power is too strong and that this can lower licensing income, the return to innovation, and growth. These models compare aggregate growth rates at steady states whereas our model studies transitional dynamics of industry evolution and two policy-relevant regimes concerning idea resale.

In our model, innovation generates a payoff that depends partly on the use of the idea in production, and partly on the value the idea yields when it is sold. Idea sales occur in bilateral meetings and our model relates to models in which agents search for a production partner after one has invested, such as Burdett and Coles (2001), Mailath, Samuelson, and Shaked (2000) and Nöldeke and Samuelson (2015). In these models, payoffs in a match depend on partners' investments and this affects investment incentives.

In the model, owners of ideas use them to compete in the product market and thus the flow value of an idea depends on how many others use it. Manea (2021) also assumes ideas are sold in bilateral meetings and uses bargaining to allocate rents, but in his model the flow value of an idea to its user does not depend on how many others have it or use it.

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[1] Our model focuses on the diffusion process driven by information spillovers from prior to future adopters, which has been a classic approach for studying diffusion in the literature (see Young 2009 for a review). There are also models where diffusion is driven by falling prices of inputs; e.g., David (1968) and Manuelli and Seshadri (2014).

The paper is organized as follows. Section 2 lays out the model and Section 3 characterizes the equilibrium. Section 4 conducts welfare analysis, and Section 5 fits the model to data from the U.S. automobile and personal computer industries. Section 6 provides additional discussions, and Section 7 concludes. The proofs of model propositions and the robustness checks of empirical studies are in the Appendix.

2   Model

Consider a competitive market in continuous time. There is a measure N of potential producers. At date 0, a measure k_0 who we call "innovators," each invest an amount c in an innovation that results in the ability to produce one unit of a new good each period at zero cost. They then immediately become producers. After that, the innovation spreads to others. At any date $t \geq 0$ the measure of producers is k_t, and the remaining $N - k_t$ agents are "outsiders." We normalize outsiders' earnings to zero and denote u_t as an outsider's option value at date t for entering the industry in the future.

The total output of the homogeneous good is k_t, and the product price is

$$p_t = A k_t^{-\beta}, \tag{1}$$

where A is a market size parameter and $\beta > 0$ is the inverse demand elasticity.

Two types of producers.— All producers have an idea and all are equally productive,[2] but some are "innovators" while the others are "imitators." An innovator has paid a direct cost c to invent an idea. An imitator who at date t has copied a producer's idea, has paid a fee equal to

$$F_t = \alpha \omega_t, \tag{2}$$

where ω_t is the value of becoming an imitator at t. The parameter $\alpha \in [0, 1]$ is an idea seller's bargaining share.[3]

2.1   Diffusion process

Diffusion occurs through random pairwise meetings between the k_t producers and the $N - k_t$ outsiders in which outsiders learn and imitate an innovation. Because each agent is too small to influence the product price in the competitive market,

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[2]Section 6.2 shows that allowing firms to have heterogeneous production capacity would not change the analysis.

[3]This bargaining protocol differs from Nash bargaining in which the innovator and imitator would split the joint surplus $\omega_t - u_t$ from the idea transfer. This alternative bargaining is easier to enforce than a Nash bargain because the courts need to know only ω_t and not the imitators' outside options. Section 6.3 shows that α coincides with the Nash bargaining share in a limiting version of the model in which $N \to \infty$.

an innovator always weakly prefers selling the idea to imitators to earn additional revenues given $\alpha \geq 0$.

The meeting function is assumed to be quadratic, and each meeting results in a new producer. An outsider can also enter as an innovator after date 0, but Proposition 1 will show that no one will want to do so. Thus, for $t > 0$ meetings are the only way that agents will in equilibrium become producers, and the number of producers then evolves as

$$\frac{dk_t}{dt} = \gamma k_t \left(N - k_t\right), \tag{3}$$

where $\gamma > 0$ is a meeting efficiency parameter. The solution to (3) is

$$k_t = \frac{N e^{\gamma N t}}{e^{\gamma N t} + \frac{N}{k_0} - 1}. \tag{4}$$

Figure 1 shows that the estimated logistic diffusion model (cf. Eq. (4)) matches the time paths of firm numbers well for U.S. automobile and PC industries.
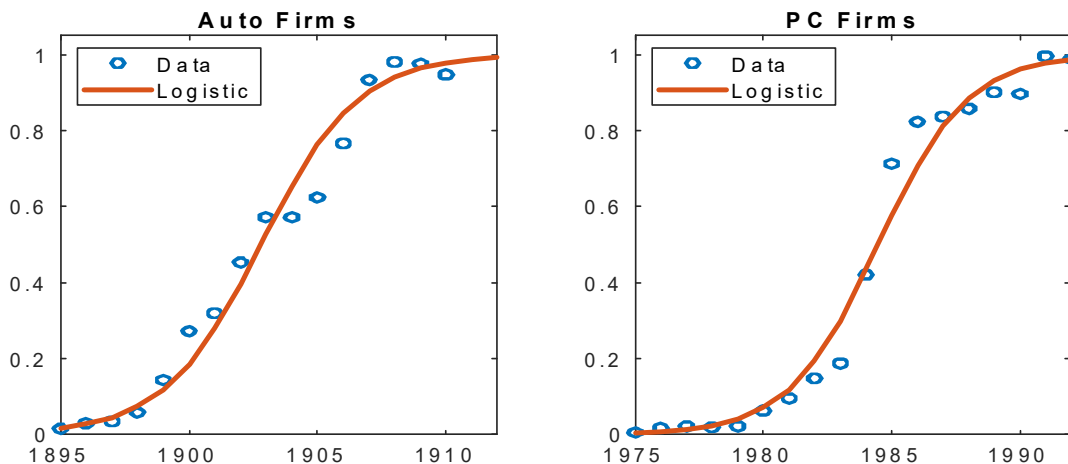

Fig. 1. Logistic Diffusion: Fitting Time Paths of Firm Numbers

2.2 Two regimes

We shall analyze two regimes that differ in how much revenue innovators get from idea sales.

Regime 1: Imitators cannot resell ideas

In Regime 1, the original innovators receive all of their ideas' sale revenues; at each date they are divided among the innovators. While an imitator may have learned an innovation from any incumbent producer, he has to pay the idea's *original* innovator. This type of idea transfer often occurs under franchising or patents that do not allow sublicensing.

Since an imitator cannot resell the innovation, his only revenue comes from selling the good, and his value ω_t satisfies

$$r\omega_t = p_t + \frac{d\omega_t}{dt}, \tag{5}$$

where r is the interest rate.

An innovator receives revenues from selling both the good and the idea. We will prove that at equilibrium, innovators only enter at date 0. Accordingly, the number of ideas sold at t is $\gamma k_t (N - k_t)$ and the total date-t revenue from these sales, $\gamma k_t (N - k_t) \alpha \omega_t$, is divided among the k_0 innovators. Therefore, the date-t value v_t of an innovator satisfies

$$rv_t = p_t + \frac{\gamma k_t (N - k_t)}{k_0} \alpha \omega_t + \frac{dv_t}{dt}. \tag{6}$$

An outsider's hazard rate for meeting a producer is $\frac{\gamma k_t (N - k_t)}{N - k_t} = \gamma k_t$. Therefore, his lifetime value at date t, u_t, satisfies

$$ru_t = \gamma k_t \left[(1 - \alpha)\omega_t - u_t \right] + \frac{du_t}{dt}. \tag{7}$$

The free entry condition requires that $v_t - u_t = c$ for $t = 0$.

Regime 2: Imitators can resell ideas

In Regime 2, imitators do get paid for ideas that they resell. An incoming idea buyer pays the agent from whom he copies the idea. This may capture the cases of patents that allow sublicensing and also the spread of non-patented know-how.

Any producer (innovator or imitator) that sells an idea can keep the proceeds. Then all producers now have the same value $v_t = \omega_t$. Again, we will prove that at equilibrium, innovators only enter at date 0. The revenue from a single idea sale is αv_t, and total revenue from idea sales, $\gamma k_t (N - k_t) \alpha v_t$, is now shared by all the k_t producers. Therefore, v_t now satisfies

$$rv_t = p_t + \gamma (N - k_t) \alpha v_t + \frac{dv_t}{dt}. \tag{8}$$

The value of an outsider becomes

$$ru_t = \gamma k_t \left((1 - \alpha)v_t - u_t \right) + \frac{du_t}{dt}. \tag{9}$$

Motivation for two regimes.—In a frictionless world, innovators would prefer Regime 2, as it does not require them to track the idea they sold and enforce the no-reselling constraint on imitators. But in Regime 2 an imitator needs to pay for an idea with a higher fee that incorporates his future revenues from reselling. This can be challenging

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for new entrants in an emerging industry who often face tight financial constraints. Regime 1 requires a smaller up-front payment by the buyer of the idea each time the idea is transferred. Of course, the use of no-reselling constraint relates to its enforceability; Regime 1 would better reflect patented innovations than non-patented ones.

In our model, imitators' ability to resell ideas does not affect the meeting process, but it affects the incentive to innovate, and it thus affects market allocation and welfare, as will be shown in the following analysis.

3   Characterization

In this section, we characterize equilibrium under each regime.

3.1   Market equilibrium

We first solve the equilibrium for each regime. We find that in either Regime 1 or 2, innovators only enter at date 0. Accordingly, the time path of firm numbers is given by Eq. (4).

Intuitively, in Regime 1 where imitators cannot resell ideas, the value of an innovator at any date t depends on the innovator's entry date τ. If a measure-0 outsider were to deviate from the equilibrium and enter as an innovator at date $\tau > 0$, its value at date $t \geq \tau$, denoted as v_t^τ, would satisfy

$$r v_t^\tau = p_t + \frac{\gamma k_t \left(N - k_t\right)}{k_\tau} \alpha \omega_t + \frac{dv_t^\tau}{dt}. \tag{10}$$

This differs from v_t in Eq. (6) because at any date $t \geq \tau$, this new entrant has a chance $1/k_\tau$ (where k_τ is the number of incumbent firms at his entry date τ) instead of $1/k_0$ to share the industry's total idea sale revenues from new imitators. The later an innovator enters the market, the fewer the offspring this innovator would have at any date $t \geq \tau$ to disseminate his idea and allow him to collect idea sale revenues. Since the number of firms continues to rise over time, it does not pay an innovator to enter after date 0.

By contrast, in Regime 2 where imitators can resell ideas, the value of an innovator at any date t does not depend on his entry date. If any innovator were to enter the industry after date 0, he would share the same value v_t as any incumbent, be it an innovator or an imitator. The free entry condition requires that $v_t - u_t = c$ for $t = 0$ and we can formally verify at equilibrium $v_t - u_t < c$ for any $t > 0$ so that even in Regime 2, innovators enter only at date 0.

In what follows we shall assume that innovation is costly enough to prevent N

agents from all innovating at date 0:[4]

$$c > \frac{(\alpha \gamma N + r)}{r(r + \gamma N)} A N^{-\beta}. \tag{11}$$

To distinguish Regimes 1 and 2, we use the superscripts **I** and **II**, and so the mass of date-0 innovators will be denoted by $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$.

Proposition 1 *Given condition (11), market equilibrium yields:*
(A) In Regime 1, innovators enter only at date 0 and $k_0^{\mathbf{I}} \in (0, N)$ solves

$$\underbrace{\frac{1}{N - k_0} \int_0^\infty e^{-rt} \left(\alpha \frac{N}{k_0} + (1 - \alpha) \frac{N}{k_t} - 1 \right) A k_t^{1-\beta} dt}_{v_0 - u_0} = c; \tag{12}$$

(B) In Regime 2, innovators enter only at date 0 and $k_0^{\mathbf{II}} \in (0, N)$ solves

$$\underbrace{\frac{1}{N - k_0} \int_0^\infty e^{-rt} \left(\left(\frac{N}{k_0} \right)^\alpha \left(\frac{N}{k_t} \right)^{1-\alpha} - 1 \right) A k_t^{1-\beta} dt}_{v_0 - u_0} = c; \tag{13}$$

where, in each regime, k_t is given by Eq. (4).

Proof. See Appendix A.1 and A.2 where we pin down $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ and derive the industry dynamic paths in each regime. ∎

In Proposition 1, condition (11) guarantees the existence of interior solution for $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$: The isoelastic form for demand in Eq. (1) implies that $k_0 > 0$ in each regime, otherwise p_0 would be infinite. Condition (11) is then needed for k_0 to be strictly below N. Equations (12) and (13) pin down the solutions for $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$, respectively. The two equations both satisfy the free entry condition that $v_0 - u_0 = c$ and are almost identical except that the idea seller's bargaining share α enters as a linear weight in Eq. (12) but as a power weight in Eq. (13).

While Eqs. (12) and (13) are implicit functions, they can be greatly simplified in special cases to deliver useful intuition. We may consider two examples under unit price elasticity ($\beta = 1$): One has $\alpha = 0$ (idea sellers are not compensated) and the other has $\alpha = 1$ (idea sellers are fully compensated). In these two cases, Eqs. (12) and (13) turn out to be equivalent:

$$k_0^{\mathbf{I}} = k_0^{\mathbf{II}} = \frac{A}{c(r + \gamma N)} \quad \text{(when } \alpha = 0\text{)}, \tag{14}$$

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[4]Note that in both Regimes 1 and 2, if all agents were to innovate at date 0 (i.e., $k_0 = N$), product price would be fixed at $AN^{-\beta}$ and each firm's present value would be $v_0 = \frac{AN^{-\beta}}{r}$. Meanwhile, each firm has the option value $u_0 = \frac{(1-\alpha)\gamma N}{r(r+\gamma N)} AN^{-\beta}$ to become an imitator, where $u_0 > 0$ for $\alpha \in [0, 1)$ or $u_0 = 0$ for $\alpha = 1$. Condition (11) hence eliminates the equilibrium in which $k_0 = N$ in both regimes by requiring $v_0 - u_0 < c$ for $k_0 = N$. See Appendix A.1 and A.2 for the proofs.

$$k_0^{\mathbf{I}} = k_0^{\mathbf{II}} = \frac{A}{rc} \qquad (\text{when } \alpha = 1), \tag{15}$$

which suggests that Regimes 1 and 2 would generate the same number of innovators in each case. This is intuitive because in the two extreme cases, where idea sellers are not compensated at all or are fully compensated, whether innovators get paid for their ideas directly (in Regime 1) or indirectly (in Regime 2) would not affect their payoffs. Also, one can easily check that condition (11) needs to hold for $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ to take interior solutions (i.e., $k_0^{\mathbf{I}} < N$ and $k_0^{\mathbf{II}} < N$). Moreover, Eqs. (14) and (15) suggest that a larger market size A encourages innovation (i.e., $\partial k_0/\partial A > 0$), while a bigger innovation cost c or a higher interest rate r does the opposite (i.e., $\partial k_0/\partial c < 0$ and $\partial k_0/\partial r < 0$). Across these two cases, k_0 is larger when idea sellers are fully compensated, which suggests α has a positive effect on innovation. The effect of the diffusion rate γ is more involved. In Eq. (14), γ shows a negative effect on k_0, but it plays no role in Eq. (15), which suggests the effect may depend on the values of α and β.[5]

Below, we show how the intuition derived from the special cases generalizes. Assuming condition (11) holds, comparing Eqs. (12) and (13) yields the findings stated in Propositions 2 and 3:

Proposition 2

$$(A) \quad k_0^{\mathbf{I}} \ \text{ and } \ k_0^{\mathbf{II}} \left\{ \begin{array}{l} \textit{increase with } \alpha \textit{ and } A, \\ \quad \textit{decrease with } c \textit{ and } r. \end{array} \right. \tag{16}$$

(B) All parameters being equal across the two regimes,

$$\frac{k_0^{\mathbf{I}}}{k_0^{\mathbf{II}}} = \left\{ \begin{array}{ll} 1 & \textit{for } \alpha \in \{0, 1\} \\ > 1 & \textit{for } \alpha \in (0, 1) \end{array} \right. . \tag{17}$$

Proof. See Appendix A.3. ∎

Proposition 2 shows that the intuition from the special examples holds generally. It also shows that for $\alpha \in (0, 1)$, all else being equal, fewer innovators enter in Regime 2 than in Regime 1. This is because innovators' revenues get discounted when they collect the payoff of ideas indirectly in Regime 2. Since the two regimes share the same diffusion process, this implies that industry output is higher for all t under Regime 1 due to its larger entry of innovators at date 0.

Next, we study how the diffusion rate γ affects innovation. The effect of γ on $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ hinges on the values of α and β as follows:

Proposition 3 *(A) For inelastic or unit elastic demand (i.e., $\beta \geq 1$),*

$$k_0^{\mathbf{I}} \textit{ and } k_0^{\mathbf{II}} \left\{ \begin{array}{l} \textit{decrease with } \gamma \textit{ when } \beta \geq 1 > \alpha, \\ \textit{do not vary with } \gamma \textit{ when } \beta = \alpha = 1. \end{array} \right.$$

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[5]Note when $\beta = 1$, the industry revenue becomes constant over time, so the diffusion rate γ would not affect the present value of industry value which is the value of innovators when $\alpha = 1$.

(B) For elastic demand (i.e., $\beta < 1$),

$$k_0^{\mathbf{I}} \begin{cases} \text{decreases with } \gamma \text{ if } 0 \leq \alpha < \frac{1}{\beta + \frac{N}{k_0^{\mathbf{I}}}(1-\beta)} < 1, \\ \text{increases with } \gamma \text{ if } 1 \geq \alpha > \beta + \frac{k_0^{\mathbf{I}}}{N}(1-\beta) > 0, \end{cases}$$

and

$$k_0^{\mathbf{II}} \begin{cases} \text{decreases with } \gamma \text{ if } 0 \leq \alpha < \beta + \frac{k_0^{\mathbf{II}}}{N}(1-\beta) < 1, \\ \text{increases with } \gamma \text{ if } 1 \geq \alpha > \beta + \left(\frac{k_0^{\mathbf{II}}}{N}\right)^{\alpha}(1-\beta). \end{cases}$$

Proof. See Appendix A.4. ∎

The intuition for these results is as follows: There are two channels through which diffusion affects innovation. One is the negative price effect, captured by β – the faster the diffusion, the lower the revenue from selling the good. The other is the positive idea-selling effect, captured by α – the faster the diffusion, the more is the revenue from selling the idea. When demand is inelastic ($\beta > 1$), a faster inflow of imitators would reduce the industry revenue stream, so the price effect dominates and the innovators' value at date 0 would drop even with the highest bargaining share ($\alpha = 1$). This is also true for the unit demand elasticity case when $\alpha < \beta = 1$. When demand is elastic ($\beta < 1$), a faster inflow of imitators would increase the industry revenue stream. If α is sufficiently high compared with β, the idea-selling effect dominates, which raises the incentive to innovate. Otherwise, the price effect dominates, which dampens innovation.

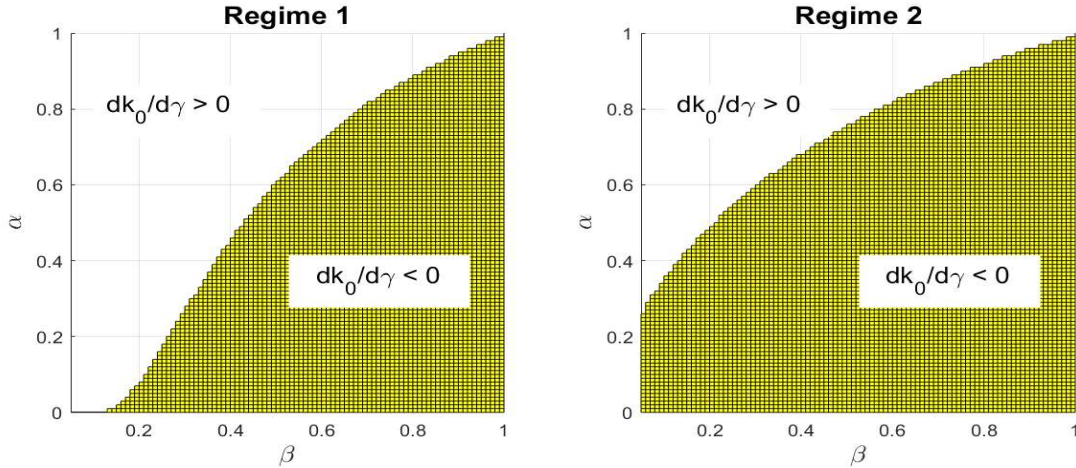Fig. 2. EFFECT OF γ ON INNOVATION UNDER ELASTIC DEMAND

Figure 2 illustrates Proposition 3(B)[6]: Under elastic demand (i.e., $\beta < 1$), a higher diffusion rate γ reduces innovation k_0 (i.e., $\partial k_0 / \partial \gamma < 0$) if β is sufficiently high or α

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[6]The conditions for $dk_0/d\gamma > 0$ or $dk_0/d\gamma < 0$ that Proposition 3(B) provides under elastic demand are only sufficient. Figure 2 takes a step further to numerically solves $dk_0/d\gamma$ for the full parameter space when $\beta < 1$. In plotting the figure, we assume that $A = 1$, $N = 1$, $r = 0.05$, $c = 30$, and $\gamma = 0.55$. The choice of parameter values is related to our empirical studies of the U.S. auto and PC industries, in which the diffusion speed γN is between 0.53 and 0.58.

is sufficiently low in Regimes 1 and 2. Comparing regimes, a higher γ is more likely to reduce innovation in Regime 2 for a given value of α because innovators are paid indirectly.

3.2 Idea diffusion and industry overtaking

The effect of the diffusion rate γ on innovation has direct implications on industry growth patterns. If a rise in γ raises k_0, it would raise output at all dates. Proposition 3 shows this can happen for a subset of parameter values when demand is elastic (i.e., $\beta < 1$) and idea sellers' bargaining share α is high, as illustrated by the unshaded areas in Fig. 2. For the remaining parameter values (as shown by the shaded areas in Fig. 2) or when demand is inelastic or unit elastic (i.e., $\beta \geq 1$), a rise in γ would reduce k_0. Proposition 4 proves that the higher-γ trajectory would then overtake the lower-γ one:

Proposition 4 *(Industry overtaking) In both Regimes 1 and 2, whenever a larger diffusion rate $\gamma^i(> \gamma^j)$ leads to a smaller $k_0^i(< k_0^j)$, there exists a date t' where*

$$t' = \frac{1}{(\gamma^i - \gamma^j)N} \ln \left(\frac{k_0^j \left(N - k_0^i \right)}{k_0^i \left(N - k_0^j \right)} \right) \tag{18}$$

such that $k_t^i > k_t^j$ for all $t > t'$.

Proof. Eq. (4) states that $k_t^l = \frac{N e^{\gamma^l N t}}{e^{\gamma^l N t} + \frac{N}{k_0^l} - 1}$ for $l \in \{i, j\}$. Therefore, $k_t^i > k_t^j \iff$

$\frac{k_t^i}{N - k_t^i} > \frac{k_t^j}{N - k_t^j} \iff \frac{e^{\gamma^i N t}}{\frac{N}{k_0^i} - 1} > \frac{e^{\gamma^j N t}}{\frac{N}{k_0^j} - 1} \iff t > t'$, where t' is given by Eq. (18). $\blacksquare$

Thus our model generates overtaking when two sectors or two locations face the same environment (i.e., same A, α, c and N) except that γ is higher in one than the other. For example, a high-tech sector may use technology based on ideas that spread faster than they do in other sectors. As a result, high-tech sectors would have a higher γ and would start smaller but would grow faster.

There also could be legal or regulatory reasons why γ differs over regions. For example, California bans non-compete contracts and therefore indirectly encourages labor turnover and spin-offs[7] whereas Massachusetts enforces those contracts. As Saxenian (1994), Gilson (1999), and Franco and Mitchell (2008) argue, this may help explain why Silicon Valley has overtaken Massachusetts' Route 128 in developing high-tech industry. Our model generates such a pattern if γ is higher in California than in Massachusetts. Under conditions in Proposition 3, Route 128 would then

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[7]Spin-offs are firms founded by former employees of incumbent firms to conduct businesses in the same industry. While our model does not include labor in production, employees could work in the same company but not be involved in directly producing the new product. And then they could learn about the idea internally.

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offer higher incentives to innovators that would result in a higher initial entry rate of firms (i.e., a higher k_0) than Silicon Valley.[8] Thus our model would predict the type of overtaking portrayed in Fig. 3.
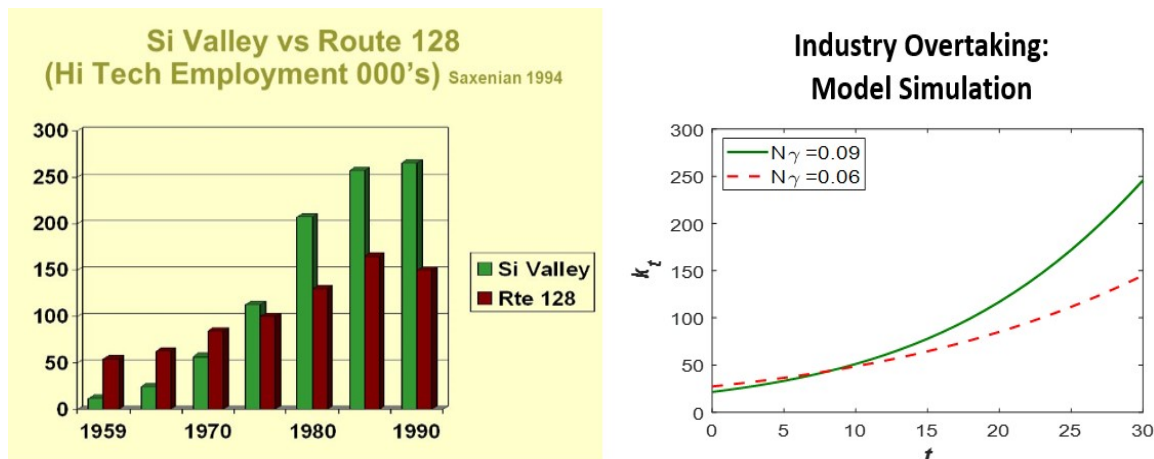

Fig. 3. Industry Overtaking: Data and Model

The left and right panels of Fig. 3 show the data from Saxenian (1994) and our model simulation, respectively. In the simulation, we assume $\alpha = 0$ in both locations, so that Regimes 1 and 2 coincide.[9] We also assume $\beta = 1$, $A/c = 3$, $N = 1000$, $r = 0.05$, and plot k_t in two locations: One with a high diffusion rate ($\gamma N = 0.09$), the other with a low one ($\gamma N = 0.06$). The stimulation is provided merely to illustrate that the model can generate the overtaking pattern seen in Saxenian (1994)'s data.

4 Welfare analysis

We now study the welfare implications of the model. Consumers' utility from consuming output k is the integral under the demand curve. For $\beta \in (0,1)$, aggregate utility at output k is

$$U\left(k\right) = \int_0^k As^{-\beta}ds = \frac{A}{1-\beta}k^{1-\beta}. \tag{19}$$

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[8]Enforcing non-compete contracts may also raise the bargaining share α of innovators (e.g., see Gottfries and Jarosch, 2023). If that happens, the entry of initial innovators will be even larger in the enforcing location and the timing of overtaking will be postponed compared with the case where both locations have the same α.

[9]See Proposition 2. One could say that when non-compete contracts are banned, innovators get no compensation from their employee spin-offs. But if non-compete contracts are strictly enforced and the bilateral negotiation to buy out those contracts are too costly for the parties involved, spin-off entry would be largely blocked. As a result, both locations would have $\alpha = 0$, but γ would differ.

For $\beta \geq 1$ the above integral is infinite; to ensure consumer surplus is finite, we put a maximum, $A\varepsilon^{-\beta}$, on the willingness to pay. Let $D(s) = \min\left(A\varepsilon^{-\beta}, As^{-\beta}\right)$ and define aggregate utility as $U(k) = \int_0^k D(s)\,ds = A\left(\int_0^\varepsilon \varepsilon^{-\beta}ds + \int_\varepsilon^k s^{-\beta}ds\right)$ where $\varepsilon \ll k$. Accordingly, for $\beta = 1$ we have

$$U(k) = A(\ln k + 1 - \ln \varepsilon), \tag{20}$$

and for $\beta > 1$, we have

$$U(k) = \frac{\beta A}{\beta - 1}\varepsilon^{1-\beta} + \frac{A}{1-\beta}k^{1-\beta}. \tag{21}$$

4.1 Planner's problem

The social planner would like to maximize social welfare W_0 given by

$$W_0 = \int_0^\infty e^{-rt}U(k_t)\,dt - ck_0, \tag{22}$$

where k_t satisfies Eq. (4).

In what follows we assume that innovation is costly enough so that the socially optimal entry number of innovators k_0^* is an interior solution (i.e., $k_0^* < N$) at date 0:

$$c > \frac{AN^{-\beta}}{(r + \gamma N)}. \tag{23}$$

Proposition 5 *Given condition (23), social optimum requires that innovators enter only at date 0 and k_0^* solves*

$$\underbrace{\int_0^\infty e^{-(r+\gamma N)t}\left(\frac{k_t}{k_0}\right)^2 Ak_t^{-\beta}dt}_{\text{marginal social return to } k_0} = c. \tag{24}$$

Proof. See Appendix A.5. ∎

The finding of Proposition 5 is intuitive. As of date $\tau \geq 0$, the social return to innovation is

$$SR_\tau = \int_\tau^\infty e^{-r(t-\tau)}U(k_t)\,dt$$

and one can verify that the marginal social return $\partial SR_\tau/\partial k_\tau$ is strictly decreasing in k_τ. So if k_0^* is chosen so that $\partial SR_0/\partial k_0 = c$ at date 0, thereafter $\partial SR_\tau/\partial k_\tau < c$ for any $\tau > 0$. Hence, it is socially optimal to innovate only at date 0. And the condition $\partial SR_0/\partial k_0 = c$ yields Eq. (24). Finally, the social welfare given by Eq. (22) is strictly concave in k_0, so for $k_0^* < N$ to hold, one needs

$$\frac{dW_0}{dk_0}\Big|_{k_0=N} < 0,$$

13

which yields condition (23). In the rest of the paper, we assume that condition (23) always holds.

Denote the socially optimal welfare by W_0^*. We have the following comparative-static results.

Proposition 6 *All else being equal,*

$$(A) \quad k_0^* \begin{cases} \text{increases with } A, \\ \text{decreases with } c \text{ and } r, \\ \text{decreases with } \gamma \text{ if } \beta > 1 - \frac{k_0^*}{N - k_0^*}. \end{cases} \tag{25}$$

$$(B) \quad W_0^* \begin{cases} \text{increases with } A \text{ and } \gamma, \\ \text{decreases with } c \text{ and } r. \end{cases} \tag{26}$$

Proof. See Appendix A.6. ∎

Thus k_0^* and W_0^* both increase in market size A but decrease in innovation cost c and interest rate r. Moreover, k_0^* decreases with the diffusion rate γ if the demand is not too elastic, while W_0^* always increases with γ.

4.2 Three policy instruments

We now show that a planner can achieve k_0^* by choosing the bargaining share of idea-sellers, α, or by choosing an innovation subsidy (or tax) s. And if the planner could raise γ by certain policies, we show that doing so would be desirable.

Optimal bargaining share.— Denote the socially optimal bargaining shares for Regimes 1 and 2 by $\alpha^{\mathbf{I}*}$ and $\alpha^{\mathbf{II}*}$, respectively. We first note that the interior solution condition (11) faced by market players is related to the interior solution condition (23) faced by the social planner by the inequality:

$$\underbrace{\frac{(\alpha\gamma N + r)}{r(r + \gamma N)} A N^{-\beta}}_{\text{RHS of (11)}} \geq \underbrace{\frac{A N^{-\beta}}{(r + \gamma N)}}_{\text{RHS of (23)}}.$$

The two are identical if $\alpha = 0$.

Assume that condition (23) always holds. If there exists an $\alpha' \in (0, 1]$ such that the inequality (11) becomes an equality (i.e., $c = \frac{\alpha'(\gamma N + r)}{r(r + \gamma N)} A N^{-\beta}$), one can prove that the planner could choose the optimal bargaining shares $\alpha^{\mathbf{I}*}$ and $\alpha^{\mathbf{II}*}$ such that $0 < \alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*} < \alpha'$ to achieve the socially optimal k^* in Regimes 1 and 2, respectively. This is illustrated by Fig. 4(A). Alternatively, if the inequality (11) holds for any $\alpha \leq 1$, one can prove $0 < \alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*} < 1$, as illustrated by Fig. 4(B). Note that $\alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*}$ results from Proposition 2: If there is a value of α that leads to $k_0^{\mathbf{I}} = k^*$ in Regime 1, the same α would lead to $k_0^{\mathbf{II}} < k^*$ in Regime 2, so a larger value of α is needed to achieve k^* in Regime 2.
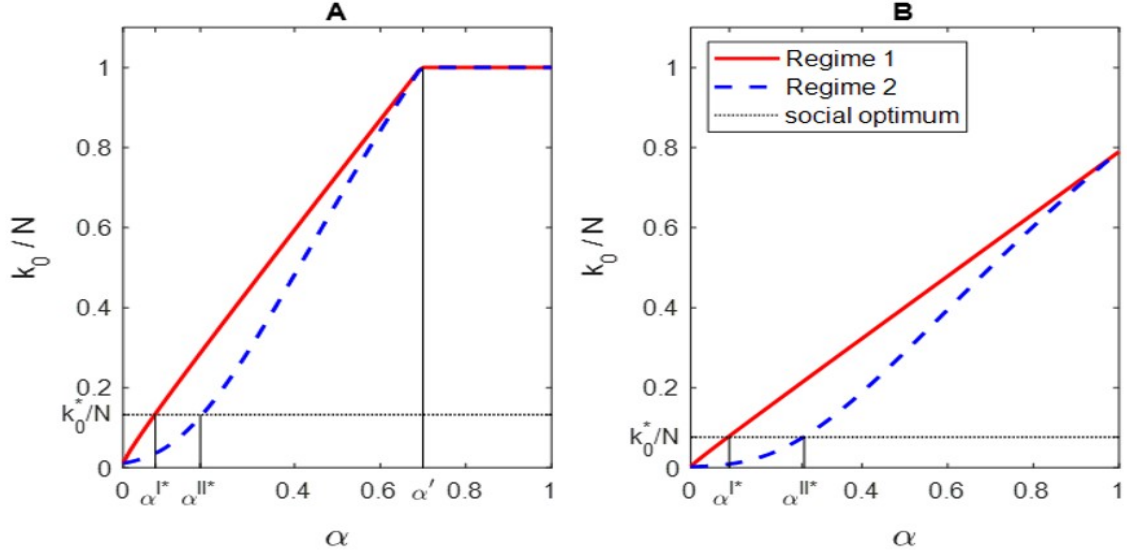
Fig. 4. Socially Optimal α for Two Regimes

The above discussion is stated formally in Proposition 7.

Proposition 7

$$0 < \alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*} < 1. \tag{27}$$

Proof. See Appendix A.7. ∎

Intuitively speaking, the findings of (27) hold because if $\alpha^{\mathbf{I}} = \alpha^{\mathbf{II}} = 0$, no innovator would internalize knowledge spillovers they create for imitators, so fewer innovators enter than the social optimum. On the other hand, if $\alpha^{\mathbf{I}} = \alpha^{\mathbf{II}} = 1$, innovators would not fully internalize the matching externality they impose on one another, so more innovators would enter than the social optimum. The matching externality arises because an innovator's meeting rate $\frac{dk_t/dt}{k_t} = \gamma(N - k_t)$ falls with k_t while an imitator's meeting rate $\frac{dk_t/dt}{N-k_t} = \gamma k_t$ rises with k_t. Therefore, an agent's decision to become an innovator imposes a negative externality on other agents' payoff to innovating.[10]

Optimal innovation subsidy or tax.—Whenever $\alpha \neq \alpha^*$ in each regime, the planner can use a subsidy (or a tax if the subsidy is negative) to achieve the social optimum. Denote the socially optimal subsidy for Regimes 1 and 2 by $s^{\mathbf{I}*}$ and $s^{\mathbf{II}*}$, respectively. We obtain the following result:

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[10]Assuming a Cobb-Douglas matching function, Hopenhayn and Shi (2020) show that at the steady state, the socially optimal compensation share for innovators coincides with the innovators' share in the matching function, as in Hosios (1990). By contrast, our study considers transitional dynamics of an industry, and in our quadratic matching function, the shares of k and $N-k$ are equal, which also happens in the Cobb-Douglas case when the innovators' share is $1/2$ in the matching function. Yet as Fig. 4 and Proposition 7 show, the socially optimal α^* in our model can be anywhere between 0 and 1 in each regime depending on parameter values.

Proposition 8 *Social optimum implies $s^{\mathbf{I}*} < s^{\mathbf{II}*}$ for $\alpha \in (0,1)$, $s^{\mathbf{I}*} = s^{\mathbf{II}*} > 0$ for $\alpha = 0$, and $s^{\mathbf{I}*} = s^{\mathbf{II}*} < 0$ for $\alpha = 1$.*

Proof. See Appendix A.8. ∎

The intuition for Proposition 8 is as follows. Whenever $\alpha \neq \alpha^*$ in each regime, the number of innovators k_0 differs from the social optimum k_0^*, in which case offering an innovation subsidy (i.e., $s^* > 0$ whenever $\alpha < \alpha^*$) or a tax (i.e., $s^* < 0$ whenever $\alpha > \alpha^*$) to adjust the innovation cost c would help restore the social optimum. Recall that when $\alpha \in \{0, 1\}$, Regimes 1 and 2 coincide. When $\alpha = 0$, too fewer innovators enter than the social optimum, so both regimes would need a positive subsidy to reduce c to achieve k_0^*. When $\alpha = 1$, a negative subsidy (i.e., a tax) is needed. Moreover, for $\alpha \in (0,1)$, according to Proposition 2(B), if a given pair of α and $(c - s^{\mathbf{I}*})$ lead to the social optimum k_0^* in Regime 1, the same parameter values would result in $k_0^{\mathbf{II}} < k_0^*$ in Regime 2. Therefore, a higher subsidy (or a smaller tax) $s^{\mathbf{II}*}$ is needed for adjusting c to achieve k_0^* in Regime 2 given that $k_0^{\mathbf{II}}$ decreases with c as shown by Proposition 2(A).

Optimal diffusion rate.—Suppose that incumbents are not compensated by imitators for spreading ideas, so that $\alpha = 0$. From the social welfare point of view, should the planner reduce the diffusion speed γ (e.g., by restricting entry of imitators) to enhance incentives for innovation?

Note that if $\alpha = 0$, Proposition 3 shows that the entry of innovators decreases with γ for any $\beta > 0$ in both regimes. Therefore, a policy that reduces the diffusion rate γ would boost the entry of innovators. Such policy, however, does not necessarily increase welfare. Next, we assume condition (11) holds so that not all agents enter at date 0 (i.e., $k_0 < N$) and we prove the welfare results for the unit demand elasticity case (i.e., $\beta = 1$):

Proposition 9 *(A) For $\beta = 1$ and all $\alpha \in [0,1]$, social welfare always increases with the diffusion rate γ in Regime 1. (B) For $\beta = 1$ and $\alpha \in \{0,1\}$, Regimes 1 and 2 coincide and social welfare always increases with the diffusion rate γ.*

Proof. See Appendix A.9. ∎

To build intuition, let us consider the two cases $\alpha \in \{0,1\}$ in Proposition 9(B), and note that

$$\frac{dW_0}{d\gamma} = \frac{\partial W_0}{\partial k_0} \frac{\partial k_0}{\partial \gamma} + \frac{\partial W_0}{\partial \gamma}. \tag{28}$$

First, the case $\alpha = 0$. From Proposition 7, we know that $\alpha = 0$ is below the socially optimal level α^*, so $\frac{\partial W_0}{\partial k_0} > 0$. Proposition 3 shows that $\frac{\partial k_0}{\partial \gamma} < 0$ for $\alpha = 0$, so $\frac{\partial W_0}{\partial k_0} \frac{\partial k_0}{\partial \gamma} < 0$. However, holding k_0 fixed, $\frac{\partial W_0}{\partial \gamma} > 0$. Ultimately, Proposition 9(B) finds that when $\alpha = 0$, the positive effect $\frac{\partial W_0}{\partial \gamma}$ (i.e., gains from knowledge spillovers) dominates the negative effect $\frac{\partial W_0}{\partial k_0} \frac{\partial k_0}{\partial \gamma}$ (i.e., disincentives to innovation). This suggests

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that in the numerical example above (cf. Fig. 3), a higher value of γ not only helps Silicon Valley overtake Route 128 in industry size, but also yields higher social welfare. Second, the case $\alpha = 1$, which is above the socially optimal level α^*, and so $\frac{\partial W_0}{\partial k_0} < 0$. However, $\frac{\partial k_0}{\partial \gamma} = 0$ for $\alpha = \beta = 1$ (cf. Proposition 3), so $\frac{\partial W_0}{\partial k_0}\frac{\partial k_0}{\partial \gamma} = 0$. Therefore, there is no industry overtaking or disincentive to innovation in this case, and $\frac{dW_0}{d\gamma} = \frac{\partial W_0}{\partial \gamma} > 0$.

This suggests that the planner may not want to slow down diffusion by lowering γ even when $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ are below k^*. Rather than lowering γ, the planner should address the compensation to innovators (i.e., α) directly. This highlights the importance of technology diffusion to welfare and lends support to public policies that accommodate diffusion. In Section 5, we empirically analyze the U.S. auto and PC industries where demands are price elastic (i.e., $\beta < 1$) and show that this finding continues to hold.[11]

5   Empirical applications

In this section, we apply our model to industry data. We consider two historically important industries: automobile and personal computer, where idea diffusion played an important role in the industries' development.[12] Using model calibration and counterfactual exercises, we evaluate and quantify our theoretical predictions.

5.1   Parameter estimation

We first estimate the model parameters using auto and PC industry data. The data come from the following sources:

Auto.— Smith (1970) lists every make of passenger cars produced commercially in the United States from 1895-1969. Smith's list of car makes is used to derive the number of auto firms each year. Thomas (1977) provides annual data of average car price and output from 1900-1929.

PC.— Firm numbers are from Stavins (1995) and the *Thomas Register of American Manufacturers*, which include desktop and portable computers. Price and quantity information is from the *Information Technology Industry Data Book*.

In addition, Williamson (2020) provides annual data of U.S. population, real GDP, and the GDP deflator.

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[11]Note that the finding does not rule out the possibility that policymakers can exploit the welfare gain of temporarily restricting γ. For example, policymakers could promise to restrict γ initially to achieve the socially optimal entry of innovators k^*, and then free up the limitation. However, such a policy is time-inconsistent and would be futile if market participants anticipate that *ex post* policymakers cannot commit to that promise (Kydland and Prescott, 1977). Presumably policy must apply more broadly, not just to one instance, but to future products and future instances of k.

[12]E.g., Klepper (2010) documents how the spawning of employee spin-offs and entry by firms in related industries drove the development of the automobile and the semiconductor industries.

5.1.1   Auto Industry

The U.S. automobile industry started in 1890s and grew from a small infant industry to a major sector of the economy in a few decades. Starting with 3 firms in 1895, the number of auto producers exceeded 200 around 1910. A shakeout then followed when a major process innovation, the assembly line, was introduced in the early 1910s. As a result, the number of firms declined sharply while the industry output expanded tremendously. Eventually, only 24 firms survived into 1930s. Figure 5 plots the number of firms and output per firm in the U.S. auto industry from 1895-1929.

Our model describes the auto industry development well for the pre-shakeout period (1895-1910). As shown in Fig. 5, during that period, the time path of firm numbers followed an S-shaped curve and the average output per firm stayed flat which reflects firms' production capacity constraint. To calibrate the model, we focus on the pre-shakeout era. We assume the shakeout to be an unexpected shock in the benchmark analysis, and we then extend the model to incorporate the shakeout as an anticipated shock in Section 5.4.2.
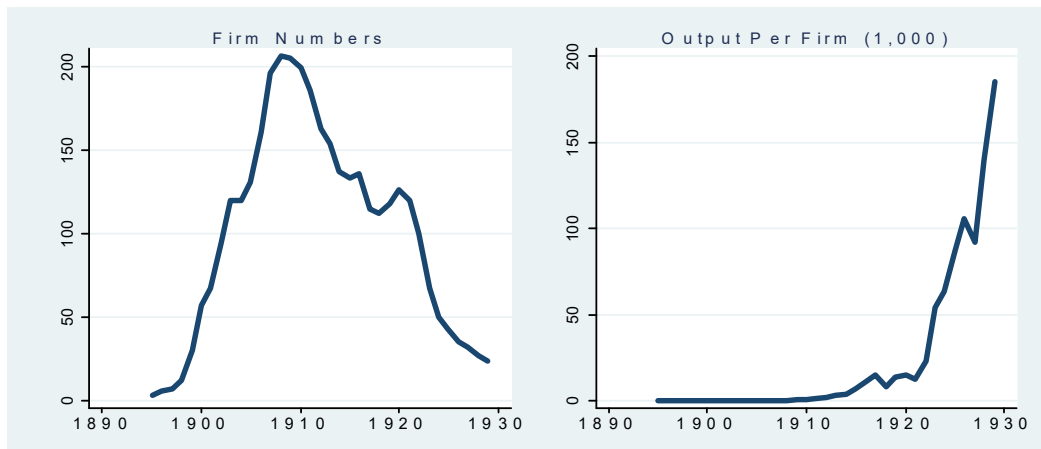

Fig. 5. Auto Firm Numbers and Output Per Firm

Diffusion estimation We first use the data of firm numbers in the pre-shakeout period, 1895-1910, to estimate the diffusion parameters. In doing so, we rewrite Eq. (4) to estimate the diffusion process of k_t as follows:

$$\ln \frac{k_t}{N - k_t} = z + \lambda t, \tag{29}$$

where $z = \ln \frac{k_0}{N - k_0}$, and $\lambda = \gamma N$.[13]

We assume that the shakeout started after almost all the potential firms had entered the industry. Accordingly, we set $N = 210$ and run the regression model.[14]

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[13]Note that Eq. (4) implies $\frac{k_t}{N - k_t} = \frac{e^{\gamma N t}}{\frac{N}{k_0} - 1}$, which leads to Eq. (29).

[14]We try an alternative assumption for N in Section 5.4.1 as a robustness check.

The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.26)^{***}}{-4.13} + \underset{(0.03)^{***}}{0.53} \; t, \tag{30}$$

and the standard errors are reported in the parentheses. The estimates of z and λ are both statistically significant at 1% level (noted by three stars), and adjusted $R^2 = 0.96$. The fit of estimation is shown in Fig. 6. Based on the estimates of diffusion parameters, we calibrate $\gamma N = 0.53$ and $k_0 = 3.31$ (i.e., $\ln \frac{k_0}{N - k_0} = -4.13$).

As a robustness check, we estimated the diffusion process using the meeting function (3) which allows differencing the data. The regression results, reported in Appendix B.1, are consistent with the estimates above.

Demand estimation We then estimate the auto demand function using annual data of real auto price p_t (in 2012 price) and industry output Q_t from 1900–1929. Equation (1) suggests a simple log-log demand function:

$$\ln(Q_t) = a - \phi \ln(p_t).$$

To address potential endogeneity of the price variable, we use the output per firm lagged by a year as an instrumental variable to estimate the demand elasticity parameter ϕ in a two-stage least-squares regression. Output per firm, while assumed fixed in our theory, did grow over the long term in data due to technological progress. If unobserved demand shocks are not serially correlated, lagged output per firm can serve as a valid supply shifter to trace out the demand curve.

The first-stage regression result (adj. $R^2 = 0.87$) is given by

$$\ln(p_t) = \underset{(0.14)^{***}}{11.37} - \underset{(0.02)^{***}}{0.24} \times \ln(\text{output per firm})_{t-1},$$

and the second-stage regression result ($R^2 = 0.82$) is

$$\ln(Q_t) = \underset{(2.75)^{***}}{47.05} - \underset{(0.29)^{***}}{3.61} \times \ln(p_t). \tag{31}$$

All the estimates are statistically significant at 1% level (noted by three stars). The fit of estimation is shown in Fig. 6.

The IV estimation gives $\phi = 3.61$ and $a = 47.05$. Because our model specifies an inverse demand function (1) that implies

$$\ln Q_t = \frac{1}{\beta} \ln \tilde{A} - \frac{1}{\beta} \ln p_t,$$

this yields that $\beta = 0.28$ (i.e., $\frac{1}{\beta} = \phi = 3.61$) and $\tilde{A} = 45,737$ (i.e., $\frac{1}{\beta} \ln \tilde{A} = 47.05$).[15] For robustness checks, we also re-ran the IV regressions by controlling changes of

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[15] In the model, we normalize a firm's output to 1, so $Q_t = k_t$ and the inverse demand function is $p_t = A k_t^{-\beta}$. In the empirical analysis, we denote a firm's output by q, so $Q_t = q k_t$ and the corresponding inverse demand function becomes $p_t = \tilde{A} Q_t^{-\beta}$.

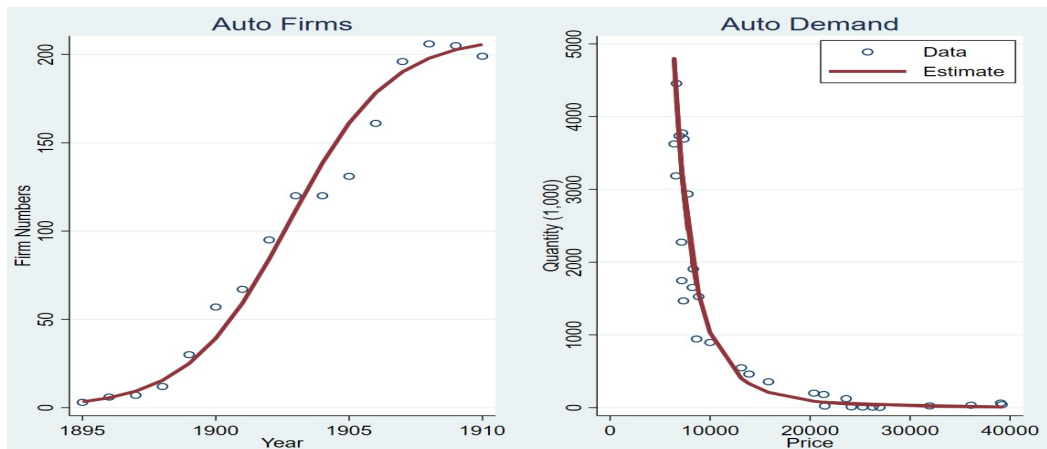population and per capita income over time, and the results are very similar (see Appendix B.2).

Fig. 6. Auto Diffusion and Demand Estimates

Moreover, to cross check if the lagged output per firm is indeed a valid instrumental variable, we follow Cabral et al. (2018) and use a different instrumental variable, the share of spin-off firms in the auto industry. Empirical studies show that the founders of spin-off firms are more experienced, so spin-off firms tend to perform better than *de novo* firms (Klepper, 2010). Therefore, the share of spin-off firms in the industry shifts the supply curve and can serve as a valid instrument to trace out the demand curve. Using this alternative instrumental variable and with the same sample range (1900-1929), the first-stage regression result (adj. $R^2 = 0.83$) is given by

$$\ln(p_t) = \underset{(0.10)^{***}}{10.51} - \underset{(0.39)^{***}}{4.65} \times (share\ of\ spin\text{-}off\ firms)_{t-1},$$

and the second-stage regression result ($R^2 = 0.82$) is

$$\ln(Q_t) = \underset{(2.82)^{***}}{47.30} - \underset{(0.30)^{***}}{3.63} \times \ln(p_t). \tag{32}$$

All the estimates are statistically significant at 1% level (noted by three stars), and the estimated demand elasticity $\phi = 3.63$ supports our estimate above.

5.1.2   PC industry

The personal computer industry was developed 80 years later than the automobile industry, but the industry evolution was not much different. Starting with two firms in 1975, the number of PC producers exceeded 430 in 1992. A shakeout then started when the number of firms fell sharply while the industry output continued to expand. Figure 7 plots the number of firms and output per firm in U.S. PC industry from 1975-1999. Like in the auto industry case, our model describes the pre-shakeout

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period (1975-1992) of the PC industry well. As shown in Fig. 7, during that period, the time path of firm numbers followed an *S*-shaped curve and the average output per firm stayed flat.
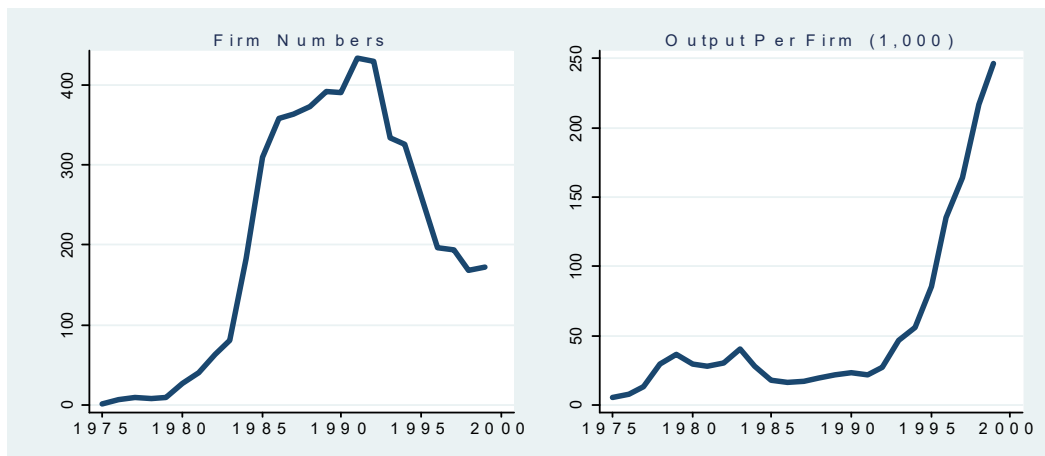

Fig. 7. PC Firm Numbers and Output Per Firm

Diffusion estimation We first use the data of firm numbers in the pre-shakeout period, 1975-1992, to estimate the diffusion parameters. We assume the shakeout started after almost all the potential PC firms had entered the industry. Accordingly, we set $N = 435$ and run the following regression model (33).[16] The result shows that

$$\ln \frac{k_t}{N - k_t} = \underset{(0.29)^{***}}{-5.49} + \underset{(0.03)^{***}}{0.58} \ t, \tag{33}$$

with the standard errors reported in the parentheses. All the coefficient estimates are statistically significant at 1% level, and adjusted $R^2 = 0.96$. The fit of estimation is shown in Fig. 8. Based on the estimates of diffusion parameters, we calibrate $\gamma N = 0.58$, and $k_0 = 1.78$ (i.e., $\ln \frac{k_0}{N-k_0} = -5.49$).

As for autos, to check robustness, we again estimated the diffusion process using Eq. (3) which allows differencing the data. The regression results, reported in Appendix B.3, are consistent with the estimates above.

Demand estimation We then estimate the PC demand function using annual data of real PC price p_t (in 2012 price) and industry output Q_t from 1975-1992. As before, in order to address potential endogeneity of the price variable, we use average output per firm (lagged by a year) as an instrumental variable to estimate the demand elasticity ϕ.

The first-stage regression result (adj. $R^2 = 0.23$) is given by

$$\ln(p_t) = \underset{(0.50)^{***}}{9.62} - \underset{(0.05)^{**}}{0.12} \times \ln(output \ per \ firm)_{t-1},$$

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[16]We try an alternative assumption for N in Section 5.4.1 as a robustness check.

and the second-stage regression result ($R^2 = 0.94$) is

$$\ln(Q_t) = \underset{(12.52)^{***}}{137.15} - \underset{(1.49)^{***}}{14.58} \times \ln(p_t). \tag{34}$$

Standard errors are reported in the parentheses, with two and three stars indicating statistical significance at 5% and 1% levels, respectively. The fit of estimation is shown in Fig. 8.

The IV estimation gives $\phi = 14.58$ and $a = 137.15$. This yields $\beta = 0.07$ (i.e., $\frac{1}{\beta} = \phi = 14.58$) and $\tilde{A} = 12,170$ (i.e., $\frac{1}{\beta} \ln \tilde{A} = a = 137.15$). For robustness checks, we also re-ran the IV regressions by controlling changes of population and per capita income over time, and the results are very similar (see Appendix B.4).
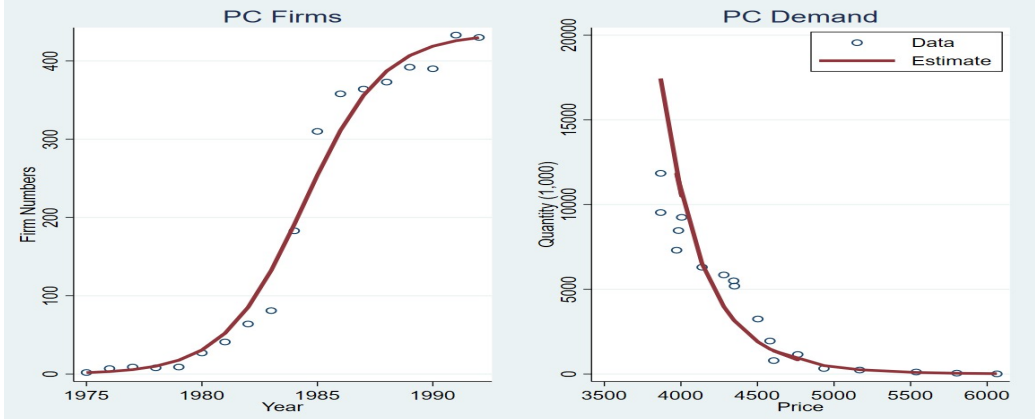

Fig. 8. PC Diffusion and Demand Estimates

5.2 Model calibration

To calibrate the model, we first pick values for N, γN and k_0 from the diffusion estimation for the auto and the PC industries, respectively. We then pick values for β and A from the demand estimation. Note that in the model, a firm's output is normalized to 1 per period. While this does not affect the theoretical analysis, we account for a firm's production size in the empirical applications. In doing so, we denote q a firm's output and Q the industry output, so $Q_t = qk_t$ at date t. Accordingly, we revise Eqs. (12), (13) and (24) as follows by replacing A with $\tilde{A}q^{1-\beta}$ (where $\tilde{A}$ and β are from the demand function estimation above):

$$\text{Regime 1:} \quad \frac{1}{N - k_0} \int_0^\infty e^{-rt} \left(\alpha \frac{N}{k_0} + (1 - \alpha) \frac{N}{k_t} - 1 \right) \tilde{A} q^{1-\beta} k_t^{1-\beta} dt = c; \tag{35}$$

$$\text{Regime 2:} \quad \frac{1}{N - k_0} \int_0^\infty e^{-rt} \left(\left(\frac{N}{k_0} \right)^\alpha \left(\frac{N}{k_t} \right)^{1-\alpha} - 1 \right) \tilde{A} q^{1-\beta} k_t^{1-\beta} dt = c; \tag{36}$$

$$\text{Social optimum:} \quad \int_0^\infty e^{-(r+\gamma N)t} \left(\frac{k_t}{k_0} \right)^2 \tilde{A} q^{1-\beta} k_t^{-\beta} dt = c. \tag{37}$$

In the auto case, a firm on average produced less than 1,000 cars a year up to 1910, and we calibrate $q = 900$ based on output per firm in 1910 and $\tilde{A}q^{1-\beta} = 61.28$ (million). In the PC case, using output per firm in 1992, we calibrate $q = 27,500$ and $\tilde{A}q^{1-\beta} = 163.63$ (million).

We then set $r = 0.05$. Because we have no direct information about two remaining parameters α and c, we assume $\alpha = 0$ to pin down c in the benchmark analysis. Since Regimes 1 and 2 coincide when $\alpha = 0$, one can use Eq. (35) or (36) to solve for c. Table 1 summarizes the benchmark parameter values calibrated for the auto and the PC industries. Because the values of α, r and N are chosen by assumption, we will consider alternative values for them in Section 5.4 for robustness checks.

Table 1. Model Parameterization

	α	r	N	γN	k_0	β	$\tilde{A}q^{1-\beta}$
Auto	0	0.05	210	0.53	3.31	0.28	61.28
PC	0	0.05	435	0.58	1.78	0.07	163.63

Figure 9 plots the calibrated model dynamics for the auto industry. The number of firms k_t grows along a logistic curve. Meanwhile, v_t decreases while u_t increases over time.[17] The initial difference $v_0 - u_0$ equals the innovation cost $c_{\text{Auto}} = \$173.73$ million (in 2012 price). By 1910, the value of a producer v_t comes down to \$274 million and the value of a future imitator rises to \$250 million. Because almost all the potential entrants N have entered the industry by then, the total value of firms $v_{1910}k_{1910}$ is very close to the present value of the industry revenue $p_{1910}Q_{1910}/r$.
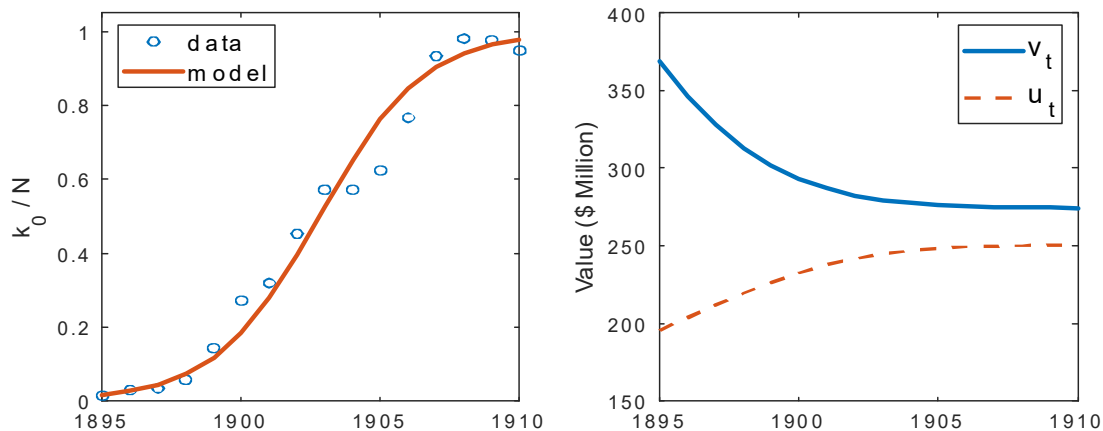

Fig. 9. MODEL CALIBRATION: AUTO

[17]Because we assume $\alpha = 0$ in the benchmark calibration, Regimes 1 and 2 coincide. Accordingly, v_t decreases in t and u_t increases in t when $\beta < 1$, as shown in the proof of Appendix A.1. More broadly, for $\alpha \in (0, 1)$, Regimes 1 and 2 do not coincide and the time paths of v_t and u_t may look differently between the two regimes. For example, with certain parameter values, v_t in Regime 1 may initially increase and later decrease in t, but v_t in Regime 2 always decreases in t. Regardless, no innovator would enter after date 0 in either regime because the entry value of an innovator minus his option value of waiting to imitate always decreases in t, as proved in Appendix A.1 and A.2.

Figure 10 plots the calibration results for the PC industry. Again, the number of firms k_t grows along a logistic curve, and v_t decreases while u_t increases over time. The initial difference $v_0 - u_0$ equals the innovation cost $c_{\mathrm{PC}} = \$\ 986.87$ million (in 2012 price). By 1992, the value of a producer v_t comes down to \$2.14 billion and the value of a future imitator rises to \$1.97 billion. Because almost all the potential entrants have entered the industry by then, the total value of firms $v_{1992}k_{1992}$ is very close to the present value of the industry revenue $p_{1992}Q_{1992}/r$.
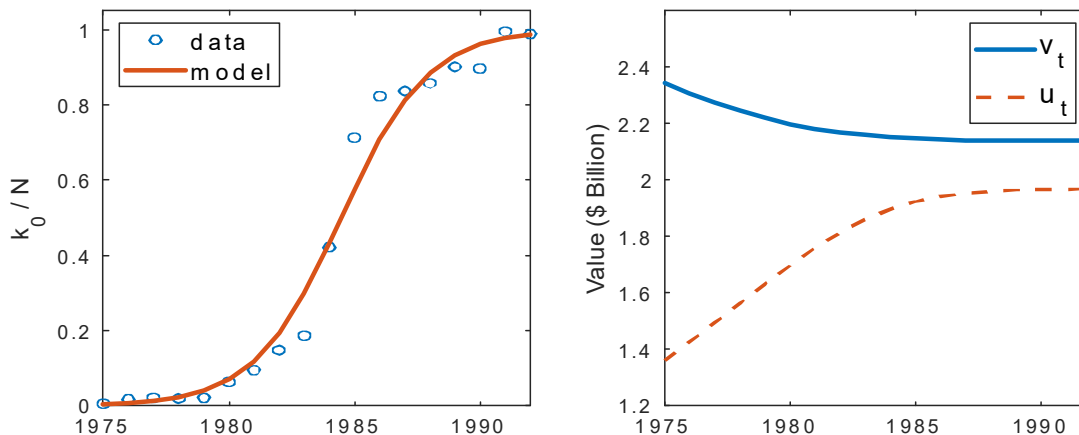

Fig. 10. MODEL CALIBRATION: PC

5.3 Counterfactual analysis

Given the calibrated model parameter values, we then conduct counterfactual analysis and evaluate welfare.

5.3.1 Optimal compensation for idea sellers

We first evaluate the effect of the compensation share α in Regimes 1 and 2, and start with the auto industry.

Given the innovation cost c_{Auto} derived from the model calibration, we solve the equilibrium industry dynamics for each counterfactual value of $\alpha \in (0, 1]$. Particularly, Eqs. (35) and (36) allow us to pin down the counterfactual entry number of innovators k_0 at date 0. Figure 11 shows that k_0 strictly increases with α for both Regimes 1 and 2 when $0 < \alpha < 0.61$ and Regime 1 has a higher value of k_0 than Regime 2. For $\alpha \geq 0.61$, the values of k_0 in both regimes reaches the corner solution $k_0 = N$. Equation (37) pins down the socially optimal entry number of innovators k_0^*/N to be 0.151, which can be achieved by choosing $\alpha_{\mathrm{Auto}}^{\mathbf{I}*} = 0.07$ in Regime 1 and $\alpha_{\mathrm{Auto}}^{\mathbf{II}*} = 0.167$ in Regime 2. The social optimum yields a social surplus $W_{0,\mathrm{Auto}}^* = \$64.45$ billion (in 2012 price).

Fig. 11. Effect of α : Auto
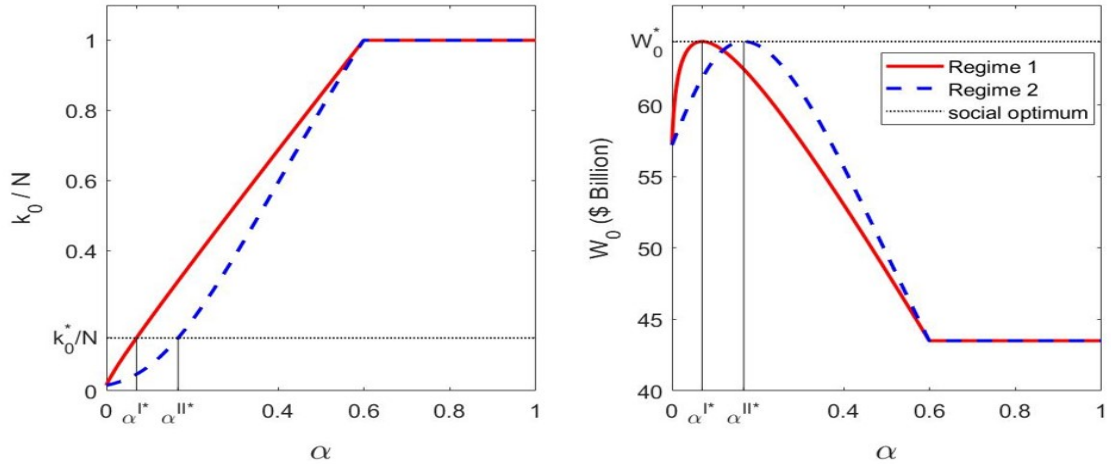
We then look into the PC industry. Given the innovation cost c_{PC} derived from the model calibration, Eqs. (35) and (36) pin down the entry number of innovators k_0 for each counterfactual value of $\alpha \in (0, 1]$. Figure 12 shows that k_0 strictly increases with α for both Regimes 1 and 2 when $0 < \alpha < 0.42$ and Regime 1 has a higher value of k_0 than Regime 2. For $\alpha \geq 0.42$, the values of k_0 in both regimes reaches the corner solution $k_0 = N$. The socially optimal entry number of innovators k_0^*/N is 0.164, which can be achieved by choosing $\alpha_{\mathrm{PC}}^{\mathbf{I}*} = 0.055$ in Regime 1 and $\alpha_{\mathrm{PC}}^{\mathbf{II}*} = 0.135$ in Regime 2. The social optimum yields a social surplus $W_{0,\mathrm{PC}}^* = \$798.9$ billion (in 2012 price).
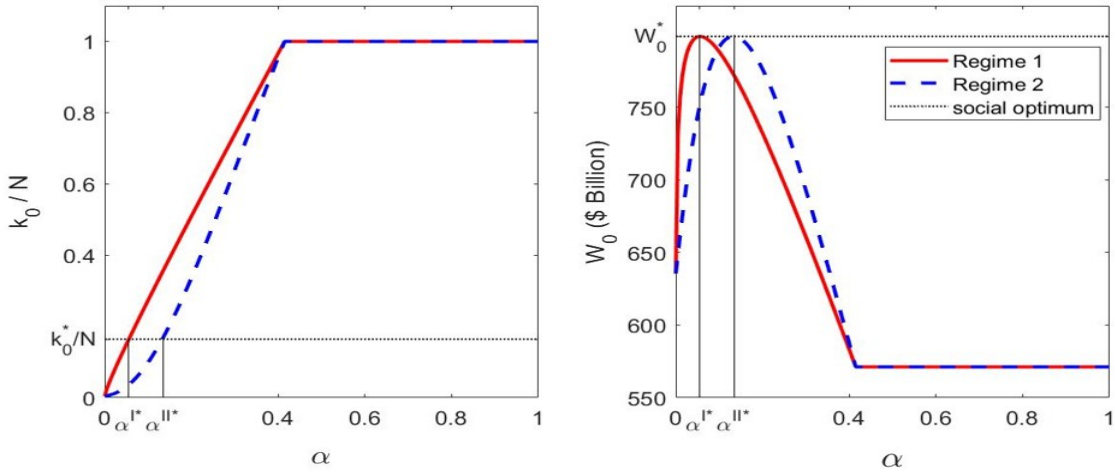

Fig. 12. Effect of α : PC

Comparative statics for α^*.—Figure 13 plots comparative statics for the socially optimal compensation share α^* under Regimes 1 and 2 based on the auto calibration. The results show the following:

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- α^* *increases with* β.—A higher β means a lower price elasticity, which leads price to decline faster which discourages k_0. This makes the matching externality less of a concern, so α^* rises.

- α^* *decreases with* γ *(holding N fixed, when γ is sufficiently large)*.—A higher γ implies a better imitation technology, so the planner would need less innovation when γ is sufficiently large and so α^* falls.

- α^* *rises with c but falls with* $\bar{A}$ $(\equiv \tilde{A}q^{1-\beta})$.—A higher c or a lower $\bar{A}$ discourages k_0. This makes the matching externality less of a concern, so α^* rises.

- α^* *rises with N* (*holding $\gamma N = \lambda$ fixed*).—A higher N leads to faster price decline which discourages k_0. This, together with a larger pool of potential adopters N, makes the matching externality less of a concern, so α^* rises.

- *Comparison of Regimes 1 and 2*.—α^* is higher under Regime 2 than under Regime 1, and the difference rises with β, γ, $c/\bar{A}$, and N.

The comparative statics help explain the difference in α^* between the auto and the PC industries. Compared to the auto, the PC industry has a smaller β and a larger γN, and these two dominate the offsetting forces of the larger $c/\bar{A}$ and larger N and hence $\alpha^*_{\text{PC}} < \alpha^*_{\text{Auto}}$ under each regime. Quantitatively, by comparing counterfactuals that let one industry take on the other industry's parameter values, we find that the smaller β (i.e., the higher price elasticity) accounts most for the smaller α^*_{PC}.
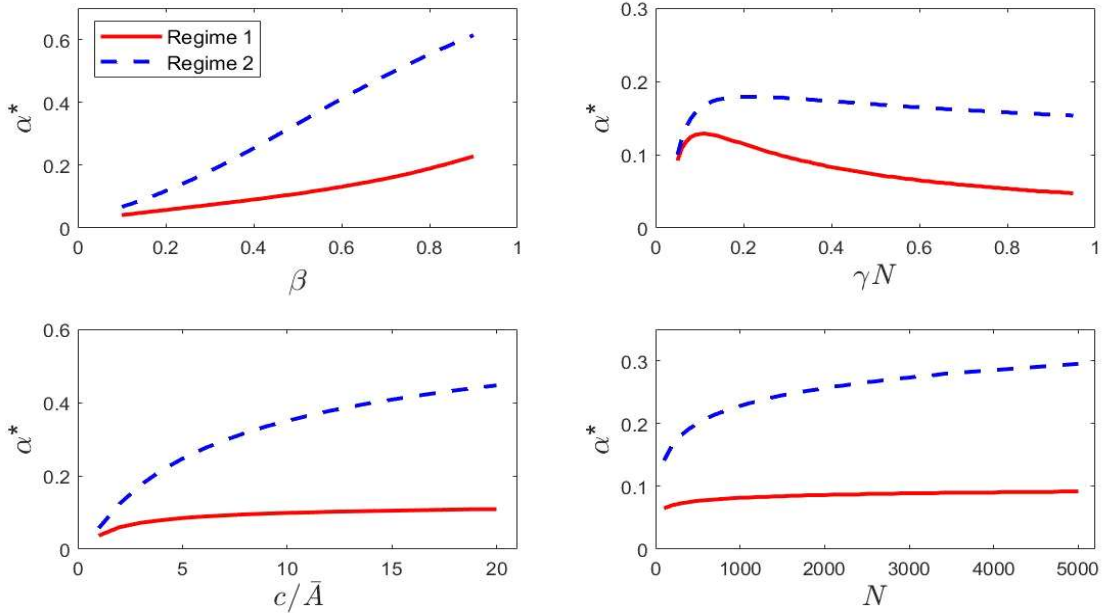

Fig. 13. COMPARATIVE STATICS FOR α^* UNDER REGIMES 1 AND 2

5.3.2 Optimal innovation subsidy

Given $\alpha = 0$, the entry of innovators is lower than the socially optimal level. Providing innovators a subsidy s, instead of setting a socially optimal α^*, can also help achieve the social optimum.

Note that with the subsidy, $c - s$ is the net entry cost for innovators. Figure 14 plots the effect of s on the entry of innovators k_0 and welfare W_0. The results show that k_0 increases with s, and the social welfare peaks at $s^*/c = 0.61$ for the auto industry and $s^*/c = 0.62$ for the PC industry.
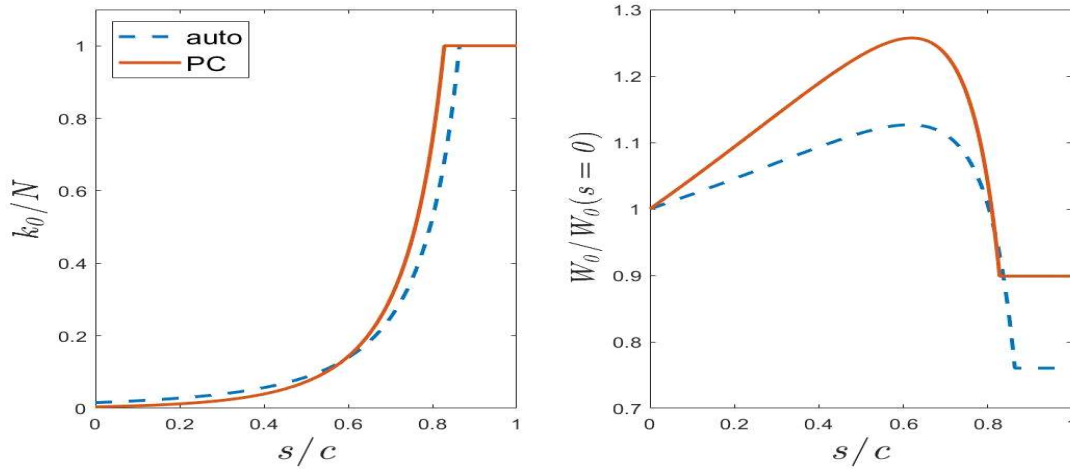

Fig. 14. EFFECT OF THE SUBSIDY s

5.3.3 Optimal diffusion rate

We can similarly evaluate the effects of varying the diffusion rate γ (holding N fixed). Consider again the scenario where innovators are not compensated by imitators, so $\alpha = 0$. Should the planner slow down the diffusion?
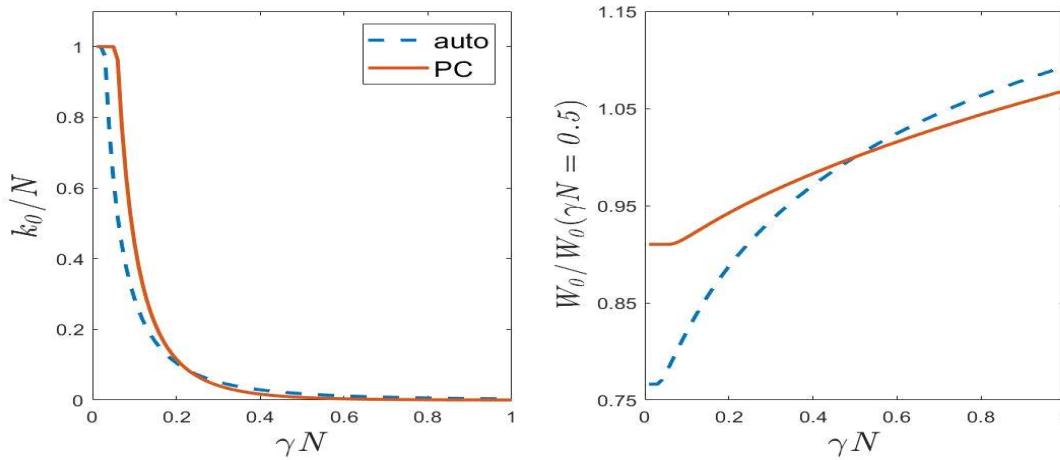

Fig. 15. EFFECT OF THE DIFFUSION RATE γ

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Figure 15 shows that for both the auto and the PC industries, k_0 decreases with γ while W_0 increases with γ. Therefore, if the planner were to push down γ, the entry of innovators k_0 would increase but social welfare would decline. The intuition is that while slowing down diffusion could encourage entry of innovators, it would forego too much free imitation and the welfare effect of the latter dominates.

5.4 Robustness checks

For robustness checks, we redo the above exercises with alternative assumptions on N, r and α. The results are consistent with our previous findings; restricting diffusion speed γ always reduces social welfare in both industries for alternative parameter values. In what follows we show how the socially optimal compensation for idea sellers α^* and the socially optimal innovation subsidy s^* depend on the parameters.

5.4.1 Pool of potential entrants

In the benchmark analysis, we assumed that the shakeout started after almost all the potential firms had entered the industry. Alternatively, one could consider that the shakeout started in the middle of the diffusion process, so there might be a larger pool of potential entrants. For example, we may assume $N = 1,000$ (instead of 210) for the auto case and $N = 2,000$ (instead of 425) for the PC case. In each case, we then obtain a smaller γN from the diffusion estimation. The new estimates imply that it would take 30 years for each industry to reach 99% adoption rate among potential producers had the shakeout not happened, doubling what is assumed in the benchmark calibration.

We then re-do the calibration and counterfactual exercises with the alternative N. Regarding the socially optimal compensation for idea sellers, we now find for the auto case, $\alpha_{\text{Auto}}^{\mathbf{I*}} = 0.134$ under Regime 1 or $\alpha_{\text{Auto}}^{\mathbf{II*}} = 0.309$ under Regime 2, while for the PC case, $\alpha_{\text{PC}}^{\mathbf{I*}} = 0.095$ under Regime 1 or $\alpha_{\text{PC}}^{\mathbf{II*}} = 0.215$ under Regime 2. These estimates of α^* are larger than those found in the benchmark analysis, due to the larger N and smaller γN and higher c from the re-calibrated models, which is consistent with the prediction of our comparative-statics analysis (cf. Fig. 13). We also find that the social optimum can be achieved by subsidizing 65.7 percent of the innovation cost in the auto case, or 62.1 percent in the PC case.

5.4.2 Anticipated shakeout

Our model can also be extended to allow the shakeout being anticipated. Specifically, we could assume that the industry expects a disruptive innovation to arrive at a Poisson rate ρ. This innovation would make obsolete existing technologies and drive firm values to zero.[18]

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[18]For example, an industry may expect a disruptive innovation (e.g., the assembly line in the auto case) to arrive at a Poisson rate ρ. This innovation would require an incumbent firm to incur a big

Accordingly, the value of an incumbent firm under Regime 2 satisfies

$$rv_t = p_t + \gamma \left(N - k_t\right) \alpha v_t - \rho v_t + \frac{dv_t}{dt},$$

i.e.,

$$\left(r + \rho\right) v_t = p_t + \gamma \left(N - k_t\right) \alpha v_t + \frac{dv_t}{dt}. \tag{38}$$

Including $\rho > 0$ in Eq. (38) is equivalent to raising r to $r + \rho$ in Eq. (8). Similarly, we can revise the value function conditions for outsiders as well as for Regime 1 and for the social planner's problem. The original functional forms of our model hold, except that r becomes $r + \rho$.

Considering that the shakeout occurred in the 16th year for the auto industry and in the 18th year for the PC industry, we take the average and calibrate $\rho = 1/17 = 0.06$. Accordingly, we set $r + \rho = 0.05 + 0.06 = 0.11$ and redo the model calibration and counterfactual analysis.

Regarding the socially optimal compensation for idea sellers, we now find in the auto case, $\alpha_{\text{Auto}}^{\textbf{I}*} = 0.147$ under Regime 1 or $\alpha_{\text{Auto}}^{\textbf{II}*} = 0.296$ under Regime 2, while in the PC case, $\alpha_{\text{PC}}^{\textbf{I}*} = 0.115$ under Regime 1 or $\alpha_{\text{PC}}^{\textbf{II}*} = 0.235$ under Regime 2. The values of α^* are larger than the benchmark analysis due to the higher discount $r + \rho$ in spite of lower c implied by the re-calibrated models. We also find that the social optimum can be achieved by subsidizing 59.2 percent of the innovation cost in the auto case or 58.4 percent in the PC case.

5.4.3   Idea sellers' bargaining share

We assumed $\alpha = 0$ in the benchmark model calibration. For robustness checks, we redo the calibration for other values $\alpha \in (0, 1]$. The results are plotted in Fig. 16.

Note that the larger value α we assume in the calibration, the higher innovation cost c must be to rationalize the observations. But then, as Fig. 16 shows, the implied rise in c means that α^* rises, consistent with our comparative-statics analysis shown in Fig. 13.

Moreover, Fig. 16 shows that $\alpha^* \in (0, 1)$ and that it crosses the 45-degree line. In the range where $\alpha^* > \alpha$, an innovation subsidy could help improve welfare; otherwise, a tax would do so.

Finally, for all $\alpha \in [0, 1]$, α^* remains small in Regime 1 between 7%-12.6% for the auto and between 5.6%-9.3% for the PC. In Regime 2, as the assumed α gets larger, α^* does increase quite a bit for both industries and the optimal α^* for the PC eventually exceeds that for the auto as the effect of c starts to dominate. However, to

capital investment to produce a newly designed product at a massive scale. When that innovation does arrive, the new (and lower) equilibrium price can only support the capital investment made by a few firms and the rest have to exit. As a result, the present value of an investing firm (net of its investment costs) is zero, and the value of an exiting firm is also zero.

the extent that Regime 2 applies naturally to scenarios of non-patented know-how, a small value of assumed α is more realistic, which would also imply a small α^*. Note that both auto and PC appear to have highly elastic demands – we estimate β_{PC} and β_{Auto} to be quite small. Imitator entry then drives prices down slowly, and that encourages innovation, and raises the matching externality that innovators impose. Then α^*_{Auto} and α^*_{PC} should both be low, especially α^*_{PC}, because PC demand is more price elastic.

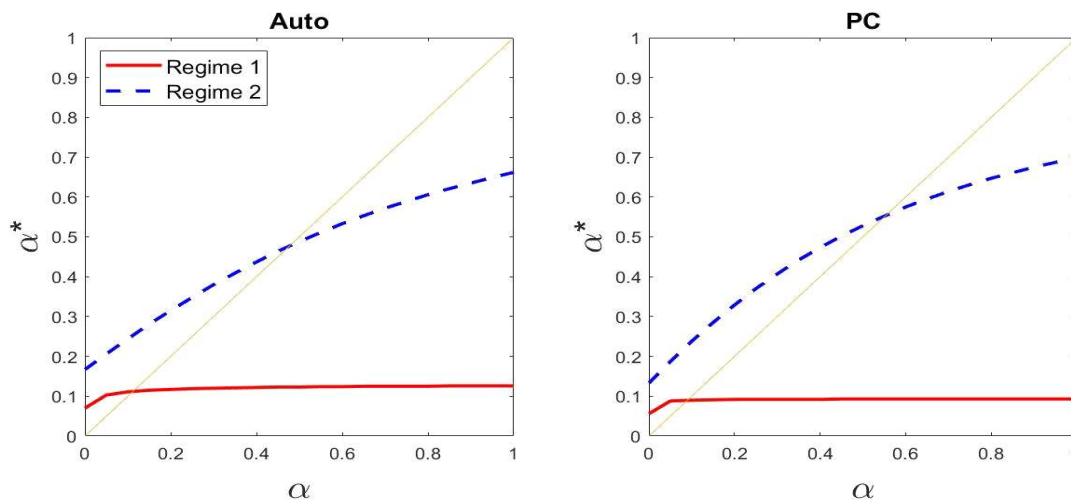Fig. 16. ASSUMED α AND IMPLIED α^*

6 Additional discussion

6.1 Matching function specification

In our model, the meeting function (3) features increasing returns to scale. However, the assumption on returns to scale is inessential for our analysis. To see why, let us generalize Eq. (3) to

$$\frac{dk_t}{dt} = \hat{\gamma} k_t \left(N - k_t \right) \qquad \text{where} \quad \hat{\gamma} = \frac{\gamma}{N^\psi}.$$

The solution of k_t then becomes

$$k_t = \frac{N e^{\hat{\gamma} N t}}{e^{\hat{\gamma} N t} + \frac{N}{k_0} - 1} = \frac{N e^{\gamma N^{1-\psi} t}}{e^{\gamma N^{1-\psi} t} + \frac{N}{k_0} - 1}.$$

By rescaling the diffusion parameter γ with a constant $\frac{1}{N^\psi}$, the meeting function features increasing returns to scale if $\psi < 1$, constant returns if $\psi = 1$, and decreasing returns if $\psi > 1$. In the model, we assume that $\psi = 0$, but our analysis and findings would hold for any ψ because a time series study takes N and $\frac{\gamma}{N^\psi}$ as given; the value

of ψ plays no role except in a counterfactual that would involve changing the value of N.

Also, the labor search literature often assumes a Cobb-Douglas matching function:

$$\frac{dk_t}{dt} = \gamma k_t^\theta \left(N - k_t \right)^{1-\theta},$$

where $0 < \theta < 1$. However, the Cobb-Douglas formulation does not appear to fit data better, and more importantly, it does not have a closed-form solution for the time path of k_t. Therefore the logistic formulation we use has analytical advantages.
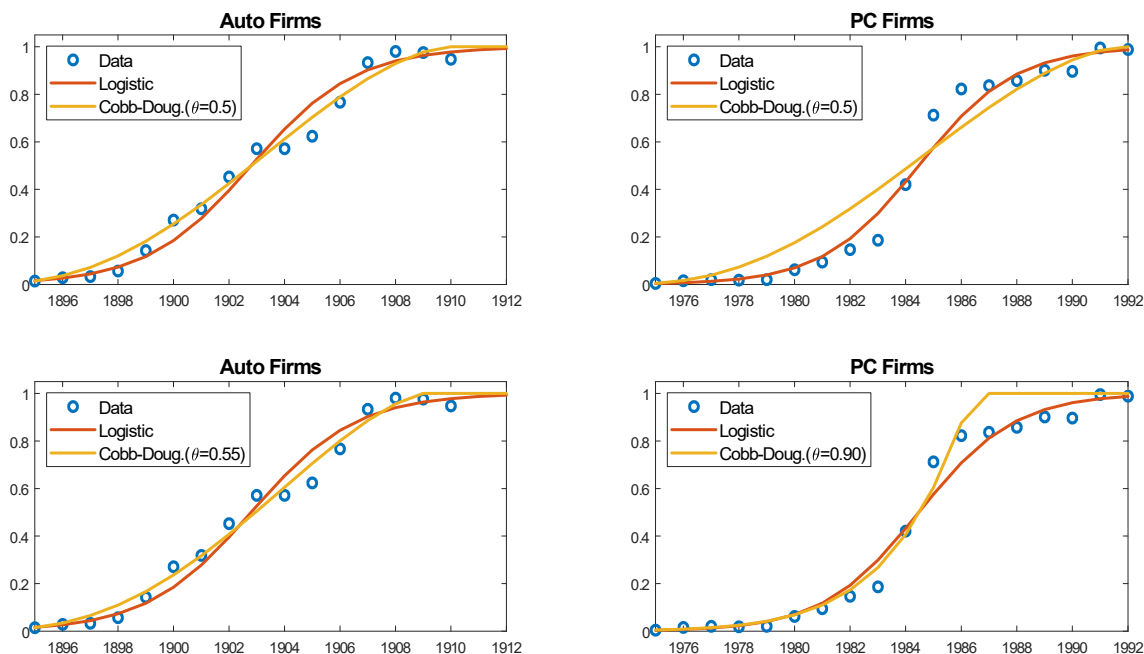

Fig. 17. Diffusion Models: Fitting Time Paths of Firm Numbers

Figure 17 shows that the estimated logistic diffusion model (cf. Eq. (4)) matches the time paths of firm numbers well for U.S. automobile and PC industries. Comparing with the symmetric Cobb-Douglas counterpart (i.e., $\theta = 0.5$), logistic diffusion shows a more pronounced inflection point and fits better for the PC industry, as shown in the top panels. In the bottom panels, we compare logistic diffusion with the best fitting Cobb-Douglas formulation for each industry without restricting θ. The former still fits better for the PC industry.[19]

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[19] The Cobb-Douglas diffusion curves plotted in Fig. 17 are ones that minimize the sum of the squares of the prediction errors. The data fitting suggests that Cobb-Douglas curves fit slightly better in the auto case ($R^2 = 0.985$ when $\theta = 0.5$ and $R^2 = 0.987$ when $\theta = 0.55$) than the logistic curve ($R^2 = 0.975$), but the logistic curve fits better in the PC case ($R^2 = 0.981$) than the Cobb-Douglas curves ($R^2 = 0.936$ when $\theta = 0.5$ and $R^2 = 0.969$ when $\theta = 0.90$).

6.2 Heterogeneous production capacity

The model assumes that all firms have the same constant production capacity. The constancy of output per firm prior to the shakeout shown in Figures 5 and 7 suggests that in the period prior to the shakeout this is not unreasonable. Still, the model can accommodate some output heterogeneity with the following minor adjustment: If k_t is the number of firms at date t, suppose the production capacity of innovator i is s_i, a random variable with CDF $H(s_i)$ over the support $(0, \bar{s})$ and mean equal to unity.

Among innovators we assume that the variables s_i are random, independent over i, but are realized after the innovator has paid the cost c. We also assume that meetings are undirected so that H pertains to the imitators too. Industry output would be

$$Q_t = k_t \int_0^{\bar{s}} s \, dH(s) = k_t.$$

Then $Q_t = k_t$, and p_t would remain the same as before. Ex ante expected revenue of firm i would be $p_t E(s_i) = p_t$. The rest of the analysis would stay unchanged.

A subset of the values of s in the support of H could then develop into "dominant designs" in the terminology of Utterback and Abernathy (1975) and survive the shakeout.

6.3 The $N \to \infty$ limit

The special case where $N \to \infty$ does not fit the industry data well, but the solutions are simpler and may help illustrate the findings of our model. In this subsection, we study a limiting version of our model with $N \to \infty$. All proofs are in Appendix A.10.

Let N get large but at the same time reduce γ so that $\gamma N \to \lambda > 0$, a constant. The logistic diffusion process (3) then converges to

$$\frac{dk_t}{dt} = \lambda k_t, \tag{39}$$

and then (4) becomes

$$k_t = k_0 e^{\lambda t}. \tag{40}$$

This essentially is the diffusion process considered by earlier models of competitive innovation, e.g., Boldrin and Levine (2002, 2008) and Quah (2002). Those studies embed such a diffusion process in a growth model, and we now incorporate it into our industry dynamic model. In what follows we shall assume that

$$\lambda < r \tag{41}$$

which is necessary for welfare to be bounded.

6.3.1 Equilibrium when $N \to \infty$

Then in Regime 1 an innovator's value at date t is

$$v_t = \frac{Ak_0^{-\beta}e^{-\beta\lambda t}}{r + \beta\lambda} + \frac{\alpha\lambda Ak_0^{-\beta}}{(r + \beta\lambda)(r + (\beta - 1)\lambda)}e^{-(\beta-1)\lambda t}, \tag{42}$$

while an imitator's value is

$$\omega_t = \frac{Ak_0^{-\beta}}{r + \beta\lambda}e^{-\beta\lambda t}. \tag{43}$$

Because the pool of outsiders is infinite, an outsider's chance of meeting an incumbent is zero so that $u_t = 0$ for all t. Finally, we pin down the entry of innovators at date 0 to be

$$k_0^{\mathbf{I}} = \left(\frac{A(r + (\beta - 1)\lambda + \alpha\lambda)}{c(r + \beta\lambda)(r + (\beta - 1)\lambda)}\right)^{\frac{1}{\beta}}, \tag{44}$$

which is valid for $\beta \geq 1$ or for all $\beta > 0$ if $\alpha = 0$. No innovator enters after date 0.

In Regime 2, the value of a producer (innovator or imitator) at date t is

$$v_t = \frac{Ak_0^{-\beta}e^{-\beta\lambda t}}{r + (\beta - \alpha)\lambda}, \tag{45}$$

which decreases with t. Again, given $N \to \infty$, an outsider's chance of meeting an incumbent is zero so that $u_t = 0$. Therefore, $v_t - u_t$ decreases with t and no innovator enters after date 0. Since the free entry condition requires $v_0 = c$, Eq. (45) evaluated at $t = 0$ yields

$$k_0^{\mathbf{II}} = \left(\frac{A}{c(r + (\beta - \alpha)\lambda)}\right)^{\frac{1}{\beta}}. \tag{46}$$

Equations (44) and (46) suggest that even when $\alpha = 0$, $k_0^{\mathbf{I}} = k_0^{\mathbf{II}} > 0$. Thus, in both regimes, the model reproduces Boldrin and Levine's (2008) claim that innovations can take place in competitive markets even with unpriced spillover externalities. Moreover, differentiation of Eqs. (44) and (46) shows that $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ both increase with α and A, but decrease with c and r.

The effect of λ.—Consistent with our finding in Proposition 3, the effect of diffusion speed on innovation depends on α and β. In Regime 1 (cf. Eq. (44)), $k_0^{\mathbf{I}}$ falls with λ when $\beta \geq 1 > \alpha$ or when $\beta > \alpha = 0$, and the effect of λ on $k_0^{\mathbf{I}}$ vanishes when $\beta = \alpha = 1$. In Eq. (46), $k_0^{\mathbf{II}}$ falls with λ if $\beta > \alpha$, or increases with λ if $\beta < \alpha$, and it vanishes if $\beta = \alpha$.

All parameters being equal across the two regimes, Eqs. (44) and (46) imply that $\left(\frac{k_0^{\mathbf{I}}}{k_0^{\mathbf{II}}}\right)^{\beta} = 1 + \frac{\alpha(1-\alpha)\lambda^2}{(r+\beta\lambda)(r+(\beta-1)\lambda)}$ so that

$$\frac{k_0^{\mathbf{I}}}{k_0^{\mathbf{II}}} = \begin{cases} 1 & \text{for } \alpha \in \{0, 1\} \\ > 1 & \text{for } \alpha \in (0, 1) \end{cases}. \tag{47}$$

Hence, Regime 1 yields more innovation unless $\alpha \in \{0, 1\}$.

6.3.2 Welfare when $N \to \infty$

Optimal compensation for idea sellers   While $\alpha = 0$ is compatible with positive innovation, it is not optimal. In fact, the limiting model implies that allowing the original innovators to extract the entire rents from succeeding imitators would yield the socially optimal incentive for innovation. This result holds for both Regimes 1 and 2.

Formally, we assume that condition (41) holds and prove that it is socially optimal to innovate only at date 0 and that the socially optimal number of innovators is

$$k_0^* = \left(\frac{A}{(r + (\beta - 1)\lambda)c} \right)^{\frac{1}{\beta}}. \tag{48}$$

Comparing Eq. (48) to Eqs. (44) and (46) shows that $k_0^* = k_0^{\mathbf{I}} = k_0^{\mathbf{II}}$ iff $\alpha = 1$. I.e., $\alpha^* = 1$ for both Regimes 1 and 2.[20]

Why does the limiting model yield an optimal compensation share for idea sellers different from that when N is finite? The key is that when N is finite, there is a matching externality that an innovator creates and ignores, which reduces other agents' innovation payoff. In the limiting model, however, there is no such externality – an innovator's meeting rate is fixed at $\frac{dk_t/dt}{k_t} = \lambda$ while an imitator's meeting rate is fixed at 0 given that $N \to \infty$. Therefore, the finite-N version fits the industry evolution pattern better and it also features a matching externality.

Optimal innovation subsidy   Given that the optimal compensation for idea sellers is $\alpha^* = 1$, the planner could offer an innovation subsidy $s^* > 0$ to achieve the social optimum whenever $\alpha < 1$. With $k_0^{\mathbf{I}}$, $k_0^{\mathbf{II}}$ and k_0^* given in Eqs. (44), (46) and (48), social optimum implies

$$\begin{aligned} 0 &< s^{\mathbf{I}*} < s^{\mathbf{II}*} & &\text{for } \alpha \in (0, 1), \\ s^{\mathbf{I}*} &= s^{\mathbf{II}*} = \frac{c\lambda}{r + \beta\lambda} > 0 & &\text{for } \alpha = 0, \\ s^{\mathbf{I}*} &= s^{\mathbf{II}*} = 0 & &\text{for } \alpha = 1. \end{aligned} \tag{49}$$

The optimal subsidy never goes negative (i.e., becomes a tax), which is in contrast to our finding with logistic diffusion. Again, this is because the limiting model incorporates only knowledge spillovers but not meeting externalities.

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[20] Earlier studies on competitive innovation by Boldrin and Levine (2002) and Quah (2002) assume that ideas are embodied in the tangible capital used to produce a new consumption good. The capital then grows at a constant rate as the result of diffusion and imitation. They show that if the capital is priced to allow the initial innovator to appropriate the entire competitive rent, the competitive market achieves the social optimum even without intellectual property right protection. Our analysis shows that $\alpha^* = 1$ yields social optimum when imitators grow at a constant rate. Because ideas in our model take intangible forms, the innovator cannot collect revenue by selling capital to new entrants and therefore intellectual property right protection is needed to achieve the social optimum. More importantly, we have shown that $\alpha^* = 1$ is not socially optimal when diffusion follows a logistic process.

Optimal diffusion rate   Parallel to the logistic diffusion case where the planner does not want to reduce γ, here the planner does not want to reduce λ:

$$\frac{\partial W_0}{\partial \lambda} > 0 \quad \text{for } \alpha \in [0,1], \text{ and } \beta > 0 \tag{50}$$

for both Regimes 1 and 2.

7   Conclusion

We modeled an innovation and its diffusion in one industry and calibrated the model to data of the U.S. automobile and personal computer industries. Though starting nearly one century apart, the two industries shared the basic feature of an S-shaped diffusion prior to the shakeout. Our empirical findings match well the expansion of firm numbers prior to the shakeout in each industry and quantify the theoretical predictions of the model.

Our analysis offers several results regarding welfare and policy. First, capacity constraints imply that licensing raises the revenues of innovators and, to a degree, licensing is socially beneficial. Second, socially optimal compensation of innovators should be only partial due to the matching externality in the meetings in which idea transfers take place. Third, the optimal bargaining share for innovators should be higher if imitators can resell ideas to other imitators.

Finally, a policy restricting diffusion always reduces welfare. And when demand is elastic and innovators' compensation share is high, slowing down diffusion reduces output at all dates. In other cases, slowing down diffusion may encourage innovation and raise initial capacity, but would lower imitation so that capacity grows more slowly. As a result, the mechanism can generate industry overtaking.

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Appendix to "Idea Diffusion and Property Rights"

Boyan Jovanovic and Zhu Wang

A. Proofs

A.1. Proof of Proposition 1(A)

Proof. In Regime 1, potential adopters can copy an idea from an imitator but the fee goes to the idea's original innovator. We first assume that at equilibrium, innovators only enter at date 0 and $k_0 < N$, so the time path of firm numbers is determined by Eq. (4) that

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + \frac{N}{k_0} - 1}.$$

We then check if any agent would want to deviate by entering as an innovator at a date $\tau > 0$.

The entry of a measure-zero innovator at $\tau > 0$ would not change the industry quantity and price through Eq. (4). Upon entry, the value of this innovator is determined by two sources: One is that he will receive a fraction $1/k_t$ of the total industry revenues $Ak_t^{1-\beta}$ at each date $t \geq \tau$ by selling goods; the other is that he will get a chance $1/k_\tau$ to collect idea-sale revenues from new imitators at each date $t \geq \tau$ (Note that k_τ is the number of all incumbent firms at his entry date τ, so $1/k_\tau$ is the probability for new imitators at each date $t \geq \tau$ to trace him as the original innovator of the idea they copy). At each date $t \geq \tau$, a fraction $\frac{k_t - k_\tau}{k_t}$ of firms in the industry are imitators who enter between date τ and date t, so this new innovator at his entry date τ expects to have $1/k_\tau$ chance to receive the discounted sum of the fraction $\alpha (k_t - k_\tau)/k_t$ of the total industry revenues $Ak_t^{1-\beta}$ as idea-sale revenues starting from date τ.[21]

Therefore, the value of a new innovator at his entry date τ, denoted by v_τ^τ, can be written as

$$v_\tau^\tau = \int_\tau^\infty e^{-r(t-\tau)} \left(\frac{1}{k_t} + \frac{\alpha}{k_\tau}(\frac{k_t - k_\tau}{k_t}) \right) Ak_t^{1-\beta}dt. \tag{51}$$

Note that the entry value of an innovator v_τ^τ varies by entry date τ because the number of existing firms k_τ increases with τ. Also, v_τ^τ is different from the value of an

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[21]Note that in Regime 1, an imitator's date-t present value ω_t is given by Eq. (5). In principle, one could characterize v_τ^τ by calculating the integral of ω_t over time, but no close-form solution exists except for special cases (e.g., $\beta = 1$). Therefore, we instead calculate the date-τ present value of imitators' shares of industry revenues.

existing innovator at date τ who entered before τ. In fact, an innovator who enters at date 0 should have the value at date τ :

$$v_\tau^0 = \int_\tau^\infty e^{-r(t-\tau)} \left(\frac{1}{k_t} + \frac{\alpha}{k_0}(\frac{k_t - k_\tau}{k_t}) \right) Ak_t^{1-\beta}dt, \tag{52}$$

so $v_\tau^\tau < v_\tau^0$ for any $\tau > 0$, and $v_\tau^\tau = v_\tau^0$ for $\tau = 0$.

Equation (4) implies that for any date $t \geq \tau$,

$$k_t = \frac{Ne^{\gamma N(t-\tau)}}{e^{\gamma N(t-\tau)} + (\frac{N}{k_\tau} - 1)}, \tag{53}$$

and

$$\frac{k_t}{k_\tau} = \frac{Ne^{\gamma N(t-\tau)}}{N + (e^{\gamma N(t-\tau)} - 1)k_\tau}. \tag{54}$$

We can rewrite Eq. (51) as

$$v_\tau^\tau = \int_\tau^\infty e^{-r(t-\tau)} \left(1 - \alpha + \alpha\frac{k_t}{k_\tau} \right) Ak_t^{-\beta}dt. \tag{55}$$

Defining $s = t - \tau$, Eq. (55) becomes

$$v_\tau^\tau = \int_0^\infty e^{-rs} \left(1 - \alpha + \alpha\frac{k_{\tau+s}}{k_\tau} \right) Ak_{\tau+s}^{-\beta}ds. \tag{56}$$

Note that Eqs. (53) and (54) imply that

$$\frac{k_{\tau+s}}{k_\tau} = \frac{Ne^{\gamma Ns}}{N + (e^{\gamma Ns} - 1)k_\tau}, \quad k_{\tau+s}^{-\beta} = \left(\frac{Ne^{\gamma Ns}}{e^{\gamma Ns} + (\frac{N}{k_\tau} - 1)} \right)^{-\beta},$$

which both decrease in k_τ. In Eq. (56), because k_τ increases in τ, $\frac{k_{\tau+s}}{k_\tau}$ and $k_{\tau+s}^{-\beta}$ decrease in τ, and hence v_τ^τ decreases in τ.

Similarly, because an imitator can keep $(1 - \alpha)$ share of his output, the total value of outsiders $u_\tau(N - k_\tau)$ at date τ equals the imitators' share of the total discounted industry revenues from date τ and onward. Therefore, we have

$$u_\tau(N - k_\tau) = \int_\tau^\infty e^{-r(t-\tau)} \left(\frac{(1-\alpha)(k_t - k_\tau)}{k_t} \right) Ak_t^{1-\beta}dt,$$

which implies

$$u_\tau = \int_\tau^\infty e^{-r(t-\tau)} \left(\frac{(1-\alpha)(k_t - k_\tau)}{k_t(N - k_\tau)}) \right) Ak_t^{1-\beta}dt. \tag{57}$$

Inserting Eq. (54) into Eq. (57), we derive

$$u_\tau = \int_\tau^\infty e^{-r(t-\tau)} \left(\frac{(1-\alpha)\left(e^{\gamma N(t-\tau)} - 1\right)}{Ne^{\gamma N(t-\tau)}} \right) Ak_t^{1-\beta}dt. \tag{58}$$

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Again, defining $s = t - \tau$, Eq. (58) becomes

$$u_\tau = \int_0^\infty e^{-rs} \left(\frac{(1-\alpha)\left(e^{\gamma Ns}-1\right)}{Ne^{\gamma Ns}} \right) Ak_{\tau+s}^{1-\beta} ds. \tag{59}$$

Equation (59) implies that if $\beta = 1$, u_τ is a constant that does not vary with τ; if $\beta > 1$, $k_{\tau+s}^{1-\beta}$ decreases with k_τ so u_τ decreases with τ; and if $\beta < 1$, $k_{\tau+s}^{1-\beta}$ increases with k_τ so u_τ increases with τ. Moreover, combining Eqs. (56) and (59), we have

$$v_\tau^\tau - u_\tau = \int_0^\infty e^{-rs} \left(1 - \alpha + \alpha \frac{k_{\tau+s}}{k_\tau} - \frac{(1-\alpha)\left(e^{\gamma Ns}-1\right)}{Ne^{\gamma Ns}} k_{\tau+s} \right) Ak_{\tau+s}^{-\beta} ds. \tag{60}$$

Within the integral of Eq. (60), both terms $\left(1 - \alpha + \alpha \frac{k_{\tau+s}}{k_\tau} - \frac{(1-\alpha)\left(e^{\gamma Ns}-1\right)}{Ne^{\gamma Ns}} k_{\tau+s} \right)$ and $k_{\tau+s}^{-\beta}$ decrease in τ, so $v_\tau^\tau - u_\tau$ decreases with τ. Therefore, given the free entry condition $v_0^0 - u_0 = c$, we have $v_\tau^\tau - u_\tau < c$ for any $\tau > 0$, so no innovator would enter the industry after date 0.

Denote $v_0 = v_0^0$. Equations (52) and (57) yield that

$$v_0 - u_0 = \frac{1}{N - k_0} \int_0^\infty e^{-rt} \left(\alpha \frac{N}{k_0} + (1-\alpha) \frac{N}{k_t} - 1 \right) Ak_t^{1-\beta} dt, \tag{61}$$

The free entry condition $v_0 - u_0 = c$ then pins down the entry of innovators k_0 at date 0, as shown by Eq. (12):

$$\frac{\int_0^\infty e^{-rt} \left(\alpha \frac{N}{k_0} + (1-\alpha) \frac{N}{k_t} - 1 \right) Ak_t^{1-\beta} dt}{N - k_0} = c.$$

Interior solution (i.e., $k_0^{\mathbf{I}} < N$).— We have shown above that $k_0^{\mathbf{I}}$ is determined by Eq. (12). Note that as $k_0 \to N$, we have $k_t \to N$. Hence, both the numerator and the denominator of the left hand side of Eq. (12) goes to 0 as $k_0 \to N$. Applying L'Hôpital's rule, the left hand side of Eq. (12) converges to $\frac{\alpha\gamma N + r}{r(r+\gamma N)} AN^{-\beta}$ as $k_0 \to N$. Proposition 2(A) shows that $dk_0^{\mathbf{I}}/dc < 0$. Therefore, the model has an interior solution $k_0^{\mathbf{I}} < N$ in Regime 1 iff condition (11) holds, i.e.,

$$c > \frac{(\alpha\gamma N + r)}{r(r + \gamma N)} AN^{-\beta}.$$

Additional details of the dynamic path.— The proof above confirms that innovators only enter at date 0. The time path of firm numbers is given by Eq. (4), and

the time path of u_t is solved by Eq. (59). Following that, the dynamic paths of ω_t, v_t and v_t^τ for any $t \geq 0$ can also be derived. Recall Eq. (5) that

$$r\omega_t = p_t + \frac{d\omega_t}{dt},$$

which yields that

$$\omega_t = \int_t^\infty e^{-r(s-t)} p_s ds = \int_t^\infty e^{-r(s-t)} A k_s^{-\beta} ds. \tag{62}$$

Because k_s increases in s, ω_t declines in t.

Consider counterfactually a marginal innovator enters at date $\tau \geq 0$. From any date $t \geq \tau$, he collects $A k_s^{-\beta}$ in each period $s \geq t$ by selling goods, and collect a fraction $\frac{\alpha(k_s - k_t)}{k_s k_\tau}$ of the total industry revenues $A k_s^{1-\beta}$ from new entrants after date t by selling ideas. Therefore, his value at date t is determined by

$$v_t^\tau = A \int_t^\infty e^{-r(s-t)} k_s^{-\beta} ds + \frac{A}{k_\tau} \int_t^\infty e^{-r(s-t)} \alpha (1 - \frac{k_t}{k_s}) k_s^{1-\beta} ds. \tag{63}$$

Because k_τ increases in τ, v_t^τ declines in τ.

Finally, Eq. (63) suggests that an innovator who entered at date 0 has value v_t for any $t \geq 0$:

$$v_t = v_t^0 = A \int_t^\infty e^{-r(s-t)} k_s^{-\beta} ds + \frac{A}{k_0} \int_t^\infty e^{-r(s-t)} \alpha (1 - \frac{k_t}{k_s}) k_s^{1-\beta} ds. \tag{64}$$

Equation (64) suggests that the time path of v_t depends on parameter values. For example, v_t may decrease in t when $\alpha = 0$ or when $\beta \geq 1$, or v_t may initially increase and later decrease in t if α is close to 1, β is close to zero, and c is large enough. $\blacksquare$

A.2. Proof of Proposition 1(B)

Proof. In Regime 2, all firms at date-t share the same value v_t regardless of their entry date or type. In this case, we can characterize the differential equations (8)-(9) directly to derive the dynamics of all variables. We first conjecture that no agent would enter as an innovator after date 0 and $k_0 < N$, so the time path of firm numbers is determined by Eq. (4) that

$$k_t = \frac{N e^{\gamma N t}}{e^{\gamma N t} + \frac{N}{k_0} - 1}.$$

Then, v_t is determined by Eq. (8) that

$$r v_t = p_t + \gamma (N - k_t) \alpha v_t + \frac{dv_t}{dt}$$

$$\Rightarrow$$

$$\frac{dv_t}{dt} - [r - \gamma(N - k_t)\alpha] v_t = -p_t. \tag{65}$$

Defining $z_t = \exp\left(\int -[r - \gamma(N - k_t)\alpha]\,dt\right)$, we can rewrite Eq. (65) as

$$\frac{d(z_t v_t)}{dt} = -z_t p_t,$$

which yields the general solution

$$v_t = z_t^{-1} \int -z_t p_t dt + z_t^{-1} C,$$

where C is a constant of integration.

Given that $k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt} + \frac{N}{k_0} - 1}$, we can solve z_t:

$$z_t = \exp\left(\int -[r - \gamma(N - k_t)\alpha]\,dt\right) = e^{-rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{-\alpha}.$$

Accordingly,

$$\begin{aligned}
v_t &= e^{rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{\alpha} \int -e^{-rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{-\alpha} p_t dt \\
&\quad + e^{rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{\alpha} C,
\end{aligned}$$

which requires $C = 0$ given that v_t needs to be bounded as $t \to \infty$. We then solve v_t as follows:

$$\begin{aligned}
v_t &= e^{rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{\alpha} \int -e^{-rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{-\alpha} p_t dt \\
&= e^{rt}\left(\frac{N - k_0}{k_0}e^{-\gamma Nt} + 1\right)^{\alpha} \int_t^{\infty} e^{-rs}\left(\frac{N - k_0}{k_0}e^{-\gamma Ns} + 1\right)^{-\alpha} p_s ds \\
&= \int_t^{\infty} e^{-r(s-t)}\left(\frac{\frac{N-k_0}{k_0}e^{-\gamma Ns} + 1}{\frac{N-k_0}{k_0}e^{-\gamma Nt} + 1}\right)^{-\alpha} p_s ds \\
&= \int_t^{\infty} e^{-r(s-t)}\left(1 - \frac{1 - e^{-\gamma N(s-t)}}{\frac{k_0}{N-k_0}e^{\gamma Nt} + 1}\right)^{-\alpha} p_s ds. \tag{66}
\end{aligned}$$

Defining $i = s - t$, we can rewrite Eq. (66) as

$$v_t = \int_0^{\infty} e^{-ri}\left(1 - \frac{1 - e^{-\gamma Ni}}{\frac{k_0}{N-k_0}e^{\gamma Nt} + 1}\right)^{-\alpha} p_{t+i}\,di.$$

In the integral, both terms $\left(1 - \frac{1-e^{-\gamma N i}}{\frac{k_0}{N-k_0}e^{\gamma N t}+1}\right)^{-\alpha}$ and $p_{t+i} = Ak_{t+i}^{-\beta}$ decrease in t. Therefore, v_t decreases with t.

Next, we show $v_t - u_t$ decreases with t. Recall that in Regime 2, u_t is determined by Eq. (9) that

$$ru_t = \gamma k_t \left((1-\alpha)v_t - u_t\right) + \frac{du_t}{dt},$$

which, together with Eq. (8), implies that

$$\frac{d\left(v_t - u_t\right)}{dt} = (r + \gamma k_t)(v_t - u_t) - (p_t + \gamma N \alpha v_t). \tag{67}$$

Defining $\psi_t \equiv v_t - u_t$, we can rewrite Eq. (67) as

$$\frac{d\psi_t}{dt} - (r + \gamma k_t)\psi_t = -(p_t + \gamma N \alpha v_t).$$

Define $z_t = \exp \int -(r + \gamma k_t)dt$. We then have

$$\frac{d(z_t\psi_t)}{dt} = -z_t(p_t + \gamma N \alpha v_t),$$

which yields the general solution

$$\psi_t = z_t^{-1} \int -z_t(p_t + \gamma N \alpha v_t)dt + z_t^{-1}C,$$

where C is a constant of integration.

Given that $k_t = \frac{Ne^{\gamma N t}}{e^{\gamma N t} + \frac{N}{k_0} - 1}$, we can solve z_t :

$$z_t = e^{-rt}\frac{k_0}{N - k_0 + k_0 e^{\gamma N t}}.$$

Again, ψ_t needs to be bounded as $t \to \infty$, so $C = 0$. We then have

$$
\begin{aligned}
\psi_t &= z_t^{-1} \int -z_t(p_t + \gamma N \alpha v_t)dt \\
&= e^{rt}\frac{N - k_0 + k_0 e^{\gamma N t}}{k_0} \int -e^{-rt}\frac{k_0}{N - k_0 + k_0 e^{\gamma N t}}(p_t + \gamma N \alpha v_t)dt \\
&= e^{rt}\frac{N - k_0 + k_0 e^{\gamma N t}}{k_0} \int_t^\infty e^{-rs}\frac{k_0}{N - k_0 + k_0 e^{\gamma N s}}(p_s + \gamma N \alpha v_s)ds \\
&= \int_t^\infty e^{-r(s-t)}\left(1 - \frac{e^{\gamma N(s-t)} - 1}{(\frac{N-k_0}{k_0})e^{-\gamma N t} + e^{\gamma N(s-t)}}\right)(p_s + \gamma N \alpha v_s)ds. \tag{68}
\end{aligned}
$$

Define $i = s - t$. We can rewrite Eq. (68) as

$$\psi_t = \int_0^\infty e^{-ri} \left(1 - \frac{e^{\gamma N i} - 1}{(\frac{N-k_0}{k_0})e^{-\gamma N t} + e^{\gamma N i}} \right) (p_{t+i} + \gamma N \alpha v_{t+i}) di.$$

Note that in the integral, both terms $\left(1 - \frac{e^{\gamma N i} - 1}{(\frac{N-k_0}{k_0})e^{-\gamma N t} + e^{\gamma N i}} \right)$ and $(p_{t+i} + \gamma N \alpha v_{t+i})$ decrease in t, hence $\psi_t = v_t - u_t$ strictly decreases with t. Given the free entry condition that $v_0 - u_0 = c$ at date 0, we know $v_t - u_t < c$ at any date $t > 0$, so no agent would enter as an innovator after date 0.

Note that Eq. (66) implies that

$$v_t = \int_t^\infty e^{-r(s-t)} \left(\frac{k_s}{k_t} \right)^\alpha p_s ds,$$

so that

$$v_0 = \int_0^\infty e^{-rt} \left(\frac{k_t}{k_0} \right)^\alpha A k_t^{-\beta} dt. \tag{69}$$

At date 0, the total industry discounted revenue, $\int_0^\infty e^{-rt} A k_t^{1-\beta} dt$, is shared by the two groups – the initial incumbents k_0 and the outsiders $N - k_0$. With the free entry condition $v_0 - c = u_0$ we have

$$\int_0^\infty e^{-rt} A k_t^{1-\beta} dt = v_0 k_0 + u_0 (N - k_0) = v_0 N - c (N - k_0). \tag{70}$$

Plugging Eq. (69) into Eq. (70) yields Eq. (13):

$$\frac{\int_0^\infty e^{-rt} \left(\left(\frac{N}{k_0} \right)^\alpha \left(\frac{N}{k_t} \right)^{1-\alpha} - 1 \right) A k_t^{1-\beta} dt}{N - k_0} = c.$$

Interior solution (i.e., $k_0^{\mathbf{II}} < N$).— We have shown that $k_0^{\mathbf{II}}$ is determined by Eq. (13). Note that as $k_0 \to N$, we have $k_t \to N$. Hence, both the numerator and the denominator of the left hand side of Eq. (13) goes to 0 as $k_0 \to N$. Applying L'Hôpital's rule, the left hand side of Eq. (13) converges to $\frac{\alpha \gamma N + r}{r(r + \gamma N)} A N^{-\beta}$ as $k_0 \to N$. Proposition 2(A) shows that $dk_0^{\mathbf{II}}/dc < 0$. Therefore, the model has an interior solution $k_0^{\mathbf{II}} < N$ in Regime 2 iff condition (11) holds, i.e.,

$$c > \frac{(\alpha \gamma N + r)}{r(r + \gamma N)} A N^{-\beta}.$$

∎

A.3. Proof of Proposition 2

Proof. (A) We first prove that $k_0^{\mathbf{I}}$ increases with α and A, but decreases with c and r. Rewrite Eq. (12) as

$$G = \int_0^\infty e^{-rt} F(t; k_0) dt - c = 0,$$

where

$$F(t; k_0) = \frac{1}{N - k_0} \left(\alpha \frac{N}{k_0} + (1 - \alpha) \frac{N}{k_t} - 1 \right) A k_t^{1-\beta},$$

and

$$k_t = \frac{N e^{\gamma Nt}}{e^{\gamma Nt} + \frac{N}{k_0} - 1}.$$

We verify that $\frac{\partial F(t; k_0)}{\partial k_0} < 0$, so $\frac{\partial G}{\partial k_0} < 0$. Following that, we can prove

$$\frac{\partial k_0^{\mathbf{I}}}{\partial \alpha} = -\frac{\partial G / \partial \alpha}{\partial G / \partial k_0^{\mathbf{I}}} > 0; \quad \frac{\partial k_0^{\mathbf{I}}}{\partial A} = -\frac{\partial G / \partial A}{\partial G / \partial k_0^{\mathbf{I}}} > 0 ;$$

$$\frac{\partial k_0^{\mathbf{I}}}{\partial c} = -\frac{\partial G / \partial c}{\partial G / \partial k_0^{\mathbf{I}}} < 0; \frac{\partial k_0^{\mathbf{I}}}{\partial r} = -\frac{\partial G / \partial r}{\partial G / \partial k_0^{\mathbf{I}}} < 0.$$

Similarly, with Eq. (13), we can prove that $k_0^{\mathbf{II}}$ increases with α and A, but decreases with c and r.

(B) First, it is straightforward to verify Eqs. (12) and (13) are identical when $\alpha \in \{0, 1\}$, so $k_0^{\mathbf{I}} = k_0^{\mathbf{II}}$.

Second, for any $\alpha \in (0, 1)$ and $t > 0$, we can apply the mean-value theorem to derive

$$\left(\frac{N}{k_0} \right)^\alpha \left(\frac{N}{k_t} \right)^{1-\alpha} = \left(\frac{N}{k_t} \right) \left(\frac{k_t}{k_0} \right)^\alpha = \left(\frac{N}{k_t} \right) \left(1 + \alpha \left(\frac{k'}{k_0} \right)^{\alpha - 1} \frac{(k_t - k_0)}{k_0} \right)$$

where $k_t > k' > k_0$. Therefore,

$$\left(\frac{N}{k_0} \right)^\alpha \left(\frac{N}{k_t} \right)^{1-\alpha} < \left(\frac{N}{k_t} \right) \left(1 + \alpha \frac{(k_t - k_0)}{k_0} \right) = \alpha \frac{N}{k_0} + (1 - \alpha) \frac{N}{k_t}. \tag{71}$$

Given that $k_0^{\mathbf{I}}$ and $k_t^{\mathbf{I}}$ satisfy Eq. (12) that

$$\int_0^\infty e^{-rt} \frac{1}{N - k_0^{\mathbf{I}}} \left(\alpha \frac{N}{k_0^{\mathbf{I}}} + (1 - \alpha) \frac{N}{k_t^{\mathbf{I}}} - 1 \right) A (k_t^{\mathbf{I}})^{1-\beta} dt = c,$$

the same $k_0^{\mathbf{I}}$ and $k_t^{\mathbf{I}}$ would not satisfy Eq. (13). Instead,

$$\int_0^\infty e^{-rt}\frac{1}{N-k_0^{\mathbf{I}}}\left(\left(\frac{N}{k_0^{\mathbf{I}}}\right)^\alpha\left(\frac{N}{k_t^{\mathbf{I}}}\right)^{1-\alpha}-1\right)A(k_t^{\mathbf{I}})^{1-\beta}dt < c, \qquad (72)$$

given the inequality (71).

The left-hand side of Eq. (13) can be written as

$$LHS = \int_0^\infty e^{-rt}F(t;k_0)dt$$

where

$$F(t;k_0) = \frac{1}{N-k_0}\left(\left(\frac{N}{k_0}\right)^\alpha\left(\frac{N}{k_t}\right)^{1-\alpha}-1\right)Ak_t^{1-\beta},$$

and

$$k_t = \frac{Ne^{\gamma Nt}}{e^{\gamma Nt}+\frac{N}{k_0}-1}.$$

We verify that $\frac{\partial F(t;k_0)}{\partial k_0} < 0$, so $\frac{\partial LHS}{\partial k_0} < 0$. Therefore, the solution $k_0^{\mathbf{II}}$ that satisfies Eq. (13) has to satisfy $k_0^{\mathbf{II}} < k_0^{\mathbf{I}}$. ∎

A.4. Proof of Proposition 3

Proof. (i) We first prove the results for Regime 1. Rewrite Eq. (12) as

$$G = \int_0^\infty e^{-rt}F(t;\gamma)dt - \frac{c}{AN^{1-\beta}} = 0,$$

where

$$F(t;\gamma) = \frac{1}{(N-k_0)}\left(\alpha\frac{N}{k_0}-1+(1-\alpha)\left(1+(\frac{N}{k_0}-1)e^{-\gamma Nt}\right)\right)\left(1+(\frac{N}{k_0}-1)e^{-\gamma Nt}\right)^{\beta-1}.$$

Note that

$$\frac{F(t;\gamma)}{\partial\gamma} \propto \left\{ \begin{array}{c} -(1-\alpha)\left(1+(\frac{N}{k_0}-1)e^{-\gamma Nt}\right) \\ -\left(\alpha\frac{N}{k_0}-1+(1-\alpha)\left(1+(\frac{N}{k_0}-1)e^{-\gamma Nt}\right)\right)(\beta-1) \end{array} \right\}.$$

Therefore, for inelastic or unit elastic demand (i.e., $\beta \geq 1$), we have $\partial G/\partial\gamma < 0$ (except $\partial G/\partial\gamma = 0$ when $\beta = \alpha = 1$). Recall that $\partial G/\partial k_0 < 0$ from the proof of Proposition 2. We then derive

$$\frac{\partial k_0^{\mathbf{I}}}{\partial\gamma} = -\frac{\partial G/\partial\gamma}{\partial G/\partial k_0^{\mathbf{I}}} < 0 \text{ (except } \frac{\partial k_0^{\mathbf{I}}}{\partial\gamma} = 0 \text{ when } \beta = \alpha = 1).$$

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We now consider the case of elastic demand (i.e., $\beta < 1$). Note that

$$\frac{F(t;\gamma)}{\partial\gamma} < 0 \iff (1-\alpha)\beta\left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right) > (\alpha\frac{N}{k_0} - 1)(1-\beta),$$

which holds for any $t \geq 0$ if

$$(1-\alpha)\beta > (\alpha\frac{N}{k_0} - 1)(1-\beta) \iff \alpha < \frac{1}{\frac{N}{k_0}(1-\beta) + \beta}.$$

Therefore, we have

$$\frac{\partial k_0^{\mathbf{I}}}{\partial\gamma} = -\frac{\partial G/\partial\gamma}{\partial G/\partial k_0^{\mathbf{I}}} < 0 \ \ \text{if} \ \ \alpha < \frac{1}{\frac{N}{k_0^{\mathbf{I}}}(1-\beta) + \beta}.$$

Similarly, we can derive

$$\frac{F(t;\gamma)}{\partial\gamma} > 0 \iff (1-\alpha)\left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)\beta < (\alpha\frac{N}{k_0} - 1)(1-\beta),$$

which holds for any $t \geq 0$ if

$$(1-\alpha)\left(\frac{N}{k_0}\right)\beta < (\alpha\frac{N}{k_0} - 1)(1-\beta) \iff \alpha > \beta + \frac{k_0}{N}(1-\beta).$$

Therefore, we have

$$\frac{\partial k_0^{\mathbf{I}}}{\partial\gamma} = -\frac{\partial G/\partial\gamma}{\partial G/\partial k_0^{\mathbf{I}}} > 0 \ \ \text{if} \ \ \alpha > \beta + \frac{k_0^{\mathbf{I}}}{N}(1-\beta).$$

(ii) Similarly, we can prove the results for Regime 2. Rewrite Eq. (13) as

$$G = \int_0^\infty e^{-rt}F(t;\gamma)dt - \frac{c}{AN^{1-\beta}} = 0,$$

where

$$F(t;\gamma) = \left(\left(\frac{N}{k_0}\right)^\alpha \left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{1-\alpha} - 1\right)\left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{\beta-1}.$$

Note that

$$\frac{\partial F(t;\gamma)}{\partial\gamma} \propto$$

$$\left\{ \begin{array}{l} -\left(\frac{N}{k_0}\right)^\alpha (1-\alpha)\left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{1-\alpha} \\ +\left(\left(\frac{N}{k_0}\right)^\alpha \left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{1-\alpha} - 1\right)(1-\beta) \end{array} \right\}.$$

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Therefore, for inelastic or unit elastic demand (i.e., $\beta \geq 1$), we have $\partial G/\partial \gamma < 0$ (except $\partial G/\partial \gamma = 0$ when $\beta = \alpha = 1$). Recall that $\partial G/\partial k_0 < 0$ from the proof of Proposition 2. We then derive

$$\frac{\partial k_0^{\mathbf{II}}}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial k_0^{\mathbf{II}}} < 0 \text{ (except } \frac{\partial k_0^{\mathbf{II}}}{\partial \gamma} = 0 \text{ when } \beta = \alpha = 1).$$

We now consider the case of elastic demand (i.e., $\beta < 1$). Note that

$$\frac{\partial F(t;\gamma)}{\partial \gamma} < 0 \iff \left(\frac{N}{k_0}\right)^{\alpha} \left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{1-\alpha} (\alpha - \beta) < (1 - \beta),$$

which holds for any $t \geq 0$ if

$$\left(\frac{N}{k_0}\right)(\alpha - \beta) < (1 - \beta) \iff \alpha < \frac{k_0}{N}(1 - \beta) + \beta.$$

Therefore, we have

$$\frac{\partial k_0^{\mathbf{II}}}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial k_0^{\mathbf{II}}} < 0 \ \text{ if } \ \alpha < \frac{k_0^{\mathbf{II}}}{N}(1 - \beta) + \beta.$$

Similarly, we can derive

$$\frac{\partial F(t;\gamma)}{\partial \gamma} > 0 \iff \left(\frac{N}{k_0}\right)^{\alpha} \left(1 + (\frac{N}{k_0} - 1)e^{-\gamma Nt}\right)^{1-\alpha} (\alpha - \beta) > (1 - \beta),$$

which holds for any $t \geq 0$ if

$$\left(\frac{N}{k_0}\right)^{\alpha}(\alpha - \beta) > (1 - \beta) \iff \alpha > \left(\frac{k_0}{N}\right)^{\alpha}(1 - \beta) + \beta.$$

Therefore, we have

$$\frac{\partial k_0^{\mathbf{II}}}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial k_0^{\mathbf{II}}} > 0 \ \text{ if } \ \alpha > \left(\frac{k_0^{\mathbf{II}}}{N}\right)^{\alpha}(1 - \beta) + \beta.$$

∎

A.5. Proof of Proposition 5

Proof. We first consider the case $\beta < 1$. For any date $\tau \geq 0$, if no further innovators enter, the number of firms at any date $t \geq \tau$ is

$$k_t = \frac{Ne^{\gamma N(t-\tau)}}{e^{\gamma N(t-\tau)} + \frac{N}{k_\tau} - 1} \qquad \text{for } t \geq \tau. \tag{73}$$

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As of date τ, the social return to innovation is

$$SR_\tau = \int_\tau^\infty e^{-r(t-\tau)} \frac{A}{1-\beta} k_t^{1-\beta} dt. \tag{74}$$

The current cost of innovation is c per unit, and its marginal social return (even if no further innovations are made) is

$$\frac{\partial SR_\tau}{\partial k_\tau} = \int_\tau^\infty e^{-r(t-\tau)} A k_t^{-\beta} \frac{\partial k_t}{\partial k_\tau} dt, \tag{75}$$

where

$$\frac{\partial k_t}{\partial k_\tau} = \frac{N^2 e^{\gamma N(t-\tau)}}{\left(N + \left(e^{\gamma N(t-\tau)} - 1\right) k_\tau\right)^2},$$

which is strictly decreasing in k_τ. And since k_t is increasing in t, $e^{-r(t-\tau)} k_t^{-\beta} \frac{\partial k_t}{\partial k_\tau}$ is also decreasing in k_τ. Therefore $\frac{\partial SR_\tau}{\partial k_\tau}$ is strictly decreasing in k_τ and so if at date zero k_0 is chosen so that $\frac{\partial SR_0}{\partial k_0} = c$, thereafter $\frac{\partial SR_\tau}{\partial k_\tau} < c$. Similarly, we can prove the result holds for $\beta \geq 1$. Hence, it is socially optimal to innovate only at date zero.

Since it is socially optimal to innovate only at date zero, the planner should choose k_0^* to maximize social welfare:

$$\max_{k_0} \left\{ \int_0^\infty e^{-rt} U(k_t) \, dt - c k_0 \right\}, \tag{76}$$

subject to Eq. (4). We can verify that the objective function is strictly concave in k_0, so the socially optimal number of innovators k_0^* is pinned down by the first-order condition which is Eq. (24) for any $\beta > 0$.

Interior solution (i.e., $k_0^* < N$).— Given that the social welfare function (76) is strictly concave in k_0, for $k_0^* < N$ to hold, one needs

$$\frac{d\left\{ \int_0^\infty e^{-rt} U(k_t) \, dt - c k_0 \right\}}{d k_0} \Big|_{k_0=N} < 0,$$

which yields condition (23):

$$c > \frac{A N^{-\beta}}{(r + \gamma N)}.$$

∎

A.6. Proof of Proposition 6

Proof. (A) Rewriting Eq. (24), we define

$$G = \int_0^\infty e^{-(r+\gamma N)t} \frac{A}{k_0^{*2}} \left(\frac{N e^{\gamma N t}}{e^{\gamma N t} + \left(\frac{N}{k_0^*} - 1\right)} \right)^{2-\beta} dt - c = 0.$$

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It follows that
$$\frac{\partial k_0^*}{\partial A} = -\frac{\partial G/\partial A}{\partial G/\partial k_0^*} > 0.$$
Similarly, we can prove $\partial k_0^*/\partial c < 0$ and $\partial k_0^*/\partial r < 0$.

The sign of $\partial k_0^*/\partial \gamma$ depends on $\partial G/\partial \gamma$ and requires some discussions.

$$\frac{\partial G}{\partial \gamma} \propto \int_0^\infty e^{-rt} \left\{ \begin{array}{c} (1-\beta)\left(1+(\frac{N}{k_0^*}-1)e^{-\gamma Nt}\right)^{\beta-2} \left(e^{\gamma Nt}+(\frac{N}{k_0^*}-1)\right)^{-1} (\frac{N}{k_0^*}-1)e^{-\gamma Nt}Nt \\ -\left(1+(\frac{N}{k_0^*}-1)e^{-\gamma Nt}\right)^{\beta-1} \left(e^{\gamma Nt}+(\frac{N}{k_0^*}-1)\right)^{-2} e^{\gamma Nt}Nt \end{array} \right\} dt.$$

This implies $\frac{\partial G}{\partial \gamma} < 0$ if $\beta \geq 1$. When $\beta < 1$, the sign of $\frac{\partial G}{\partial \gamma} < 0$ if $(1-\beta)(\frac{N}{k_0^*}-1) < e^{\gamma Nt}$. A sufficient condition is that

$$(1-\beta)(\frac{N}{k_0^*}-1) < 1$$

$$\Longleftrightarrow$$

$$\beta > 1 - \frac{k_0^*}{N-k_0^*}.$$

(B) Social planner's problem (76) requires $\frac{\partial W_0^*}{\partial k_0} = 0$. Applying the envelope theorem, we have

$$\frac{dW_0^*}{d\gamma} = \frac{\partial W_0^*}{\partial k_0}\frac{\partial k_0}{\partial \gamma} + \frac{\partial W_0^*}{\partial \gamma} = \int_0^\infty e^{-rt}\frac{\partial U(k_t)}{\partial \gamma}dt > 0$$

for any $\beta > 0$. Similarly, we can prove $dW_0^*/dA > 0$, $dW_0^*/dc < 0$, and $dW_0^*/dr < 0$. ∎

A.7. Proof of Proposition 7

Proof. Given that condition (23) holds, we have $k_0^* < N$. Proposition 5 shows that the socially optimal innovation k_0^* satisfies Eq. (24) that

$$\int_0^\infty e^{-(r+\gamma N)t}\left(\frac{k_t}{k_0}\right)^2 Ak_t^{-\beta}dt = c.$$

When $\alpha = 0$, condition (23) is equivalent to condition (11). Accordingly, we have $k_0^{\mathbf{I}} = k_0^{\mathbf{II}} < N$ and they satisfy Eq. (12) or Eq. (13) evaluated at $\alpha = 0$ so that

$$\int_0^\infty e^{-rt}\frac{1}{N-k_0}\left(\frac{N}{k_t}-1\right)Ak_t^{1-\beta}dt = c.$$

$$\Longleftrightarrow$$

$$\int_0^\infty e^{-(r+\gamma N)t}\left(\frac{k_0}{k_t}\right)\left(\frac{k_t}{k_0}\right)^2 Ak_t^{-\beta}dt = c. \tag{77}$$

Note that the left hand side of Eq. (77) is smaller than the left hand side of Eq. (24) given that $\frac{k_0}{k_t} < 1$ for $t > 0$. Therefore, the solution $k_0^{\mathbf{I}} = k_0^{\mathbf{II}}$ to Eq. (77) would cause the left hand side of Eq. (24) greater than c. Because the left hand side of Eq. (24) decreases with k_0, this implies $k_0^{\mathbf{I}} = k_0^{\mathbf{II}} < k_0^* < N$ for $\alpha = 0$.

Given that the right-hand side of condition (11) increases with α, if there exists an $\alpha' \in (0, 1]$ such that the inequality (11) becomes an equality (i.e., $c = \frac{\alpha'(\gamma N + r)}{r(r+\gamma N)} A N^{-\beta}$), we would then have $k^* < k_0^{\mathbf{I}} = k_0^{\mathbf{II}} = N$ for $\alpha = \alpha'$. In this case, the planner could choose optimal shares $\alpha^{\mathbf{I}*}$ and $\alpha^{\mathbf{II}*}$ such that $0 < \alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*} < \alpha' \leq 1$ to achieve the socially optimal k^* in Regimes 1 and 2, respectively. Note that $\alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*}$ results from Proposition 2: If there is a value of α leads to $k_0^{\mathbf{I}} = k^*$ in Regime 1, the same α would lead to $k_0^{\mathbf{II}} < k^*$ in Regime 2, so a larger value of α is needed to achieve k^* in Regime 2.

Alternatively, if the inequality (11) holds for any $\alpha \leq 1$, we have $k_0^{\mathbf{I}} = k_0^{\mathbf{II}} < N$ for $\alpha = 1$ and they satisfy Eq. (12) or Eq. (13) evaluated at $\alpha = 1$ so that

$$\int_0^\infty e^{-(r+\gamma N)t} \left(\frac{k_t}{k_0}\right)^2 A k_t^{-\beta} dt = c.$$

$\Longleftrightarrow$

$$\int_0^\infty e^{-(r+\gamma N)t} \left(\frac{k_0}{N} e^{\gamma N t} - \frac{k_0}{N} + 1\right) \left(\frac{k_t}{k_0}\right)^2 A k_t^{-\beta} dt = c. \tag{78}$$

Note that the left hand side of Eq. (78) is greater than the left hand side of Eq. (24) given that $\frac{k_0}{N} e^{\gamma N t} - \frac{k_0}{N} + 1 > 1$ for $t > 0$. Therefore, the solution $k_0^{\mathbf{I}} = k_0^{\mathbf{II}}$ to Eq. (78) would cause the left hand side of Eq. (24) smaller than c. Because the left hand side of Eq. (24) decreases with k_0, this implies $k_0^* < k_0^{\mathbf{I}} = k_0^{\mathbf{II}} < N$ for $\alpha = 1$. In this case, the social planner could choose optimal shares $\alpha^{\mathbf{I}*}$ and $\alpha^{\mathbf{II}*}$ such that $0 < \alpha^{\mathbf{I}*} < \alpha^{\mathbf{II}*} < 1$ to achieve the socially optimal k^* in Regimes 1 and 2, respectively. ∎

A.8. Proof of Proposition 8

Proof. In Regime 1, for a given value of α, Eq. (12) yields the market equilibrium entry of innovators $k_0^{\mathbf{I}}$. Proposition 7 suggests that whenever $\alpha \neq \alpha^{\mathbf{I}*}$, the number of innovators $k_0^{\mathbf{I}}$ from Eq. (12) differs from the social optimum k_0^*, in which case offering an innovation subsidy (or tax) to adjust the innovation cost c would help restore the social optimum. This implies that k_0^* can be achieved by a subsidy (or a tax if the subsidy is negative) $s^{\mathbf{I}*}$ as follows:

$$\frac{1}{N - k_0^*} \int_0^\infty e^{-rt} \left(\alpha \frac{N}{k_0^*} + (1-\alpha)\frac{N}{k_t^*} - 1\right) A k_t^{*1-\beta} dt = c - s^{\mathbf{I}*}.$$

The same logic applies to Regime 2 that

$$\frac{1}{N - k_0^*} \int_0^\infty e^{-rt} \left(\left(\frac{N}{k_0^*}\right)^\alpha \left(\frac{N}{k_t^*}\right)^{1-\alpha} - 1\right) A k_t^{*1-\beta} dt = c - s^{\mathbf{II}*}.$$

Recall that when $\alpha \in \{0, 1\}$, Regimes 1 and 2 coincide. When $\alpha = 0$, both regimes would need more entry of innovators, so a positive subsidy is needed to achieve that, and when $\alpha = 1$, a negative subsidy (tax) is needed. Moreover, for $\alpha \in (0, 1)$, according to Proposition 2(B), if a given pair of α and $(c - s^{\mathbf{I}*})$ lead to the social optimum k_0^* in Regime 1, the same parameter values would result in $k_0^{\mathbf{II}} < k_0^*$ in Regime 2. Therefore, a higher subsidy (or a smaller tax) $s^{\mathbf{II}*}$ is needed for adjusting c to achieve k_0^* in Regime 2 given that $k_0^{\mathbf{II}}$ decreases with c as shown by Proposition 2(A). $\blacksquare$

A.9. Proof of Proposition 9

Proof. (A) With $\beta = 1$ under Regime 1, Eq. (12) can be simplified as

$$k_0^{\mathbf{I}} = \frac{A(\alpha\gamma N + r)}{cr(r + \gamma N)}. \tag{79}$$

Given Eq. (79) and $\beta = 1$, social surplus is

$$W_0 = -\frac{A(1-\alpha)}{(r+\gamma N)} + A \int_0^\infty e^{-rt} \left[\gamma Nt - \ln\left(e^{\gamma Nt} + \frac{Ncr(r+\gamma N)}{A(\alpha\gamma N + r)} - 1 \right) \right] dt + \text{constant.}$$

This suggests that

$$\frac{dW_0}{d\gamma} = \frac{A(1-\alpha)N}{(r+\gamma N)^2} + A \int_0^\infty e^{-rt} Nt \, dt - A \int_0^\infty e^{-rt} \left[\frac{Nte^{\gamma Nt} + \frac{crN^2}{A(\alpha\gamma N+r)} - \frac{\alpha cr(r+\gamma N)N^2}{A(\alpha\gamma N+r)^2}}{e^{\gamma Nt} + \frac{Ncr(r+\gamma N)}{A(\alpha\gamma N+r)} - 1} \right] dt.$$

We then verify that

$$d\left[\frac{Nte^{\gamma Nt} + \frac{crN^2}{A(\alpha\gamma N+r)} - \frac{\alpha cr(r+\gamma N)N^2}{A(\alpha\gamma N+r)^2}}{e^{\gamma Nt} + \frac{Ncr(r+\gamma N)}{A(\alpha\gamma N+r)} - 1} \right] / dc$$

$$\propto \left(\frac{(1-\alpha)r}{(\alpha\gamma N + r)} \right) \left(e^{\gamma Nt} + \frac{Ncr(r+\gamma N)}{A(\alpha\gamma N + r)} - 1 \right)$$

$$- \left(te^{\gamma Nt} + \frac{crN}{A(\alpha\gamma N + r)} - \frac{\alpha cr(r+\gamma N)N}{A(\alpha\gamma N + r)^2} \right)(r + \gamma N)$$

$$< 0$$

for any $t > 0$. Equation (79) implies that $c > \frac{A(\alpha\gamma N+r)}{Nr(r+\gamma N)}$ for $k_0^{\mathbf{I}}$ to be interior solution (i.e., $k_0^{\mathbf{I}} < N$). Accordingly, given that $c > \frac{A(\alpha\gamma N+r)}{Nr(r+\gamma N)}$, we have

$$A \int_0^\infty e^{-rt} \left[\frac{Nte^{\gamma Nt} + \frac{crN^2}{A(\alpha\gamma N+r)} - \frac{\alpha cr(r+\gamma N)N^2}{A(\alpha\gamma N+r)^2}}{e^{\gamma Nt} + \frac{Ncr(r+\gamma N)}{A(\alpha\gamma N+r)} - 1} \right] dt < A \int_0^\infty e^{-rt} \left[\frac{Nte^{\gamma Nt} + \frac{N}{r+\gamma N} - \frac{\alpha N}{(\alpha\gamma N+r)}}{e^{\gamma Nt}} \right] dt.$$

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Therefore,

$$
\begin{aligned}
\frac{dW_0}{d\gamma} &> \frac{A(1-\alpha)N}{(r+\gamma N)^2} + A\int_0^\infty e^{-rt}Nt\,dt - A\int_0^\infty e^{-rt}\left[\frac{Nte^{\gamma Nt} + \frac{N}{r+\gamma N} - \frac{\alpha N}{(\alpha\gamma N+r)}}{e^{\gamma Nt}}\right]dt \\
&= \frac{A(1-\alpha)N}{(r+\gamma N)^2} - \frac{A}{(r+\gamma N)}\left(\frac{N}{r+\gamma N} - \frac{\alpha N}{(\alpha\gamma N+r)}\right) \\
&= \frac{A\alpha N}{(r+\gamma N)}\left(\frac{1}{r+\alpha\gamma N} - \frac{1}{r+\gamma N}\right) \geq 0 \text{ for any } \alpha \in [0,1].
\end{aligned}
$$

(B) Proposition 9(B) is an application of Proposition 9(A) by taking $\alpha = 0$ or $\alpha = 1$, and Regimes 1 and 2 coincide in those cases (cf. Proposition 2). ∎

A.10. Proofs of Section 6.3

Derivation of Eq. (39).—Equations (3) and (4) imply that for a given k_t,

$$
\frac{dk_t/dt}{k_t} = \gamma\left(N - k_t\right) = \gamma N(1 - \frac{e^{\lambda t}}{e^{\lambda t} + \frac{N}{k_0} - 1}). \tag{80}
$$

Given the inverse demand function (1), k_0 has to be finite as $N \to \infty$; otherwise $p_0 \to 0$, and no innovator would enter at date 0. Therefore, Eq. (80) implies that

$$
\left.\frac{dk_t/dt}{k_t}\right|_{N\to\infty} \to \gamma N \to \lambda. \tag{81}
$$

Derivation of Eqs. (42), (43) and (44).—We conjecture that no agent would enter as an innovator after date 0, so k_t is given by Eq. (40). Given that an imitator cannot resell the idea, his only revenue comes from selling the good, and his value ω_t satisfies the ordinary differential equation (ODE):

$$
r\omega_t = p_t + \frac{d\omega_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \frac{d\omega_t}{dt}. \tag{82}
$$

The ODE has the unique bounded solution

$$
\omega_t = \frac{Ak_0^{-\beta}}{r+\beta\lambda}e^{-\beta\lambda t}, \tag{83}
$$

i.e., Eq. (43).

An innovator receives revenues from selling both the good and the idea. The number of ideas sold at t is λk_t and the total date-t revenue from these sales, $\lambda k_t \alpha \omega_t$ is divided among the k_0 innovators. Thus v_t, the value of being an innovator at date t, follows the ODE:

$$
rv_t = p_t + \frac{\lambda k_t}{k_0}\alpha\omega_t + \frac{dv_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \frac{\alpha\lambda Ak_0^{-\beta}e^{(1-\beta)\lambda t}}{r+\beta\lambda} + \frac{dv_t}{dt}. \tag{84}
$$

Unless $\alpha = 0$, innovators receive a fraction of revenues from idea sales, and we shall need to restrict the elasticity of demand to be below unity which means $\beta \geq 1$. Imposing the boundary condition $v_{t\to\infty} < \infty$ yields the unique solution to Eq. (84), i.e., Eq. (42).

Recall that u_t denotes the option value of becoming a future imitator. At $t = 0$, the free entry condition requires $v_0 - u_0 = c$. The pool of outsiders being infinite, an outsider's chance of meeting an incumbent is zero so that $u_t = 0$ for all t, implying that $v_0 = c$. Since v_t decreases over time, we verify the conjecture that no one would pay c to become an innovator at any date $t > 0$. Note that if an agent deviates from the equilibrium and enters at date $t > 0$, he would have a lower valuation than an innovator who entered at date 0 (i.e., $v_t^t < v_t^0$) because the latter would have a larger family of imitators to disseminate his idea and collect idea-sale revenues. Therefore, the finding that $v_t - u_t$ declines in t implies that $v_t^t - u_t < c$ at any date $t > 0$.

Combining $v_0 = c$ with Eq. (42) yields

$$v_0 = \frac{Ak_0^{-\beta}}{r + \beta\lambda} + \frac{\alpha\lambda Ak_0^{-\beta}}{(r + \beta\lambda)(r + (\beta-1)\lambda)} = c. \tag{85}$$

Equation (85) then determines the entry of innovators at date 0 to be (44).

Derivation of Eq. (45).—Assume that condition (41) holds (i.e., $\lambda < r$) so that social welfare derived from the innovation is bounded. We conjecture that no agent would enter as an innovator after date 0. Given that imitators can resell the innovation, all the incumbents (be they innovators or imitators) share the same value v_t. The revenue from an idea sale is αv_t and the total date-t revenue from these sales, $\lambda k_t \alpha v_t$, is shared equally among all the incumbents. Then v_t follows the ODE:

$$rv_t = p_t + \lambda\alpha v_t + \frac{dv_t}{dt} = A(k_0 e^{\lambda t})^{-\beta} + \lambda\alpha v_t + \frac{dv_t}{dt}. \tag{86}$$

The general solution of Eq. (86) is

$$v_t = \frac{Ak_0^{-\beta} e^{-\beta\lambda t}}{r + \beta\lambda - \lambda\alpha} + Ak_0^{-\beta} C e^{(r-\lambda\alpha)t},$$

where C is a constant of integration. Given that $\lambda < r$, the boundary condition $v_{t\to\infty} < \infty$ requires $C = 0$ and yields Eq. (45).

Derivation of Eq. (48).—The planner maximizes

$$W_0 = \int_0^\infty e^{-rt} U(k_t)\, dt - ck_0,$$

where

$$U(k_t) = \begin{cases} \frac{A}{1-\beta} k_t^{1-\beta} & \text{if } \beta \in (0,1), \\ A(\ln k_t + 1 - \ln\varepsilon) & \text{if } \beta = 1, \\ \frac{A}{1-\beta} k_t^{1-\beta} + \frac{\beta A}{\beta-1}\varepsilon^{1-\beta} & \text{if } \beta > 1. \end{cases}$$

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and $k_t = k_0 e^{\lambda t}$. We maintain condition (41).

Consider the case $\beta < 1$ first. For any date $\tau \geq 0$, if no further innovators enter, the number of firms at dates $t \geq \tau$ is

$$k_t = k_\tau \, e^{\lambda(t-\tau)} \quad \text{for } t \geq \tau.$$

As of date τ, the social return to innovation is

$$SR_\tau = \int_\tau^\infty e^{-r(t-\tau)} \frac{A}{1-\beta} k_t^{1-\beta} dt.$$

The current cost of innovation is c per unit, and its marginal social return (even if no further innovations are made) is

$$\frac{\partial SR_\tau}{\partial k_\tau} = \int_\tau^\infty e^{-r(t-\tau)} A k_t^{-\beta} \frac{\partial k_t}{\partial k_\tau} dt = \int_\tau^\infty e^{-(r-\lambda)(t-\tau)} A k_t^{-\beta} dt. \tag{87}$$

Define $s = t - \tau$, we can rewrite Eq. (87) as

$$\frac{\partial SR_\tau}{\partial k_\tau} = \int_0^\infty e^{-(r-\lambda)s} A k_{\tau+s}^{-\beta} ds = \int_0^\infty e^{-(r-\lambda)s} A \left(k_\tau e^{\lambda s} \right)^{-\beta} ds,$$

which is strictly decreasing in k_τ. Therefore, if at date zero k_0 is chosen so that $\frac{\partial SR_0}{\partial k_0} = c$, thereafter $\frac{\partial SR_\tau}{\partial k_\tau} < c$. Similarly, we can prove the result holds for $\beta \geq 1$. Hence, it is socially optimal to innovate only at date 0.

Accordingly, the social planner chooses k_0 to maximize social welfare:

$$W_0 = \int_0^\infty e^{-rt} U(k_t) \, dt - c k_0.$$

We verify that the social welfare function is strictly concave in k_0, so the first-order condition yields Eq. (48).

Derivation of (49).—Recall that the solutions for $k_0^{\mathbf{I}}$, $k_0^{\mathbf{II}}$ and k_0^* are given by Eqs. (44), (46) and (48), respectively. Given that the optimal compensation for idea sellers is $\alpha^* = 1$, the planner could improve social welfare by offering an innovation subsidy s whenever $\alpha < 1$. Accordingly, the net entry cost of innovators becomes $c - s$. Under Regime 1, Eqs. (44) and (48) pin down the optimal subsidy $s^{\mathbf{I}*}$ so that

$$c \left[r + (\beta - 1)\lambda + \alpha\lambda \right] = (c - s^{\mathbf{I}*}) (r + \beta\lambda),$$

which yields

$$s^{\mathbf{I}*} = \frac{c(1-\alpha)\lambda}{r + \beta\lambda}. \tag{88}$$

Under Regime 2, Eqs. (46) and (48) pin down the optimal subsidy $s^{\mathbf{II}*}$ so that

$$c \left[r + (\beta - 1)\lambda \right] = (c - s^{\mathbf{II}*})[r + (\beta - \alpha)\lambda],$$

which yields

$$s^{\mathbf{II}*} = \frac{c(1-\alpha)\lambda}{r + (\beta - \alpha)\lambda}. \tag{89}$$

The results (88) and (89) yield (49).

Derivation of the finding (50).—Recall that the solutions for $k_0^{\mathbf{I}}$ and $k_0^{\mathbf{II}}$ are given by Eqs. (44), (46):

$$k_0^{\mathbf{I}} = \left(\frac{A(r + (\beta - 1)\lambda + \alpha\lambda)}{c\,(r + \beta\lambda)\,(r + (\beta - 1)\lambda)} \right)^{\frac{1}{\beta}}; \quad k_0^{\mathbf{II}} = \left(\frac{A}{c(r + (\beta - \alpha)\lambda)} \right)^{\frac{1}{\beta}}.$$

In each regime, the number of firms grows at a constant rate λ (i.e., $k_t^{\mathbf{I}} = k_0^{\mathbf{I}} e^{\lambda t}$ and $k_t^{\mathbf{II}} = k_0^{\mathbf{II}} e^{\lambda t}$). The planner would like to maximize social welfare

$$W_0 = \int_0^\infty e^{-rt} U\left(k_t\right) dt - ck_0,$$

where

$$U\left(k_t\right) = \begin{cases} \frac{A}{1-\beta} k_t^{1-\beta} & \text{if } \beta \in (0,1), \\ A(\ln k_t + 1 - \ln\varepsilon) & \text{if } \beta = 1, \\ \frac{A}{1-\beta} k_t^{1-\beta} + \frac{\beta A}{\beta - 1}\varepsilon^{1-\beta} & \text{if } \beta > 1. \end{cases}$$

We again assume that condition (41) holds and denote $W_0^{\mathbf{I}}$ and $W_0^{\mathbf{II}}$ as the social welfare under Regimes 1 and 2, respectively. With free knowledge spillovers ($\alpha = 0$), we have $W_0^{\mathbf{I}} = W_0^{\mathbf{II}} = W_0$, where

$$W_0 = \begin{cases} A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+\beta\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+\beta\lambda)^{-\frac{1}{\beta}} \right) & \text{if } \beta \in (0,1), \\ \frac{A}{r}\ln\frac{A}{c(r+\lambda)} + \frac{A\lambda}{r^2} - \frac{A}{(r+\lambda)} + \frac{A(1-\ln\varepsilon)}{r} & \text{if } \beta = 1, \\ A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+\beta\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+\beta\lambda)^{-\frac{1}{\beta}} \right) + \frac{\beta A}{r(\beta-1)}\varepsilon^{1-\beta} & \text{if } \beta > 1. \end{cases} \tag{90}$$

It is straightforward to show that for any $\beta > 0$, $\frac{\partial W_0}{\partial \lambda} > 0$.

This finding extends to any $\alpha \in (0,1]$, for which we have

$$W_0^{\mathbf{I}} = \begin{cases} \text{N/A} & \text{if } \beta \in (0,1), \\ \frac{A}{r}\ln\left(\frac{A(r+\alpha\lambda)}{c(r+\lambda)r}\right) + \frac{A\lambda}{r^2} - \frac{A(r+\alpha\lambda)}{(r+\lambda)r} + \frac{A(1-\ln\varepsilon)}{r} & \text{if } \beta = 1, \\ \begin{aligned} & A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{1}{(1-\beta)} \left(\frac{(r+(\beta-1)\lambda+\alpha\lambda)}{(r+\beta\lambda)} \right)^{\frac{1-\beta}{\beta}} (r+(\beta-1)\lambda)^{-\frac{1}{\beta}} \right. \\ & \qquad\qquad \left. - \left(\frac{r+(\beta-1)\lambda+\alpha\lambda}{(r+\beta\lambda)(r+(\beta-1)\lambda)} \right)^{\frac{1}{\beta}} \right) \\ & + \frac{\beta A}{r(\beta-1)}\varepsilon^{1-\beta} \end{aligned} & \text{if } \beta > 1. \end{cases} \tag{91}$$

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$$W_0^{\mathbf{II}} = \begin{cases} A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+(\beta-\alpha)\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+(\beta-\alpha)\lambda)^{-\frac{1}{\beta}} \right) & \text{if } \beta \in (0,1), \\[2ex] \frac{A}{r} \ln \frac{A}{c(r+(1-\alpha)\lambda)} + \frac{A\lambda}{r^2} - \frac{A}{r+(1-\alpha)\lambda} + \frac{A}{r}(1-\ln\varepsilon) & \text{if } \beta = 1, \\[2ex] A^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}} \left(\frac{(r+(\beta-\alpha)\lambda)^{1-\frac{1}{\beta}}}{(1-\beta)(r-\lambda(1-\beta))} - (r+(\beta-\alpha)\lambda)^{-\frac{1}{\beta}} \right) & \\ \quad + \frac{\beta A \varepsilon^{1-\beta}}{r(\beta-1)} & \text{if } \beta > 1. \end{cases} \tag{92}$$

We then confirm from Eqs. (91) and (92) that $\frac{\partial W_0^{\mathbf{I}}}{\partial \lambda} > 0$ for $\beta \geq 1$ and $\frac{\partial W_0^{\mathbf{II}}}{\partial \lambda} > 0$ for $\beta > 0$.

B. Additional regressions

B.1. Auto diffusion estimation: Robustness checks

For robustness checks, we estimate the meeting function directly by rewriting Eq. (3) into a discrete-time version:

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \gamma k_{t-1}. \tag{93}$$

Note that the left-hand side of Eq. (93) is the hazard rate of adopting the new product. We set $N = 210$ and run the regression model (93) using auto firm number data from 1895-1908. The result shows that

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \underset{(0.0004)^{***}}{0.0028} \; k_{t-1},$$

and the standard error is reported in the parentheses. The estimate of γ is statistically significant at 1% level and adjusted $R^2 = 0.77$. The estimate $\gamma = 0.0028$ implies that $\gamma N = 0.59$, which is similar to the estimate from Eq. (30).

We also redo the exercise by estimating an extended version of Eq. (93) that

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \eta + \gamma k_{t-1}, \tag{94}$$

proposed by Bass (1969). The Bass model allows the hazard rate of adoption to be influenced by both the coefficient of innovation η and the coefficient of imitation γ. In our context, η captures the hazard rate of entry by innovators independent of incumbents while γk_{t-1} captures the hazard rate of entry by imitators. The regression result shows that

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \underset{(0.069)}{-0.0292} + \underset{(.0007)^{***}}{.0031} \; k_{t-1},$$

and the standard errors are reported in the parentheses. The estimate $\gamma = 0.0031$ (which implies $\gamma N = 0.64$) is statistically significant at 1% level, but the estimate

of η is not statistically significant, which is consistent with our theoretical prediction that innovators only enter at the beginning of the industry.

B.2. Auto demand estimation: Robustness checks

For robustness checks, we estimate the auto industry demand function by controlling for changes of population and per capita income over time. In doing so, we use annual data of auto price p_t and output Q_t from 1900–1929 to estimate a per capita demand function:
$$\ln(\frac{Q_t}{pop_t}) = a_t - \phi \ln(p_t).$$
The dependent variable is auto demand per capita (where pop_t is U.S. population at year t), and we control for log U.S. GDP per capita (as a proxy for income) in the demand intercept a_t. Both auto price and GDP per capita are in real terms.

As before, to address potential endogeneity of the price variable, we use the output per firm (lagged by a year) as an instrumental variable to estimate the demand elasticity parameter ϕ in a two-stage least-squares regression.

The first-stage regression result (adj. $R^2 = 0.89$) is given by

$$\ln(p_t) = \underset{(1.09)^{***}}{8.56} + \underset{(0.64)^{**}}{1.66} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.03)^{***}}{0.29} \times \ln(\textit{output per firm})_{t-1},$$

and the second-stage regression result ($R^2 = 0.83$) is

$$\ln(\frac{Q_t}{pop_t}) = \underset{(7.16)^{***}}{32.37} + \underset{(2.10)}{0.28} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.38)^{***}}{3.33} \times \ln(p_t).$$

Standard errors are reported in the parentheses, with three stars and two stars representing statistical significance at 1% and 5% level, respectively. The estimate $\phi = 3.33$ is highly statistically significant and the implied inverse demand elasticity $\beta = \frac{1}{\phi} = 0.3$ is similar to the estimate from Eq. (31).

B.3. PC diffusion estimation: Robustness checks

For robustness checks, we estimate the meeting function (93) for the PC industry:
$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \gamma k_{t-1}.$$
We set $N = 435$ and run the regression using PC firm number data from 1975-1991. The result shows that
$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \underset{(0.00025)^{***}}{0.00092} k_{t-1},$$
and the standard error is reported in the parentheses. The estimate of γ is statistically significant at 1% level and adjusted $R^2 = 0.44$. The estimate $\gamma = 0.00092$ implies that $\gamma N = 0.40$.

We also redo the exercise by estimating a more general version (cf. Eq. (94)) that

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \eta + \gamma k_{t-1},$$

proposed by Bass (1969). In our context, η captures the hazard rate of entry by innovators independent of incumbents, while γk_{t-1} captures the hazard rate of entry by imitators. The regression result shows that

$$\frac{k_t - k_{t-1}}{N - k_{t-1}} = \underset{(0.08398)}{0.04721} + \underset{(0.00036)^*}{0.00078} k_{t-1},$$

and the standard errors are reported in the parentheses. The estimate $\gamma = 0.00078$ (which implies $\gamma N = 0.34$) is statistically significant at 5.1% level, but the estimate of η is not statistically significant, which is consistent with our theoretical prediction that innovators only enter at the beginning of the industry.

B.4. PC demand estimation: Robustness checks

For robustness checks, we estimate the PC industry demand function by controlling for changes of population and per capita income over time. In doing so, we use annual data of PC price p_t and output Q_t from 1975–1992 to estimate a per capita demand function:

$$\ln(\frac{Q_t}{pop_t}) = a_t - \phi \ln(p_t).$$

The dependent variable is PC demand per capita (where pop_t is U.S. population at year t), and we control for log U.S. GDP per capita (as a proxy for income) in the demand intercept a_t. Both auto price and GDP per capita are in real terms.

As before, to address potential endogeneity of the price variable, we use the output per firm (lagged by a year) as an instrumental variable to estimate the demand elasticity parameter ϕ in a two-stage least-squares regression.

The first-stage regression result (adj. $R^2 = 0.95$) is given by

$$\ln(p_t) = \underset{(0.23)^{***}}{12.44} - \underset{(0.06)^{***}}{0.95} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(0.01)^{***}}{0.07} \times \ln(output\ per\ firm)_{t-1},$$

and the second-stage regression result ($R^2 = 0.95$) is

$$\ln(\frac{Q_t}{pop_t}) = \underset{(29.00)^{***}}{143.18} - \underset{(2.63)}{2.90} \times \ln(\frac{GDP_t}{pop_t}) - \underset{(2.40)^{***}}{15.57} \times \ln(p_t).$$

Standard errors are reported in the parentheses, with three stars indicating statistical significance at 1% level. The estimate $\phi = 15.57$ is highly statistically significant and the implied inverse demand elasticity $\beta = \frac{1}{\phi} = 0.06$ is similar to the estimate from Eq. (34).