Working Paper Series

Multilateral Comovement in a New Keynesian World: A Little Trade Goes a Long Way

WP 22-10

Paul Ho
Federal Reserve Bank of Richmond

Pierre-Daniel Sarte
Federal Reserve Bank of Richmond

Felipe Schwartzman
Federal Reserve Bank of Richmond

FEDERAL RESERVE BANK
OF RICHMOND®
Richmond ▪ Baltimore ▪ Charlotte

Multilateral Comovement in a New Keynesian World: A Little Trade Goes a Long Way*

Paul Ho Pierre-Daniel Sarte Felipe Schwartzman

Federal Reserve Bank of Richmond

November 16, 2022

Abstract

We study how international linkages and nominal price rigidities jointly shape the dynamics of inflation and output across multiple large economies. We describe how these features produce a global system of Phillips curves explicitly connected by multilateral trade relationships. In equilibrium, disturbances abroad propagate to domestic variables not only directly, through pairwise trade between countries, but also indirectly through third-country effects arising from the network structure of trade. The combined propagation mechanisms imply that country-specific shocks alone explain almost 90 percent of the observed average pairwise comovement in output growth between countries. These idiosyncratic shocks also explain more than 1/2 the cross-country comovement in inflation, and between output and inflation. We estimate that a European inflationary shock results in significant U.S. inflation accompanied by lower output, and that these responses transpire almost entirely from the network effects of trade. In addition, a tightening of U.S. monetary policy generates a percentage decline in output globally that is comparable to 1/2 the domestic response.

Keywords: International Comovement, Multilateral Trade, New Keynesian Phillips Curve
JEL Codes: E31, E32, F41, F44

*The views expressed herein are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System. We thank David Baqaee, Zhen Huo, Ricardo Reis, and Stephanie Schmitt-Grohé, for discussions. We also thank participants at the Richmond Fed-UVA-Duke Research Workshop and the Marvin Goodfriend Memorial Conference, as well as seminar participants at Pennsylvania State University and the Brazilian Macroeconomics Group for their comments. Nick Garvey, Colton Lapp, James Lee, and Erin Henry provided outstanding research assistance.
Paul Ho: paul.ho@rich.frb.org.
Pierre-Daniel Sarte: pierre.sarte@rich.frb.org.
Felipe Schwartzman felipe.schwartzman@rich.frb.org.

1 Introduction

Inflation and output growth exhibit substantial comovement between countries. Remarkably, the correlation between inflation and output growth at business cycle frequencies, a hallmark of the New Keynesian framework, is at times as pronounced *across* countries as it is *within* countries. However, given that large economies including the U.S. or the European Union (E.U.) purchase goods and services mostly from themselves, with shares of domestic purchases exceeding 80 percent of total purchases, the quantitative study of inflation dynamics has largely proceeded within the context of closed-economy models. Does it then follow that international linkages have only a small role in explaining this comovement, and that global shocks are the main drivers of common variations in output growth or inflation? In this paper, we develop a structural framework featuring multiple large open economies with nominal rigidities that suggests otherwise. In other words, a little trade goes a long way in explaining global comovement across countries.

The structural framework we develop brings together nominal rigidities in the tradition of New Keynesian models (e.g., Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007); Justiniano, Primiceri, and Tambalotti (2010); Del Negro, Giannoni, and Schorfheide (2015)) with the literature on multilateral trade (e.g., Eaton and Kortum (2002), Dekle, Eaton, and Kortum (2007)). The network structure of trade, in particular, plays a critical role in increasing the significance of foreign shocks, including foreign demand shocks, for the domestic economy. These network effects arise by way of both *backward* trade linkages, summarizing a country's import network, and *forward* trade linkages, summarizing its export network. The end result is that, despite relatively low observed trade shares across countries, country-specific shocks generate sizeable cross-country comovement in output growth, in inflation, and between output growth and inflation.[1]

International linkages between countries subject to nominal rigidities imply important departures from the conventional closed-economy New Keynesian Phillips curve (NKPC). First, whereas marginal costs are only a function of domestic input costs in a closed economy, they now also depend on foreign input costs and exchange rates of all trading partner countries. Second, these input costs, and notably wages, in part reflect demand conditions abroad.

Both these departures from the conventional NKPC imply a global system of Phillips curves explicitly connected by weights reflecting multilateral trade relationships. This system relates inflation in one country to variations in output growth, consumption, and exchange rates in every other country. The coefficients in this system of Phillips curves, therefore, provide a structural interpretation for the weights used in empirical work relating domestic inflation to foreign real activity (e.g., Bianchi and Civelli (2015)). In equilibrium, cross-country variations in output growth, consumption, and exchange rates reflect the effects of foreign disturbances not only directly, through pairwise trade between countries, but also indirectly through third-country effects. It is these third-

[1] In that sense, our analysis generalizes the seminal contributions of Clarida, Gali, and Gertler (2002), and Galí and Monacelli (2005), to a setting with multiple large countries with network implications from trade for global comovement across countries.

country effects, in particular, that help generate sizeable comovement between domestic and foreign variables.

The literature has studied various sources of international comovement. The importance of global factors for the comovement in real activity is explored empirically by Kose, Otrok, and Whiteman (2003), Stock and Watson (2005), and Del Negro and Otrok (2008), among others. In Ciccarelli and Mojon (2010), Mumtaz and Surico (2012), and Andrés, Schmitt-Grohé, and Uribe (2017), global factors also account for some of the observed comovement in inflation across countries. Finally, Mumtaz, Simonelli, and Surico (2011) and Ferroni and Klaus (2015) establish empirically that global factors play another role, connecting nominal and real variables. Aside from global factors, the comovement in variables between countries can arise more structurally from the endogenous propagation of idiosyncratic country shocks through trade, as explored recently with respect to real activity in Huo, Levchenko, and Pandalai-Nayar (2019), as well as Boeckelmann, Imbs, and Pauwels (2022), or through spillovers in global financial markets as in Imbs, Mumtaz, Ravn, and Rey (2005), Imbs (2006), or Itskhoki and Mukhin (2021).

This paper merges these different strands of literature within a single structural framework that highlights their interplay in a parsimonious set of equations. In the absence of trade, this set of equations collapses to the canonical closed-economy New Keynesian model country by country. More generally, however, New Keynesian forces interact with the effects of foreign wages and exchange rates on marginal costs, of trade flows on local demand, and of financial flows on the trade balance and exchange rates.

We use our structural model to quantify the role of both global factors and international linkages for the cross-country comovement in output growth, in inflation, and between output growth and inflation. Our findings derive from a full-information Bayesian estimation of all structural parameters (e.g., Justiniano and Primiceri (2008); Schmitt-Grohé and Uribe (2012); Herbst and Schorfheide (2015); Bianchi and Melosi (2017)). This approach allows us to estimate the cross-country effects of shocks while accounting for the network effects of trade, financial flows and other general equilibrium channels. Our data cover five world regions, namely Canada, China, the E.U., Japan, and the U.S., and we allow for heterogeneous structural parameters, such as in the degree of nominal rigidities across countries. Estimating a New Keynesian model with multiple large countries poses two key computational challenges. First, there are 2.5 times as many parameters and about double the number of equations relative to Smets and Wouters (2007).[2] Second, steady state calculations are more involved than typical closed-economy New Keynesian models, requiring an iterative procedure that can take half the time needed to compute the likelihood for a given set of parameters. We show that recent state-of-the-art adaptive sequential Monte Carlo algorithms (Cai, Del Negro, Herbst, Matlin, Sarfati, and Schorfheide (2021)) can help overcome those challenges.

The multi-country New Keynesian framework we develop implies that country-specific shocks

[2]This particular curse of dimensionality is noted by Guerron-Quintana (2013) as a significant obstacle to the estimation of New Keynesian models with more than two countries.

alone explain almost 90 percent of the observed average pairwise cross-country correlation in output growth. These idiosyncratic shocks also explain more than 1/2 the cross-country comovement in inflation, as well as between output and inflation. Our findings, therefore, highlight how trade linkages, given nominal rigidities, are critical in propagating country specific shocks globally both directly and indirectly.

Consider, for example, the effects of a European stimulative monetary shock on the U.S. Since demand increases in the E.U., and E.U. consumption also consists of foreign goods, U.S. exports to the E.U. rise, and so does U.S. production (i.e., a forward linkage). To the extent that higher demand also increases E.U. production and its associated costs, including wages, the composition of E.U. expenditures shifts towards U.S. goods, further increasing U.S. output. Importantly, these direct effects on U.S. exports and production are complemented by indirect effects through third countries. For example, suppose that Japan (the third country in this example) also trades with the E.U. Then, higher E.U. wages lead Japan to substitute away from E.U.-produced goods and towards imports from the U.S. (i.e., a backward linkage). Finally, the propagation of the initial stimulative European shock continues further afield as higher U.S. export activity and production raise U.S. wages, thus prompting higher U.S. imports from, say, China. Higher U.S. wages also lead China to redirect some of its imports from the U.S. back towards the E.U., raising E.U. wages and thus creating a new round of trade from the E.U. and so on.

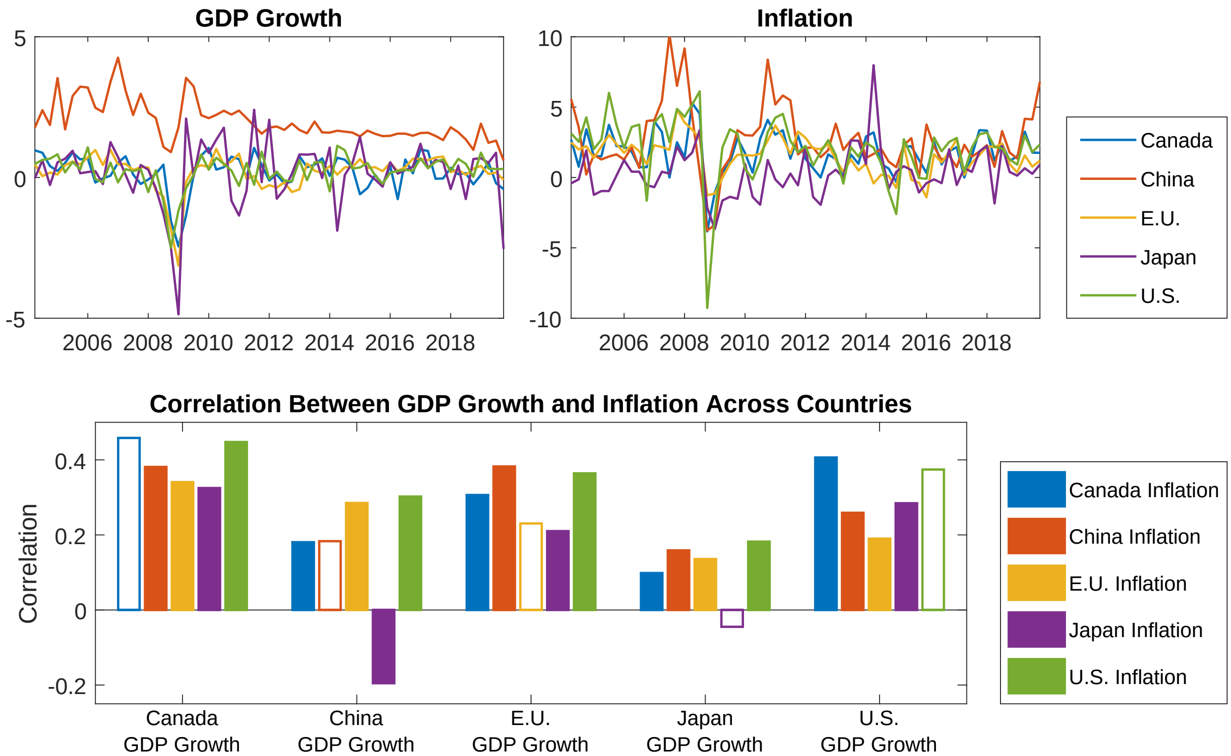
We use our model to quantitatively explore two applications related to the domestic effects of foreign shocks in the post-pandemic world. First, we estimate that an inflationary supply shock in Europe spills over into a significant rise in U.S. inflation and fall in U.S. output, and that these responses almost entirely transpire from third-country network effects. Second, we find that a tightening of U.S. monetary policy results in a percentage output decline globally that is comparable to 1/2 the domestic response.

This paper is organized as follows. Section 2 reviews some motivating data. Section 3 presents the economic environment. Section 4 highlights key equilibrium conditions that describe the interplay between sticky prices and multilateral trade. Section 5 discusses the model solution in linearized form. Section 6 explores the mechanisms underlying global spillovers from independent country-specific shocks. Section 7 describes our quantitative findings. Section 8 concludes. A detailed online Appendix contains derivations of all results discussed in the text, a comprehensive description of the data and statistical methods, discussions of departures from our benchmark assumptions, and additional figures and tables complementing our main analysis.

2 GDP Growth and Inflation Across Countries

We begin by documenting some key stylized facts that motivate the paper using data for five world regions, namely Canada, China, the E.U., Japan and the United States, over the period 2004Q2 through 2019Q4. These regions cover roughly 62 percent of world GDP over that period.

Figure 1: Comovement in Real GDP and Inflation Within and Across Regions



Notes: In upper panels, GDP growth rates are quarter-over-quarter and deflated by actual inflation, while inflation rates are quarter-over-quarter and annualized. In lower panel, each bar corresponds to correlation between GDP growth in country i with inflation in country j. Filled bars correspond to cross-country correlations (i.e., $i \neq j$), empty bars correspond to own-country correlations (i.e., $i = j$) without trade. Each group of bars corresponds to a country i's GDP growth and each color corresponds to a country j's inflation.

The top left-hand panel of Figure 1 illustrates the behavior of real GDP growth in each region. While there are differences across regions, particularly with respect to the level of GDP growth in China, the panel shows positive comovement in real activity across countries. The mean pairwise correlation in GDP growth across countries is around 0.4, with Europe and Japan exhibiting the largest correlation in real activity at 0.6.

The top right-hand panel of Figure 1 similarly shows that inflation rates tend to move together across countries, albeit with a somewhat lower mean pairwise correlation of 0.25. The degree of comovement in inflation between any two countries, however, can be quite large, as in the case of Canada and the U.S. with a correlation of 0.8.

Finally, while both of these observations have been well documented for a wide range of countries and time periods, the bottom panel of Figure 1 shows a less known fact - inflation and output comove positively not only *within* countries but also *across* countries. Interestingly, the correlation between real activity and inflation may be as large or larger across countries than it is within countries. For instance, the correlation of E.U. output growth with E.U. inflation is around 0.2 but well above 0.3

with U.S. inflation and almost 0.4 with Chinese inflation. Similarly, Canadian real GDP growth comoves as much with Canadian inflation as it does with U.S. inflation. Overall, mean cross-country and own-country correlations are of similar magnitude, at 0.25 and 0.24, respectively. The notion of a Phillips curve, therefore, if pertinent within countries is potentially equally relevant across countries.

Two main possible mechanisms can underlie the comovement between countries depicted in Figure 1. One relates to global factors in productivity, policy, or other disturbances. Another is spillovers across countries that arise from the endogenous propagation of country-specific shocks through international linkages. How does the interplay between global factors and the propagation of country-specific shocks shape the dynamics of output and inflation across countries? What do these interactions imply for the comovement shown in Figure 1? This paper presents a quantitative framework aimed at addressing these questions.

3 Economic Environment

This section presents a New Keynesian world economy with international trade in goods and imperfectly integrated asset markets. There are N countries indexed by $n = 1, ..., N$. As in the prototypical New Keynesian closed-economy model, each country has three types of agents: firms that produce various types of outputs, households that consume final goods aggregated from these outputs, and a monetary authority that sets policy according to a Taylor rule. However, we depart from the closed-economy literature by introducing multilateral trade in goods and financial assets.

We model the interplay between sticky prices and multilateral trade by having two layers of intermediate inputs rather than the single layer of monopolistically competitive firms in the standard model. The first layer consists of different varieties that may be purchased from different countries. We refer to this first layer of intermediate varieties as *tradable goods*. The second layer consists of intermediate inputs that are subject to nominal rigidities, as in Calvo (1983), and assembled into final goods. We refer to this second layer of intermediate goods as *sticky-price goods*.

Frictions in global financial markets determine the extent to which countries can run trade deficits or surpluses, thereby influencing the behavior of exchange rates and the way that shocks spill over across countries. Thus, we introduce trade in financial assets by allowing households not only to purchase and sell bonds domestically but also to trade state-contingent assets internationally. Securities traded in global financial markets are subject to portfolio adjustment costs and exogenous disturbances that, in equilibrium, influence the behavior of exchange rates.

Finally, a monetary authority in each country sets policy according to a country-specific Taylor rule. To the degree that some coordination exists between different central banks, we allow for common factors across countries' monetary policy shocks.

Multilateral trade in goods and global financial markets means that disturbances abroad spill over to domestic variables through the network effects of trade. We show that these departures imply a global Phillips curve for vectors of output, inflation, consumption, and real exchange rates that are interconnected across countries.

3.1 Households

The representative household in country n has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_{n,t}/A_{n,t})^{1-\sigma_n} - 1}{1-\sigma_n} - \omega_n \frac{L_{n,t}^{1+\eta_n}}{1+\eta_n} \right\},\ 0 < \beta < 1,\ \sigma_n, \omega_n, \eta_n > 0, \tag{1}$$

where $L_{n,t}$ denotes labor supplied by the household, $C_{n,t}$ is a bundle of consumption goods, and $A_{n,t}$ summarizes country n's technical progress. As in Lubik and Schorfheide (2005), the presence of the term $A_{n,t}$ in equation (1) ensures that each country evolves along a balanced growth path in the long run, allowing hours worked, $L_{n,t}$, to remain stationary even if the intertemporal elasticity of substitution, $1/\sigma_n$, differs from 1.

Households in country n face the budget constraint

$$\begin{aligned} P_{n,t}C_{n,t} + \frac{E_t\left[V_{t+1}S_{n,t+1}\right]}{\mathcal{E}_{rn,t}} + E_t\left[P_{n,t}A_{n,t}\Theta_n\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right] + \frac{B_{n,t}}{R_{n,t}} \\ = W_{n,t}L_{n,t} + \Pi_{n,t} + \frac{1}{\mathcal{E}_{rn,t}}S_{n,t} + B_{n,t-1}, \end{aligned} \tag{2}$$

where $P_{n,t}$ is the nominal price of the consumption bundle (i.e., the price of final goods), $W_{n,t}$ denotes nominal wages, and $\Pi_{n,t}$ represents nominal firm profits. All prices in country n are expressed in units of the local currency. The term $B_{n,t}$ denotes the quantity of a bond traded domestically by households at price $1/R_{n,t}$ at date t which pays 1 unit of the local currency in the following period.

The budget constraint (2) differs from the usual closed-economy household budget constraint in that countries trade a state-contingent security internationally, $S_{n,t+1}$, subject to portfolio adjustment costs, $\Theta_n(\cdot)$. The security is purchased at date t by country n at price V_{t+1}, expressed in the currency of a reference country r. It pays off 1 unit of the reference country currency, translated into local currency units by the exchange rate $\mathcal{E}_{rn,t}$ in the budget constraint (2).[3]

The function $\Theta_n(\cdot)$ reflects a cost incurred by country n when adjusting its financial position, similar to Schmitt-Grohé and Uribe (2003). We assume that $\Theta_n(0) = \Theta_n'(0) = 0$ and $\Theta_n''(\cdot) = \tau > 0$ so that portfolio adjustment costs increase at an increasing rate the larger country n's financial position is in absolute value. The curvature parameter τ captures the degree of financial frictions in global financial markets: $\tau \to 0$ corresponds to perfect capital markets, while $\tau \to \infty$ represents financial autarky. The function Θ_n varies with two objects. First, portfolio adjustment costs incurred by country n depend on $\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}}$, the real value of state-contingent securities purchased

[3] Thus, if reference country r uses, say, the Euro and country n is the U.S., $\mathcal{E}_{rn,t}$ is expressed in €/\$. Moreover, $\frac{1}{\mathcal{E}_{rn,t}}S_{n,t}$ then represents a payoff in dollars from $S_{n,t}$ shares of the state-contingent security purchased at date $t-1$.

in global financial markets in units of local goods (i.e., the consumption bundle, $C_{n,t}$), normalized by $A_{n,t}$ to account for country n's rate of technical progress. Second, the term $\frac{1}{\tau}\xi_{n,t}$ captures exogenous disturbances that affect country n's ability to trade in international financial markets.

Substantively, these adjustment costs, Θ_n, and the shocks, $\xi_{n,t}$, ensure that we match exchange rate movements when estimating the model. These frictions imply deviations from Uncovered Interest Parity (UIP) that are an important feature of the data. As emphasized by Clarida, Gali, and Gertler (2002) and Galí and Monacelli (2005), exchange rates generally affect the behavior of domestic inflation in an international setting. Those deviations, therefore, become part of global inflation dynamics.

3.2 Firms

The production side consists of final goods and two layers of intermediate goods. As in the conventional New Keynesian framework, final goods are produced using intermediate inputs subject to nominal rigidities. We thus refer to this layer of intermediate goods as sticky-price goods. However, sticky-price goods are now produced using another layer of intermediate varieties that can be purchased from other countries.

Final Goods: We denote final goods in each country by $\mathcal{C}_{n,t}$. These goods may be used by households either for consumption proper or to pay for portfolio rebalancing costs, so that $\mathcal{C}_{n,t} = C_{n,t} + E_t\left[A_{n,t}\Theta_n\left(\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right]$ in the household's budget constraint (2).

Final goods are assembled by firms using sticky-price intermediate goods, $y_{n,t}(j)$, with the technology

$$\mathcal{C}_{n,t} = \left[\int_0^1 y_{n,t}(j)^{\frac{\varepsilon_{n,t}-1}{\varepsilon_{n,t}}} dj\right]^{\frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}}, \ \varepsilon_{n,t} > 1.$$

We index the elasticity of substitution between intermediate varieties by t to account for potential time-varying markups, denoted by $\mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1} > 1$. Firms that produce final goods are competitive and, given the constant-returns-to-scale technology, make zero profits. Given a price $P_{n,t}(j)$ for variety j, cost minimization implies the demand function

$$y_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t}, \tag{3}$$

where $P_{n,t} = \left[\int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj\right]^{\frac{1}{1-\varepsilon_{n,t}}}$ such that $P_{n,t}\mathcal{C}_{n,t} = \int_0^1 P_{n,t}(j)y_{n,t}(j)dj$.

Non-Tradable Sticky-Price Intermediate Goods: In country n, each variety j of sticky-price goods is produced using a composite input bundle, $Q_{n,t}(j)$, according to

$$y_{n,t}(j) = A_{n,t}Q_{n,t}(j). \tag{4}$$

This input bundle consists of different varieties (described below) that are traded across countries. Crucially, shocks affecting production costs abroad will then affect firm choices at home. We denote the price index associated with the composite bundle by $P^Q_{n,t}$ so that $P^Q_{n,t}Q_{n,t}(j) = MC_{n,t}y_{n,t}(j)$, where $MC_{n,t} = P^Q_{n,t}/A_{n,t}$ is the nominal marginal cost faced by sticky-price goods producers. Their real marginal cost is then given by

$$mc_{n,t} = \frac{P^Q_{n,t}}{P_{n,t}A_{n,t}}, \tag{5}$$

which, in contrast to its closed-economy counterpart, can reflect foreign production costs through the price index of tradable goods, $P^Q_{n,t}$.

As in Calvo (1983), in each period a firm producing sticky-price goods in country n is unable to choose its price optimally with probability θ_n. It then sets its price using the indexation rule, $P_{n,t}(j) = (1+\pi_n)P_{n,t-1}(j)$, where π_n denotes country n's long-run or steady-state inflation rate.[4] With probability $1-\theta_n$, the firm is able to reset its price and optimally chooses a price, $P^*_{n,t}$, that maximizes its expected present discounted value of future profits,

$$E_t\left[\sum_{k=0}^{\infty}\beta^{t+k}\frac{\Lambda_{n,t+k}}{\Lambda_{n,t}}\theta^k\left(\frac{P^*_{n,t}(1+\pi_n)^k}{P_{n,t+k}}\right)^{-\varepsilon_{n,t}}\left(\frac{P^*_{n,t}(1+\pi_n)^k}{P_{n,t+k}} - mc_{n,t+k}\right)\mathcal{C}_{n,t+k}\right], \tag{6}$$

where $\beta\frac{\Lambda_{n,t+k}}{\Lambda_{n,t}}$ is the discount factor from t to $t+k$.

The implied first-order conditions indicate that under constant markups, $\mu = \frac{\varepsilon}{\varepsilon-1} > 1$, the nominal price chosen by sticky-price producers at date t, $P^*_{n,t}$, is proportional to a weighted average of their expected future nominal marginal costs. Inflation, therefore, will tend to rise when those marginal costs are expected to increase. Importantly, because domestic marginal costs depend on conditions abroad, foreign shocks will affect domestic prices and inflation.

Tradable Intermediate Goods: The second layer of intermediate goods consists of tradables whose production follows, for the most part, the framework originally laid out in Eaton and Kortum (2002). Tradable goods come in varieties that differ in their production technology. In particular, varieties are indexed by a vector of country-specific productivity parameters, $\mathbf{z} = \{z_1, z_2, ..., z_N\}$, where each element z_n reflects the productivity of firms producing that variety in country n. Formally, the production of tradable variety $\mathbf{z}$ in country n is given by

$$q_{n,t}(\mathbf{z}) = z_{n,t}\ell_{n,t}(\mathbf{z}), \tag{7}$$

[4]In general, this indexation rule may also include lagged inflation, denoted $1+\pi_{n,t-1} = P_{n,t-1}/P_{n,t-2}$. In that case, firms that are unable to choose their price set $P_{n,t}(j) = (1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}P_{n,t-1}(j)$, $\varrho_n \in [0,1]$, where ϱ_n and $1-\varrho_n$ indicate the degree of indexation to lagged inflation and long-run inflation respectively. In Section D of the Appendix, we show that this more general formulation introduces greater inflation persistence in the set of interconnected country-specific Phillips curves that arises in this setting.

where $\ell_{n,t}(\mathbf{z})$ denotes local labor used in the production of that variety.[5]

Producers of sticky-price goods in country n can acquire any tradable variety $\mathbf{z}$ necessary to produce those goods from any country including their own. From here on, we assume that sticky-price goods $j \in [0,1]$ are symmetric and abstract from the j index, $Q_{n,t}(j) = Q_{n,t}$. Thus, $\forall j \in [0,1]$, the demand for tradable variety $\mathbf{z}$ in country n is

$$Q_{n,t}(\mathbf{z}) = \sum_{n'} Q_{nn',t}(\mathbf{z}),$$

where $Q_{nn',t}(\mathbf{z})$ denotes traded varieties produced by country n' and used by country n. These tradable varieties are then aggregated into a single composite tradable bundle according to a CES aggregator, $Q_{n,t} = \left[\int Q_{n,t}(\mathbf{z})^{\frac{\gamma-1}{\gamma}} d\Phi(\mathbf{z})\right]^{\frac{\gamma}{\gamma-1}}$, $\gamma > 1$, which is then used as an input by sticky-price firms in equation (4). The function Φ denotes the joint *c.d.f.* from which $\mathbf{z}$ is drawn.

As in Eaton and Kortum (2002), the elements of $\mathbf{z}$ are jointly drawn from a Fréchet distribution with shape parameter φ, $\Phi(\mathbf{z}) = \exp\left\{-\sum_n (z_n)^{-\varphi}\right\}$. This distributional assumption ensures analytically tractable aggregate import demand functions (discussed in detail below). The shape parameter, φ, governs how much a given variety $\mathbf{z}$ is likely to differ in productivity across countries. Low values of φ imply large cross-country variations in productivity. In contrast, as $\varphi \to \infty$, the level of productivity associated with tradable varieties becomes common across countries.

Sticky-price producers in country n acquire each variety $\mathbf{z}$ from the country that supplies it most cheaply given its production costs and bilateral trade costs. The technology in equation (7) implies nominal production costs in country n' of $W_{n',t}/z_{n',t}$, where $W_{n't}$ is the nominal wage in n' in local currency. The cost of acquiring tradable varieties from abroad also includes an allowance for shipping. In particular, goods imported by country n from country n' are subject to iceberg shipping costs such that only a fraction $1/\kappa_{nn'} \in (0,1)$ of those goods make it to their destination. This implies that the price paid for variety $\mathbf{z}$ in country n, $P_{n,t}(\mathbf{z})$, satisfies

$$P_{n,t}(\mathbf{z}) = \min_{n'} \kappa_{nn'} \mathcal{E}_{nn',t} \frac{W_{n',t}}{z_{n',t}}.$$

Consistent with our definition of nominal exchange rates above, $\mathcal{E}_{nn',t}$ converts prices quoted in the currency of country n' into that of country n.[6] It also follows that $q_{n,t}(\mathbf{z}) = \sum_{n'} \kappa_{n'n} Q_{n'n,t}(\mathbf{z})$ since actual output of any tradable variety $\mathbf{z}$, $q_{n,t}(\mathbf{z})$, must account for losses in transit in meeting the demand for that variety.

[5] We follow the convention adopted by Eaton and Kortum (2002) in indexing varieties by their productivity vectors, $\mathbf{z}$, rather than an arbitrary label $i \in [0,1]$. Observe that any differences in production and prices across varieties reflect only productivity differences. Therefore, denoting the output of variety i by $q_{n,t}(i)$, and letting $\mathbf{z}(i)$ represent its corresponding productivity vector, it follows that $q_{n,t}(i) = q_{n,t}(\mathbf{z}(i))$. Thus, one need only refer to $q_{n,t}(\mathbf{z})$.

[6] Observe that no-arbitrage in currency markets implies that $\mathcal{E}_{nn',t} = \mathcal{E}_{nn'',t}\mathcal{E}_{n''n',t}$.

3.3 Monetary Authority

As in Taylor (1993, 1999), monetary policy in country n is described by the standard monetary policy rule (e.g., Smets and Wouters (2007); Justiniano et al. (2010); Del Negro et al. (2015)),

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^{*} = R_n \left(\frac{1+\pi_{n,t}}{1+\pi_n}\right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\overline{Y}_{n,t}}\right)^{\phi_{n,Y}}, \tag{8}$$

$R_{n,t}$ is the nominal interest rate, denoted by R_n in the steady state, $\overline{Y}_{n,t}$ denotes trend output,[7] and $\nu_{n,t}$ is an exogenous monetary policy shock. With $\phi_{n,\pi} > 0$ and $\phi_{n,Y} > 0$, the monetary authority increases nominal interest rates in response to high inflation or output relative to their long-run paths. We allow the monetary policy rule to differ across countries to account for different responses to inflation or the output gap. Unlike closed-economy settings, we also allow the shock, $\nu_{n,t}$, to be correlated across countries to account for potential coordination in policy.[8]

4 Equilibrium Conditions in a World with Trade and Sticky Prices

4.1 Some Key Equilibrium Relationships

We now highlight several key equilibrium conditions that describe the role of trade in an environment with nominal rigidities. Details of all derivations are provided in the Appendix.

Aggregate Import Shares: The economic environment we have just presented yields tractable expressions that describe aggregate trade shares as functions of their underlying determinants.

In the notation introduced above, $\kappa_{nn'}Q_{nn',t}$ represents real imports of variety $\mathbf{z}$ by country n. Letting $p_{n',t} = W_{n',t}/z_{n',t}$ denote the price charged by country n' for variety $\mathbf{z}$ quoted in local currency, $\mathcal{E}_{nn',t}p_{n',t}(\mathbf{z})\kappa_{nn'}Q_{nn',t}$ is the value of country n's imports from n' in n's currency. Then,

$$X_{nn',t} = \int \mathcal{E}_{nn',t}p_{n',t}(\mathbf{z})\kappa_{nn'}Q_{nn',t}(\mathbf{z})d\Phi(\mathbf{z})$$

represents expenditures by country n on all traded intermediate varieties from country n'. It follows that $X_{n,t} = \sum_{n'} X_{nn',t}$ represents country n's total expenditures on varieties from abroad.

Under the maintained assumptions, each country purchases all tradable varieties. It acquires any one variety $\mathbf{z}$ from a single other country (frequently itself). The choice of whom to purchase a given variety from depends on the production cost of that variety in different countries as well as shipping costs. The assumption that $\mathbf{z}$ is Fréchet distributed ensures that the total value of trade

[7] In our model, trend output is proportional to the stochastic trend A_t.

[8] Similar to Cúrdia, Del Negro, and Greenwald (2014) and Leeper, Traum, and Walker (2017), we abstract from the zero lower bound, keeping the system log-linear. To the extent that the zero lower bound amplifies the effects of demand shocks, it would also reinforce the effect of 'third countries' in trade and their implications for global comovement.

between any country pair is a smooth function of foreign marginal production costs. In particular, the share of country n's expenditures on varieties from country n' at date t, $\varpi_{nn',t}$, is given by

$$\varpi_{nn',t} = \frac{X_{nn',t}}{X_{n,t}} = \frac{\left(\kappa_{nn'}\mathcal{E}_{nn',t}W_{n',t}\right)^{-\varphi}}{\sum_{n''}\left(\kappa_{nn''}\mathcal{E}_{nn'',t}W_{n'',t}\right)^{-\varphi}}. \tag{9}$$

Thus, the share of country n's expenditures on imports from country n' reflects the costs of acquiring varieties from n', including production costs, $W_{n',t}$ and bilateral shipping costs, $\kappa_{n,n'}$, relative to all other countries (all expressed in n's currency).

We briefly highlight three important features of the matrix of import shares, $\varpi_t = \{\varpi_{nn',t}\}$. First, ϖ_t has the properties of stochastic matrix in the sense that it is a real square matrix where each row sums to one, a feature that will prove useful below. Second, the larger the Fréchet shape parameter φ, the more import shares depend on relative wages and exchange rates. Put another way, when φ is large, different varieties are produced with similar productivities in all countries. In that case, small shifts in wages or exchange rates lead to large changes in the number of varieties traded between country pairs. In contrast, $\lim_{\varphi\to 0}\varpi_{nn't} = 1/N\ \forall n, n'$. Finally, equation (9) implies that, conditional on wages and exchange rates, trade costs, $\kappa_{nn'}$, can be directly inferred from long-run import shares, $\varpi_{nn'}$. This feature makes it possible to condition the quantitative analysis on those shares directly rather than trade costs.

Marginal Cost and the Price of the Input Bundle: Recall that the marginal cost faced by producers of sticky-price goods in country n depends on the price of their input bundle, $P^Q_{n,t}$, in equation (5). The assumption in Eaton and Kortum (2002) that $\mathbf{z}$ is drawn from a Fréchet distribution with shape parameter φ implies that the price of the input bundle is

$$P^Q_{n,t} = \Gamma\left(\Upsilon\right)^{\frac{1}{1-\gamma}}\left(\sum_{n'}\left(\kappa_{nn'}\mathcal{E}_{nn',t}W_{n',t}\right)^{-\varphi}\right)^{-\frac{1}{\varphi}}, \tag{10}$$

where $\Gamma(\Upsilon)$ is the Gamma function evaluated at $\Upsilon = 1 + \frac{1-\gamma}{\varphi}$. Equation (10) illustrates how production costs, $W_{n't}$, trade costs, $\kappa_{nn'}$, and exchange rates, $\mathcal{E}_{nn',t}$, across different countries all factor into the price of the input bundle, $P^Q_{n,t}$, paid by country n firms. This mechanism is analogous to how these same factors determine import shares in equation (9). In that sense, equation (10) captures the role of *backward trade linkages*, or alternatively country n's import network, in determining domestic marginal production costs. Because these multilateral trade linkages then directly impact real profits of country n firms, they also affect their pricing decisions.

Aggregate Market Clearing for Traded Intermediate Goods: Given perfectly competitive markets, the total cost of producing tradable varieties in country n, which is to say its wage bill, $W_{n,t}L_{n,t} = W_{n,t}\int \ell_{n,t}(\mathbf{z})d\Phi(\mathbf{z})$, exhausts the value of varieties it produces and exports to foreign countries. In equilibrium, these exports must equate to the demand for country n's domestic product by economies abroad. Foreign demand for n's goods in turn amounts to expenditures abroad on country n's varieties, or $\mathcal{E}_{nn',t}X_{n',t}$ for trading partner n' (in n's currency), weighted by their import share in country n's goods, $\varpi_{n'n,t}$. It follows that

$$W_{n,t}L_{n,t} = \sum_{n'} \varpi_{n'n,t}\mathcal{E}_{nn',t}X_{n',t}. \tag{11}$$

Equation (11), therefore, highlights how *forward trade linkages*, or alternatively country n's export network, help determine the value of domestic output.

4.2 Steady State

As in much of the New Keynesian closed-economy literature, our objective is in part to estimate our multi-country framework approximated around its steady state and explore its quantitative implications. However, unlike much of the related literature, the New Keynesian steady state we study possesses unique features that arise from international trade linkages. We discuss salient implications of these features here and refer the reader to the Appendix for details.

Country-Specific Balanced Growth Paths: In departing from the closed-economy environment, we focus on a world balanced growth path that has two features. First, steady state growth rates for variables such as consumption, real wages, and inflation are country-specific. Second, the world steady-state growth path nevertheless features constant country shares in the world economy so that no country's share of the world economy grows or shrinks indefinitely.

For a given economy n, the derivation of country-specific balanced growth paths for real quantities and steady-state inflation follows that of closed-economy models. In particular, real quantities grow at rate $g_n = E\left[g_{n,t}\right]$, which denotes the long-run mean of technical progress, $g_{n,t} = \frac{A_{n,t}}{A_{n,t-1}}$. In addition, prices increase at the rate $1+\pi_n$, which also determines the steady state interest rate through the Fisher equation, $1+\pi_n = \frac{\beta R_n}{g_n}$.

The existence of a global balanced growth path where no country eventually dominates or becomes irrelevant in the world economy is shown in the Appendix. This requires, among other conditions, that the real exchange rate for each country n relative to the reference country, $\mathcal{E}_{rn,t}\frac{P_{n,t}}{P_{r,t}}$, grow in the long run at a rate that reflects the difference in technical progress between country n and the reference country.[9]

[9]The derivations of these paths, and the conditions that must hold for global balanced growth in that environment, open up applications beyond nominal rigidities, for example with respect to the study of long-run global change. See Müller, Stock, and Watson (2020) for a statistical analysis of the joint long-run evolution of GDP per capita for 113 countries over 118 years from 1900 to 2017. From a structural modeling standpoint, the assumption that varieties $\mathbf{z}$ are drawn from a Fréchet distribution in much of the trade literature is not innocuous. In some instances, country-specific balanced growth paths within a stable world economy, where country shares are constant in the long run, will not be feasible given constant trade costs, $1/\kappa_{nn'} \in (0,1)$. See Section B of the Appendix.

Trade Balances: While the model features transition dynamics whereby different countries can run trade deficits or surpluses, the form of portfolio adjustment costs does not ultimately pin down countries' long-run net asset positions. In other words, analogous to the static framework of Eaton and Kortum (2002), the environment does not provide a theory of countries' steady-state trade deficits or surpluses. As such, we adopt a benchmark defined by balanced trade in steady state.

However, even over periods as long as our sample (16 years), a country's net asset position can differ from zero on average. We address this observation following the approach first proposed by Dekle, Eaton, and Kortum (2007) in a static environment but now applied to the steady state of a dynamic setting. Specifically, we first write a variant of the economic environment whose steady state can be matched to observed average regional trade imbalances over our entire sample, 2004Q2–2019Q4. This can be done by allowing for exogenous frictions in international asset markets even in the long-run, that is $\xi_n \neq 0$. Then, for the set of parameters implied by a world with long-run trade imbalances, we calculate counterfactual steady-state allocations and prices corresponding to zero trade imbalances. More specifically, for the implied set of parameters, we compute counterfactual steady-state allocations consistent with $\xi_n = 0\ \forall n$, in which case real net asset positions are zero for all countries, $S_n = 0\ \forall n$. Analogously to Dekle et al. (2007), the resulting counterfactual world steady-state allocations without imbalances can then be used to produce a linearized model for estimation and analysis using standard linear rational expectations toolkits.

5 The Linearized Global New Keynesian Model

This section describes the solution to our model economy, casting some of its equations in terms of expressions familiar from international trade. We use these expressions to provide intuition for the mechanisms and forces underlying the New Keynesian world economy.

To obtain intuitive expressions, we follow the New Keynesian literature and log-linearize the deflated-detrended equilibrium equations around their steady state. Thus, all variables in the expressions below refer to their appropriately deflated-detrended counterparts.[10] We use a 'hat' over variables to denote log-deviations from steady-state (i.e., $\widehat{x}_{n,t} = \log(x_{n,t}) - \log(x_n)$), and use dx to denote level deviations from steady state, (i.e., $dx_{n,t} = x_{n,t} - x_n$). We simplify these expressions further by assuming that the demand faced by sticky-price firms is subsidized at rate $1 + \tau_n$, set to the inverse of the markup to free our discussion from steady-state distortions arising from monopolistic pricing.[11] Thus, the long-run real marginal cost of sticky price goods is 1 in all countries (i.e., $mc_n = 1\ \forall n$). In the quantitative analysis, however, we allow for pricing distortions. We let $\mathcal{E}_{n,t} \equiv \mathcal{E}_{rn,t}$ for notational convenience and, without loss of generality, take country 1 to be the reference country, (i.e., $r = 1$). Under no arbitrage in currency markets, $\mathcal{E}_{1,t} = \mathcal{E}_{r,t} = \mathcal{E}_{rr,t} = 1$.

[10]See Sections B.1 and B.2 of the Appendix for details of the model normalization with respect to country-specific inflation, $1 + \pi_n$, and technical progress, g_n.

[11]Specifically, firms' profits are then given by $\frac{\Pi_{n,t}(j)}{P_{n,t}} = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t} \left((1+\tau_n)\frac{P_{n,t}(j)}{P_{n,t}} - mc_{n,t}\right)$, where the subsidy to sticky-price firms is financed by lump sum taxes, $T_{n,t}$.

Similarly, we define the trend-adjusted real exchange rate, which converts country n final goods into units of country 1 final goods, as $e_{n,t} = \mathcal{E}_{n,t}\frac{A_{n,t}P_{n,t}}{A_{1,t}P_{1,t}}$. Finally, while we allow for country-specific preference and technology parameters in the estimation, σ_n, η_n, ω_n, θ_n, etc., we set these parameters to be the same in this section to simplify notation. To that end, we also express all equations in vector form, stacking country-specific variables into vectors $\widehat{x}_t = (\widehat{x}_{1,t}, ..., \widehat{x}_{N,t})$.

The interaction between nominal rigidities, international trade, and international financial flows can be summarized by two main blocks of equations. The first block describes conventional New Keynesian mechanisms in all countries. The second block captures forces underlying international trade in goods and financial assets. Crucially, these two blocks are linked through the marginal cost of sticky price goods, a central element of the New Keynesian Phillips curve (NKPC), that now reflects export prices of trading partner countries and the fundamentals that drive them.

5.1 The New Keynesian Block

The New Keynesian block reflects intertemporal mechanisms at home along with a description of monetary policy and domestic labor supply,

$$\widehat{C}_t = E_t\widehat{C}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{g}_{t+1}\right), \tag{12}$$

$$\widehat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{mc}_t + \widehat{\mu}_t) + \beta(E_t[\widehat{\pi}_{t+1}]), \tag{13}$$

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)(\phi_\pi\widehat{\pi}_t + \phi_Y\widehat{Y}_t) + \widehat{\nu}_t, \tag{14}$$

$$\widehat{w}_t = \eta\widehat{L}_t + \sigma\widehat{C}_t, \tag{15}$$

where $\widehat{C}_t$ is a vector that contains consumption deviations from steady state for each country, $\widehat{C}_t = (\widehat{C}_{1,t}, ..., \widehat{C}_{N,t})$, and similarly for $\widehat{\pi}_t$, $\widehat{R}_t$, etc. Equations (12)-(14) replicate familiar expressions from the New Keynesian literature in vector form: equation (12) is a standard household Euler equation for every country, equation (13) describes a vectorized NKPC that highlights the role of marginal cost, markup shocks, and expected inflation, and equation (14) describes a monetary policy rule for each country. Finally, equation (15) describes labor supply in all countries with its attendant income and substitution effects. The model includes three exogenous shocks per country: shocks to technical progress, $\widehat{g}_t$, markup shocks, $\widehat{\mu}_t$, and monetary policy shocks, $\widehat{\nu}_t$.

In the absence of trade, the New Keynesian block reduces to a set of conventional closed-economy three-equation New Keynesian models (e.g., An and Schorfheide (2007); Galí (2008)), thus yielding isolated New Keynesian economies (i.e., autarky). In particular, absent trade, the real marginal cost of sticky-price goods, $\widehat{mc}_t$, is determined by payments to domestic inputs, in this case wages, $\widehat{w}_t = \eta\widehat{L}_t + \sigma\widehat{C}_t$. Since, absent trade, linearized output, $\widehat{Y}_t$, equates to consumption, $\widehat{C}_t$, the marginal cost faced by domestic firms ultimately reflects a domestic output gap, $\widehat{mc}_t = (\eta + \sigma)\widehat{Y}_t$.

When countries are instead engaged in multilateral trade, the marginal cost of sticky-price goods no longer equates to payments for domestic inputs. This marginal cost now reflects global and country-specific forces operating through adjustments in countries' trade balances and net asset positions along with global market clearing. These international forces are described in the second block of equations.

5.2 The International Block

The second block of equations relates to relationships that arise under international trade. These include i) a set of equations describing the marginal cost of producing sticky-price goods in each country as a function of cost conditions abroad, ii) a set of equations describing market clearing conditions for traded goods, iii) a set of equations describing the balance of payments in each country, and iv) a set of equations describing the evolution of countries' financial assets.

i) Marginal Cost of Producing Sticky-Price Goods:

Unlike a closed economy where marginal costs are determined only by the price of domestic inputs, marginal costs generally depend on costs in all trading partner countries. In particular, equation (10) implies that

$$\widehat{e}_t + \widehat{mc}_t = \varpi(\widehat{e}_t + \widehat{w}_t), \tag{16}$$

where $\widehat{e}_t$, $\widehat{mc}_t$, and $\widehat{w}_t$ are vectors of exchange rates, marginal costs, and wages, respectively, in log-linearized form. The left-hand side of equation (16) is the marginal cost of producing sticky-price goods for each country, expressed in real units of the reference country's goods. This is determined by wages in each country, expressed in the reference country's goods, weighted by the matrix of import shares, $\varpi = \{\varpi_{nn'}\}$. The influence that country n' has on country n increases in proportion to how much n imports from n', $\varpi_{nn'}$. As such, $\varpi_{nn'}$ captures the role of backward trade linkages in determining marginal cost.

ii) Market Clearing for Traded Goods:

There must be N market clearing conditions for traded goods, one for each country. These can be expressed in vector form in a way that describes the makeup of the real trade balance. From equation (11),

$$\underbrace{\widehat{Y}_t - \widehat{C}_t}_{\text{real trade balance}} = \underbrace{(\psi^T - I)\widehat{C}_t}_{\substack{\text{relative demand} \\ \text{in export markets}}} - \underbrace{(1+\varphi)}_{\substack{\text{price elasticity} \\ \text{of trade}}} \underbrace{(I - \psi^T\varpi)(\widehat{e}_t + \widehat{w}_t)}_{\substack{\text{relative price} \\ \text{at destination}}}, \tag{17}$$

where $\psi = \{\psi_{n'n}\}$ denotes the matrix of (steady state) export shares from country n to n'.[12]

The left-hand side of the equation, $\widehat{Y}_t - \widehat{C}_t$, captures real quantities produced and consumed, defining the real trade balance. Deviations in countries' real detrended GDP from their steady state are $\widehat{Y}_t = \widehat{L}_t$. Naturally, the real trade balance depends on the relative demand in export markets, the

[12] Formally, $\psi_{nn'} = \frac{\varpi_{nn'} e_n mc_n \mathcal{C}_n}{\sum_{n''} \varpi_{n''n'} e_{n''} mc_{n''} \mathcal{C}_{n''}}$.

first term on the right-hand side of equation (17). It moves towards a surplus when the consumption of home goods abroad, $\psi^T \widehat{C}_t$, increases. Here, each foreign country's consumption of home-produced goods is weighted by the share of exports it purchases from the home country. Conversely, the real trade balance moves towards a deficit when domestic consumption, $I\widehat{C}_t$, increases.

The real trade balance also depends on the relative price of exports in foreign markets, the second term on the right-hand side of equation (17). The term compares the price of exports in each country (in real units of the reference country), $\widehat{e}_t + \widehat{w}_t$, with the corresponding prices of goods produced in other countries weighted by $\psi^T \varpi$. As such, the term $(I - \psi^T \varpi)(\widehat{e}_t + \widehat{w}_t)$ represents the substitution from countries with higher costs to those with lower costs.

Importantly, this substitution means that the trade balance in country n depends on country n' not only through economic conditions in country n', but also through conditions in other countries that trade with n'. The import matrix, ϖ, indicates that in response to an increase in wages in a 'third' country n'', country n' will substitute toward country n (and thereby increase country n's trade balance according to how much n' imports from n). Country n' reacts more strongly to real wages in places from which it imports heavily, substituting away from these 'third' countries and towards country n proportionally to its import shares with n''. Concretely, suppose that real wages in China increase, $\widehat{e}_{n'',t} + \widehat{w}_{n'',t} > 0$. All else equal, Japan reacts by substituting toward U.S. exports and away from China in proportion to its import share from China (captured in ϖ). This shift in trade increases the U.S. trade balance in proportion to its exports towards Japan (captured in ψ^T). Equation (17) thus emphasizes the role of both backward trade linkages, ϖ, and forward linkages, ψ^T, in driving the effects of foreign prices on the trade balance.[13]

The Fréchet parameter, φ, governs the scale of real trade adjustments to relative export prices. In other words, φ ultimately plays the role of a price elasticity of trade. Larger values of φ imply less dispersion in the productivity of any given variety across countries. In that case, firms are more sensitive to a given relative price change and substitute inputs more readily across countries resulting in larger trade effects.

iii) Balance of Payments:

Rearranging countries' household budget constraints (2), linearized around a steady state with no monopolistic trade distortions, yields the following expression relating the trade balance and the terms of trade to financial flows

[13]These third-country effects stand in contrast to much of the open economy New Keynesian literature modeling either two large countries or individual small open economies. Guerron-Quintana (2013) and Aggarwal, Auclert, Rognlie, and Straub (2022) go a step further by considering multiple small open economies, but Guerron-Quintana (2013) acknowledges that such models abstract from the multiple layers of interactions across countries that are central in an environment with multiple large countries and highlighted herein.

$$\underbrace{\widehat{Y}_t - \widehat{C}_t}_{\text{real trade balance}} + \underbrace{(I-\varpi)(\widehat{e}_t + \widehat{w}_t)}_{\substack{\text{terms of trade}\\ \times \text{ s.s. import shares}}} = \underbrace{Vg_1(1+\pi_1)E_t\left[\frac{ds_{t+1}}{eC}\right] - \frac{ds_t}{eC}}_{\text{financial flows}}. \tag{18}$$

The left-hand side of the equation is a vector of trade balances for each country in local units. It can be decomposed into a real trade balance of quantities produced and consumed, $\widehat{Y}_t - \widehat{C}_t$, and a measure of the terms of trade for each country, scaled by import shares, $(I-\varpi)(\widehat{e}_t + \widehat{w}_t)$.[14] The terms of trade captures the wedge between production costs (which are a function of local factor prices) and consumption costs (which are a function of foreign factor prices and exchange rates). As emphasized by Kehoe and Ruhl (2008), while terms of trade fluctuations do not affect real GDP, they do affect welfare. Importantly, the equation highlights that the terms of trade also affect the domestic value of the trade balance that needs to be covered by financial flows.

The right-hand side captures financial flows, normalized by consumption. Keeping track of financial flows becomes necessary when international financial markets are subject to frictions. These frictions imply that adjustments to countries' net asset positions are costly and so distort consumption allocations across time and states of nature. To the extent that portfolio rebalancing costs affect financial flows, the balance-of-payments equation implies that they also affect the behavior of the real trade balance and the terms of trade. In the limiting case, where portfolio adjustment costs become infinitely high, trade balance holds period by period and creates a tight link between consumption, output, and exchange rates.

iv) Financial Asset Allocations:

Finally, around a steady state with balanced trade, the Euler equation governing the evolution of countries' financial assets satisfies

$$\underbrace{\widehat{C}_{1,t} - \widehat{C}_t}_{\text{relative consumption}} = \frac{1}{\sigma}\underbrace{\widehat{e}_t}_{\substack{\text{real}\\ \text{exchange rate}}} + \underbrace{\beta\frac{\tau}{\sigma}\overbrace{\left(\frac{1}{e}dS_t - dS_{1,t}\right)}^{\substack{\text{cumulative financial}\\ \text{asset position}}} + \overbrace{d\Xi_t - d\Xi_{1,t}}^{\substack{\text{cumulative portfolio}\\ \text{adjustment shocks}}}}_{\text{asset pricing wedge}}, \tag{19}$$

$$\text{where } dS_t = \sum_{k=-\infty}^{t} ds_k;\ d\Xi_t = \frac{1}{\sigma}\sum_{k=-\infty}^{t-1} d\xi_k.$$

Absent frictions in global financial markets, defined by $\tau = 0$ and $d\xi_{t-k} = 0\ \forall k$, equation (19) reduces to the expression first emphasized by Backus and Smith (1993). In that case, the (log) ratio of consumption between any two countries, $\widehat{C}_{1,t} - \widehat{C}_t$, reflects variations in their real exchange

[14] The terms of trade is the ratio of a country's export prices to import prices (in common units, here the reference country's final goods). Thus country n's linearized terms of trade are given by $\widehat{e}_{n,t} + \widehat{w}_{n,t} - \frac{1}{1-\varpi_{nn}}\sum_{n'\neq n}\varpi_{nn'}(\widehat{e}_{n',t} + \widehat{w}_{n',t}) = \frac{1}{1-\varpi_{nn}}\left(\widehat{e}_{n,t} + \widehat{w}_{n,t} - \sum_{n'}\varpi_{nn'}(\widehat{e}_{n',t} + \widehat{w}_{n',t})\right)$, or in matrix form, $\left(diag(I-\varpi)^{-1}\right)(I-\varpi)(\widehat{e}_{n,t} + \widehat{w}_{n,t})$.

rate, $(1/\sigma)\widehat{e}_t$, (i.e., the relative price of consumption between the two countries). Put another way, higher consumption at home goes hand in hand with a depreciating exchange rate (as consumption goods abroad become more expensive).

However, as also noted by Backus and Smith (1993), the data do not support a clear connection between countries' relative consumption and their real exchange rate. This 'consumption-exchange rate disconnect puzzle' is suggestive of potential financial frictions that drive a wedge between cross-country differences in consumption and their corresponding real exchange rates.[15] Here, we allow for two dimensions in this wedge on the right-hand side of equation (19): one relates to exogenous disturbances in financial markets (relative to the reference country), $d\Xi_t - d\Xi_{1,t}$, and the other relates to endogenous portfolio adjustments (relative to the reference country), $dS_t/e - dS_{1,t}$, where dS_t is a vector representing the history of countries' net asset positions (in linearized form). This history matters because financial frictions affect consumption growth through the Euler equation, and thus have a permanent effect on consumption levels. For example, following a one-time financial disturbance that lowers consumption in country n relative to the reference country, country n households will from then on keep consumption smooth at a level lower than they otherwise would have absent the shock.[16]

6 Foreign Spillovers to the Domestic Economy

Given the equilibrium conditions characterizing the interaction between nominal rigidities and international linkages, we now combine the two blocks of equations to derive the implications for cross-country correlations in output and inflation. First, we show that our model implies a set of interconnected Phillips curves that characterize how each country's inflation depends not only on domestic factors but also on real activity and the exchange rates of its trading partners. Next, we present a special case of our model whose analytical solution makes the role of third-country network effects for cross-country comovement transparent.

6.1 The Global Phillips Curve

Implicit in the two blocks of equations above is a system of interrelated Phillips curves. This system defines a *global Phillips curve* for vectors of variables pertaining to all countries, highlighting how domestic inflation in each country is connected to real activity and exchange rates of trading partners. The global Phillips curve thus provides a useful summary of how trade connects nominal and real variables across countries, similar to how the conventional Phillips curve summarizes the trade-off between output and inflation in a closed economy.

[15] See Kollmann (1995), Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008), for examples of papers that specifically address the Backus and Smith (1993) puzzle through financial frictions.

[16] An interesting implication of this history dependence is that the model can imply very persistent real exchange rate fluctuations in response to transitory financial or monetary shocks, in line with the 'purchasing power parity puzzle' documented in Rogoff (1996).

In the NKPC for each country, described by equation (13), the marginal cost of producing sticky-price goods depends on multilateral trade considerations. It follows from the labor supply equation (15) in the New Keynesian block and the expression for marginal cost (16) in the international block that

$$\widehat{mc}_t = \varpi(\eta \widehat{Y}_t + \sigma \widehat{C}_t) + (\varpi - I)\widehat{e}_t.$$

In other words, in equation (16), the marginal cost at home depends not only on domestic production costs (domestic wages in this case), but also on the real exchange rate and production costs of imported goods (wages in trading partner countries). These in turn reflect real activity embodied in the determination of labor supply abroad, $\widehat{w}_t = \eta \widehat{Y}_t + \sigma \widehat{C}_t$.

The global Phillips curve is then given, in vector form, by

$$\widehat{\pi}_t = \kappa \left[\varpi(\eta \widehat{Y}_t + \sigma \widehat{C}_t) + (\varpi - I)\widehat{e}_t + \widehat{\mu}_t \right] + \beta(E_t[\widehat{\pi}_{t+1}]), \tag{20}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is the coefficient of inflation on the output gap in the conventional closed economy model. In this context, however, the coefficient, κ, applies to an import-share weighted average of domestic and foreign production costs, $\varpi(\eta \widehat{Y}_t + \sigma \widehat{C}_t)$, and also applies to variations in real exchange rates, $(\varpi - I)\widehat{e}_t$. Put another way, inflation in country n no longer moves one-for-one with country n's own output gap (conditional on expected inflation), but also depends on real activity and exchange rates in trading partner countries. Specifically, the comovement between inflation in country n and real activity and exchange rates in country n' is proportional to country n's import share from country n', $\varpi_{nn'}$.[17]

The global Phillips curve (20) imposes theoretical structure on the weights that different foreign countries have in the determination of domestic inflation. The choice of appropriate weights is a problem that Borio and Filardo (2007) and Bianchi and Civelli (2015), among others, had to contend with in their study of global determinants of domestic inflation. The global Phillips curve implies that conditional on international output and consumption data, import-weighted exchange rates are appropriate.

To quantify the role of foreign spillovers for domestic inflation, one approach is to estimate equation (20) using direct observations or proxies of the variables. However, in equilibrium, variations in exchange rates and output are ultimately determined by global or country-specific shocks operating through adjustments in countries' trade balances and asset positions as well as global market clearing. Simply put, the variables on the right-hand side of (20) are endogenous. Real

[17] The global Phillips curve as written in equation (20) depends not only on output in all countries, but also on expenditures (here, consumption) and exchange rates. Therefore, it differs from more compact expressions developed in frameworks with complete markets such as Galí and Monacelli (2005) and Clarida et al. (2002), where equilibrium trade relationships and financial market equilibrium conditions may be used to substitute for consumption and exchange rates. Importantly, our multi-country setup allows the expression to highlight the role of heterogeneous trade linkages. Another difference is that our expression holds for total headline inflation, rather than just inflation in domestically produced goods. This difference stems from our assumption that local sticky price producers use goods produced by workers in all countries rather than local labor. Our framework also assumes local currency pricing by construction.

exchange rates, in particular, will deviate from uncovered interest rate parity (UIP) according to

$$
\begin{aligned}
(E_t\widehat{e}_{t+1} - \widehat{e}_t) = & - \widehat{R}_t + E_t[\widehat{\pi}_{t+1} + \widehat{g}_{t+1}] + \left(\widehat{R}_{1,t} - E_t[\widehat{\pi}_{1,t+1} + \widehat{g}_{1,t+1}]\right) \\
& - \beta\tau E_t\left(\frac{1}{e}ds_{t+1} - ds_{1,t+1}\right) - (d\xi_t - d\xi_{1,t}),
\end{aligned} \tag{21}
$$

where $\beta\tau E_t\left(\frac{1}{e}ds_{t+1} - ds_{1,t+1}\right)$ are costs arising from endogenous portfolio adjustments and $(d\xi_t - d\xi_{1,t})$ are exogenous disturbances that affect countries' trading in global financial markets. Therefore, in a generalized open economy setting, financial frictions create an additional challenge to directly estimating New Keynesian Phillips curves. Moreover, in equilibrium, spillovers from output on the right-hand side of (20) are driven in part by third-country effects involving export shares that do not directly appear in that equation, and that we illustrate explicitly in the next section. These endogeneity issues add to existing complications in using limited-information approaches to estimate closed-economy NKPCs, including ambiguous interpretations of the output gap (e.g., Galí and Gertler (1999); Neiss and Nelson (2005)) or measurements of inflation expectations (e.g., Roberts (1995); Sbordone (2005)). Mavroeidis, Plagborg-Møller, and Stock (2014) highlight and summarize how these complications translate into a broad range of estimates in the literature. Given these challenges, our quantitative analysis in Section 7 takes a full-information Bayesian approach, utilizing the complete set of cross-equation restrictions from our model for identification.

6.2 Spillovers from Idiosyncratic Demand Shocks: An Analytical Example

We now provide further intuition on the nature of cross-country spillovers through an analytical example. As in closed-economy models, this example highlights the importance of demand shocks with nominal rigidities. To obtain closed-form solutions, we make two key assumptions. First, all shocks are *i.i.d.* across countries and over time. In particular, all comovement in this example arises from idiosyncratic country-specific shocks through endogenous spillovers rather than exogenous global shocks. Second, following Clarida et al. (2002), we replace the monetary policy rule (8) with a simple forward-looking inflation-centric rule in every country,

$$
\widehat{R}_t = \phi E_t\widehat{\pi}_{t+1} + \sigma\nu_t.
$$

For transparency, we let all elements of the monetary shocks, ν_t, have unit variances. However, for the discussion in this section, this is without loss of generality.

These assumptions imply a solution for consumption of the form,

$$
\widehat{C}_t = -\nu_t.
$$

That is, $\widehat{C}_{n,t}$ depends only on its own-country monetary policy shocks. Since these shocks are independent across countries in this example, consumption is also uncorrelated across countries,

$E[\widehat{C}_t\widehat{C}_t^T] = I$. As expected, lower nominal rates in a given country, $\nu_{n,t} < 0$ say, raises its consumption relative to trend. In what follows, we delve into the mechanisms at work by focusing on the limit case with no frictions in global financial markets, $\tau = 0$ and $d\xi_t = 0 \ \forall t$.

International Comovement from Demand Shocks: Under the above assumptions, analytical expressions for output and inflation provide intuition for how the model reproduces the positive correlations described in Section 2.

The solution for output is, in vector form,

$$\widehat{Y}_t = -\mathcal{A}_{YY}^{-1}\psi^T\nu_t, \text{ where } \mathcal{A}_{YY}^{-1} = \frac{1}{a}(I - b\psi^T\varpi)^{-1}, \tag{22}$$

with $a = 1 + (1+\varphi)\eta$ and $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$. Absent trade, $\varpi = \psi^T = I$ and equation (22) reduces to $\widehat{Y} = -\nu_t = \widehat{C}_t$, so that each country's production adjusts to meet its own local demand. With trade, the export matrix ψ^T captures the first layer of trade spillovers: when demand in country n' increases through an expansionary monetary shock, $\nu_{n',t} < 0$, output in country n increases in proportion to its exports to country n'. The matrix $\mathcal{A}_{YY}^{-1}$ summarizes subsequent rounds of trade in the form of a Leontief inverse capturing the effects of backward linkages, ϖ, and forward linkages, ψ^T. The strength of these effects depends on model elasticities with respect to labor, $1/\eta$, and trade, φ. We discuss these successive rounds of trade below after deriving additional results pertaining to the model solution.

The solution for inflation also reflects spillovers from foreign demand shocks,

$$\widehat{\pi}_t = \kappa\eta\varpi\widehat{Y}_t - \kappa\sigma\nu_t + \kappa\mu_t, \tag{23}$$

which stands for the global Phillips curve (20) in this case. As before, domestic inflation comoves with output abroad to the extent that countries are connected through the import matrix ϖ. However, consumption increases in country n' are also offset by a depreciating exchange rate, as implied by equation (19) when $\tau = 0$.[18]

The above expressions for output, (22), and inflation, (23), imply the following proposition:

Proposition 1. *For any matrix M, let $M \geq 0$ if and only if $m_{ij} \geq 0 \ \forall ij$. Then, under the maintained assumptions, $E[\widehat{Y}_t\widehat{Y}_t^T] \geq 0$, $E[\widehat{\pi}_t\widehat{\pi}_t^T] \geq 0$ and $E[\widehat{\pi}_t\widehat{Y}_t^T] \geq 0$.*

Proof. From equation (22), the variance-covariance matrix of output across countries is given by,

$$E\left[\widehat{Y}_t\widehat{Y}_t^T\right] = \mathcal{A}_{YY}^{-1}\psi^T\psi\left(\mathcal{A}_{YY}^{-1}\right)^T.$$

[18] The net effect reduces to the term $-\kappa\sigma\nu_t$, as the depreciating exchange rate increases inflation in country n in response to an expansionary monetary shock. In a closed economy, (23) becomes the familiar expression, $\widehat{\pi}_t = \kappa(\eta+\sigma)\widehat{Y}_t + \kappa\mu_t$, with $\widehat{Y}_t = \widehat{C}_t = -\nu_t$ as inflation now only depends on domestic conditions.

Since $\psi^T \geq 0$, it immediately follows that $E\left[\widehat{Y}_t\widehat{Y}_t^T\right] \geq 0$ whenever $\mathcal{A}_{YY}^{-1} \geq 0$.

To show that $\mathcal{A}_{YY}^{-1} \geq 0$, recall that $\mathcal{A}_{YY} = a(I - b\psi^T\varpi)$ where a and b are scalars such that $a = 1 + (1+\varphi)\eta$ and $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$. Because the matrices of import and export shares, ϖ and ψ^T respectively, are stochastic matrices, so is their product, $\psi^T\varpi$. It follows that $\rho(\psi^T\varpi) = 1$ where $\rho(\psi^T\varpi) = \max_{\Lambda(\psi^T\varpi)}\{|\lambda|\}$ such that $\Lambda(\psi^T\varpi) = \{\lambda | \lambda \text{ is an eigenvalue of } \psi^T\varpi\}$, i.e., the spectral radius of $\psi^T\varpi$ is 1. In addition, since $b < 1$, the spectral radius of $b\psi^T\varpi$ is strictly less than 1, $\rho(b\psi^T\varpi) < 1$. We can then write $\mathcal{A}_{YY}^{-1}$ as

$$\mathcal{A}_{YY}^{-1} = \{a(I - b\psi^T\varpi)\}^{-1} = \frac{1}{a}\sum_{m=0}^{\infty} b^m(\psi^T\varpi)^m.$$

Given that $\psi^T\varpi \geq 0$, $\mathcal{A}_{YY}^{-1} \geq 0$ and $E\left[\widehat{Y}_t\widehat{Y}_t^T\right] \geq 0$.

The solution for inflation above is given by $\widehat{\pi}_t = \kappa\eta\varpi\widehat{Y}_t - \kappa\sigma\nu_t + \kappa\mu_t$. Therefore, $E[\widehat{\pi}_t\widehat{\pi}_t^T] = \kappa\eta\varpi E[\widehat{Y}_t\widehat{Y}_t^T]\varpi^T\ \eta\kappa + 2\kappa^2\eta\sigma\varpi\mathcal{A}_{YY}^{-1}\psi^T I + \kappa^2\sigma^2 I + \kappa^2 I$ since $E[\nu_t\mu_t^T] = 0$. Moreover, given that $E[\widehat{Y}_t\widehat{Y}_t^T] \geq 0$, it then follows that $E[\widehat{\pi}_t\widehat{\pi}_t^T] \geq 0$. It also follows that $E[\widehat{\pi}_t\widehat{Y}_t^T] = \kappa\eta\varpi E[\widehat{Y}_t\widehat{Y}_t^T] + \kappa\sigma\psi(\mathcal{A}_{YY}^{-1})^T \geq 0$. □

The proposition states that under frictionless asset markets, demand shocks imply that output comoves positively across countries, inflation comoves positively across countries, and inflation comoves positively with output across countries, as in Figure 1. These comovement properties derive from three channels: i) the direct effects of shocks on the demand for goods both domestically and from abroad, ii) their effects on wages, and iii) their effects on capital flows.

Comovement arises first as spillovers from multilateral trade in goods. Consider the effects on country n of an expansionary monetary shock in country n', $\nu_{n',t} < 0$, that raises consumption in n', $\widehat{C}_{n',t} = -\nu_{n',t} > 0$. From equation (22) and the Neumann series expression for A_{YY}^{-1} in Proposition 1, we can summarize the effects of this increase in demand on output growth as

$$\widehat{Y}_t = \underbrace{-\frac{1}{a}\psi^T\nu_t}_{\text{Direct Effects}} \underbrace{-\frac{1}{a}\left(b\psi^T\varpi + b^2(\psi^T\varpi)^2 + b^3(\psi^T\varpi)^3 + ...\right)\nu_t}_{\text{Indirect Effects}}, \tag{24}$$

which illustrates both direct and indirect spillovers. These direct and indirect effects then extend to inflation via equation (23).

Since consumption in country n' includes foreign goods, there is a *direct* positive demand spillover onto its trading partners, including country n. The effect on country n is greater the more country n exports to country n', as reflected in the matrix of export shares, ψ^T. More generally, production in each country immediately increases in proportion to its exports to country n' via forward linkages, $-(1/a)\psi^T\nu_t > 0$.

However, additional *indirect* effects also arise through repeated rounds of trade created by import or export substitution in third countries, thus involving both backward and forward linkages, the terms $-(1/a)b^m(\psi^T\varpi)^m\nu_t > 0$ on the right-hand-side of (24). These terms make the multiple rounds of third-country effects captured in equation (17) even more transparent. Specifically, with the rise in the demand for goods in country n' (arising from its expansionary shock) comes an increase in labor demand and thus wages in country n'. Therefore, a 'third' country, say n'', that imports from country n' will substitute away from n' towards other countries including n, more so if a large fraction of imports in country n'' originates from n' in the import matrix, ϖ. This effect induces a further increase in the demand for goods produced by country n, the extent of which will be larger the more n exports to n'' in the export matrix, ψ^T. Finally, the propagation of the initial stimulative shock in country n' continues further afield as higher export activity and production in country n raises its wages so that its trading partners, country n''' say, redirect some of their imports from n back towards n'. This redirection induces higher labor demand and wages in country n' thus generating new demand for foreign goods from n' and creating a new round of trade, $-(1/a)b^{m+1}(\psi^T\varpi)^{m+1}\nu_t > 0$, and so on.

The strength of these spillovers depend on the trade and labor elasticities, φ and $1/\eta$, respectively, through $a = 1 + (1 + \varphi)\eta$ and $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$ in the solution for output growth and inflation. The greater the Fréchet parameter, φ, the more sensitive countries are to changes in export prices and the more easily they substitute across trading partners. The smaller the labor elasticity, $1/\eta$, the more wages increase in country n' as the demand for its goods rises, thus magnifying the extent of trade substitutions in third countries.

Finally, multilateral trade in assets also plays an important role. A positive demand shock in country n' increases its budget deficit in order for households to smooth consumption over time. The demand for state-contingent assets in country n', therefore, falls and implies a depreciation of its currency in equilibrium. Equation (19) indicates that the extent of this depreciation is smaller the lower the intertemporal elasticity of substitution, $1/\sigma$. As $1/\sigma$ falls, households are less willing to smooth consumption intertemporally so that equilibrium requires a larger depreciation of the exchange rate.

Besides the positive cross-country comovement in real activity or inflation implied by nominal rigidities and trade, these features also mean that inflation *at home* comoves positively with real activity *abroad*, captured in part by the global Phillips curve (20). In this setting, higher employment at home and abroad imposes a higher disutility of labor and higher wages everywhere. Therefore, higher employment in any given country potentially induces higher inflation in all of its trading partners.

Implications for Other Variables and Shocks: We end this section with two observations. First, output is more correlated across countries than consumption. This observation is contrary to that implied by international real business cycle models (IRBCs) with no trade costs (Backus, Kehoe, and Kydland (1992)). Proposition 1 implies that this feature of IRBCs need not be the case in a setting with multilateral trade costs and sticky prices.[19] In such a setting, uncorrelated demand shocks can generate orthogonal consumption movements that spill over into correlated output fluctuations.

The second observation is that *i.i.d.* shocks to the growth rate of a country's TFP do not spill over to other countries in this example. This is because the model admits country-specific balanced growth paths so that it can be written entirely in terms of detrended variables. Therefore, an *i.i.d.* shock that changes the level of TFP permanently in a country will only scale other variables in that country to the same extent (including bilateral exchange rates). In other words, a one-off change in a country's TFP generates offsetting wealth and substitution effects that keep its labor unchanged along the balanced growth path. This shock also generates an offsetting real exchange rate appreciation that keeps trade patterns from changing along that path, and keeps output in other countries unchanged. These results are specific to growth rate shocks that do not persist. If TFP were instead expected to keep rising after an initial unanticipated increase, consumption decisions would then be affected. The effects of variations in consumption relative to trend will then propagate across countries through the same channels highlighted in Proposition 1.

7 Quantitative Findings

To study the quantitative implications of our model, we calibrate the steady-state values of productivity growth, inflation, employment, and the wage bill to their averages over the period 2004Q2 through 2019Q4, as well as those of the trade matrix over the same period. We then estimate the remaining parameters using Bayesian methods.

7.1 Shock Structure

Our model includes four types of shocks: TFP growth shocks, $g_{n,t}$, markup shocks, $\mu_{n,t}$, monetary policy shocks, $\nu_{n,t}$, and shocks to portfolio costs, $\xi_{n,t}$. The first three shocks are standard in the New Keynesian literature and the last one ensures that we match the behavior of real exchange rates, as discussed in Section 6, through the modified UIP condition, (21).

To allow for common shocks across countries, we consider a factor structure for the first three shocks.[20] In particular, for a generic shock $\mathsf{x}_{n,t}$, $\mathsf{x} \in \{g, \mu, \nu\}$:

$$\mathsf{x}_{n,t} = \lambda_{\mathsf{x},n}\mathsf{x}_t^G + \tilde{\mathsf{x}}_{n,t}, \tag{25}$$

$$\mathsf{x}_{n,t}^G = \rho_{\mathsf{x}}^G \mathsf{x}_{t-1}^G + \sigma_{\mathsf{x}}^G \epsilon_t^G, \tag{26}$$

$$\tilde{\mathsf{x}}_{n,t} = \tilde{\rho}_{\mathsf{x},n}\tilde{\mathsf{x}}_{n,t-1} + \tilde{\sigma}_{\mathsf{x},n}\tilde{\epsilon}_{n,t}. \tag{27}$$

[19] Trade costs are necessary to generate real exchange rate fluctuations, allowing consumption to vary across countries even when $\tau = \xi_{n,t} = 0$ (see equation (19)).

[20] Shocks to portfolio costs only matter to the degree that they are country specific. Global shocks in international financial markets cancel out in equation (19) unless they impact each country differently.

Thus, each shock consists of a global factor, x_t^G, and an idiosyncratic country component, $\tilde{\mathsf{x}}_{n,t}$, both of which follow AR(1) processes. Since $\lambda_{\mathsf{x},n}$ and σ_{x}^G cannot be separately identified, we normalize $\sigma_{\mathsf{x}}^G = 1/100$. The sign of $\lambda_{\mathsf{x},n}$ is identified through its prior.[21]

With this factor structure, comovement across countries can arise from either common global shocks or spillovers from country-specific shocks through trade. Note that trade also affects the propagation of global shocks.

7.2 Data and Calibrated Parameters

In the quantitative application, we rely on data for Canada, China, the E.U., Japan, and the U.S. over the period 2004Q2 through 2019Q4. The latter four countries capture the world's largest economies, while Canada is included as a major trading partner of the U.S. Altogether, these countries and regions represent on average 53 percent of world imports and 50 percent of world exports. The time period used in the quantitative analysis allows us to have a balanced sample for the Bayesian estimation and captures most of the period following China's integration in the World Trade Organization. Details pertaining to all data sources and construction as well as estimation are included in Section G of the Appendix.

The calibrated parameters are summarized in Table 1. In the steady state, these parameters reflect time series means for each country over the sample period. In calibrating the matrix of import shares, we set its diagonal elements to the share of domestic goods in consumption for each country. The off-diagonal elements, $\varpi_{nn'}$, are proportional to import shares of country n from country n' for every country pair (n, n'), scaled so that each row sums to one (because the model does not capture the entire world economy). The import trade matrix implies that most countries purchase goods mostly from themselves.

7.3 Estimation

We estimate the remaining parameters of the model using Bayesian methods. The estimation relies on observable variables given by per capita GDP growth, inflation, interest rates, and exchange rates in the measurement equations for each country. GDP growth, inflation, and interest rates are standard observables in the estimation of closed-economy New Keynesian models (e.g., An and Schorfheide (2007); Aruoba, Cuba-Borda, and Schorfheide (2018)). Here, we keep to this conventional approach for the choice of observables but take into account data from multiple countries simultaneously. In addition, we include exchange rates as observables to account for their role in determining trade patterns and, as suggested by the global Phillips curve (20), in determining variations in cross-country inflation.

[21]The approach of using a prior for identification is similar to Baumeister and Hamilton (2015) and Matthes and Schwartzman (2021), who apply the idea in the context of shock identification in vector autoregressions. Unlike those applications, the sign of $\lambda_{\mathsf{x},n}$ is purely a normalization without economic content, making the identification scheme innocuous.

Table 1: Calibrated Parameters

Steady States

		Canada	**China**	**E.U.**	**Japan**	**U.S.**
$400\,(g_n - 1)$	Productivity Growth	0.76	7.93	0.80	0.70	0.92
$400\pi_n$	Inflation	1.76	2.68	1.55	0.31	2.05
L_n	Employment	0.03	1.70	0.22	0.12	0.26
$\mathcal{E}_n W_n L_n$	Wage Bill	1.56	7.83	12.04	5.14	16.39

Notes: Productivity growth and inflation are annualized and calibrated based on quarterly averages of per capita GDP growth and inflation, respectively, over the 2004Q2 to 2019Q4 sample period. Total employment, L_n, is in trillions of hours, calibrated to the product of employment and average hours, using annual data from the Penn World Table over the 2004 to 2019 sample period. The wage bill, $\mathcal{E}_n W_n L_n$, is calibrated to average GDP in trillions of current U.S. dollars, using annual data from the World Bank and OECD over the 2004 to 2019 sample period.

Import Trade Matrix (ϖ)

	Canada	**China**	**E.U.**	**Japan**	**U.S.**
Canada	0.674	0.047	0.036	0.014	0.230
China	0.009	0.775	0.074	0.086	0.057
E.U.	0.007	0.105	0.779	0.029	0.080
Japan	0.006	0.082	0.031	0.843	0.039
U.S.	0.043	0.055	0.040	0.019	0.842

Notes: Import shares correspond to average Import Partner Shares over the 2004Q2 through 2019Q4 sample period, using data from World Integrated Trade Solution compiled by the World Bank. Diagonal elements are calibrated to match the share of domestically produced output. Off-diagonal elements are proportional to import shares from respective countries, scaled so that each row sums to one.

Model parameter priors are summarized in Table 2. The prior for the price elasticity of trade, φ, covers the range of estimates in the literature.[22] We use a prior that is relatively flat for the degree of adjustment costs in global financial markets, τ. Priors for the factor loadings are chosen so that global factors account for approximately half of the variation of each shock at the prior mode. As mentioned earlier, these priors also help normalize the sign of the factor loadings by placing a small probability that factor loadings for a given shock are negative for a majority of the shock realizations. The remaining priors are comparable to the existing literature (e.g., Lubik and Schorfheide (2005); Adolfson, Laséen, Lindé, and Villani (2007); An and Schorfheide (2007)).

Accounting for international linkages within a multi-country sticky price framework raises two computational challenges relative to existing work. First, with five countries, there are a total of 90 parameters to be estimated, with 63 equations characterizing the equilibrium.[23] At the same time, for quantitative purposes, the model we lay out is in some sense the smallest version of an environment with nominal rigidities and multilateral trade that one would reasonably consider. Second, a key related challenge in estimating the model is the calculation of the steady state,

[22]Estimates of trade elasticities center around 4 but are as large as 17. See Footnote 44 in Caliendo and Parro (2015) as well as section 4.2 in Head and Mayer (2014) for comprehensive summaries of estimates.

[23]In comparison, the medium-scale New Keynesian model in Smets and Wouters (2007) has 36 parameters and 32 equations.

Table 2: Priors

Parameter		**Distribution**	**Mean**	**Std Dev**
Global Parameters				
$100(\beta^{-1}-1)$	Discount Rate	Gamma	0.20	0.10
φ	Fréchet Parameter	Gamma	4.00	2.00
τ	Financial Frictions	Gamma	5.00	3.00
$\rho^G_{\mathsf{x},n}$	Global Shock Persistence	Beta	0.50	0.20
Country-Specific Parameters				
σ_n	Inverse Intertemporal Elasticity of Substitution	Gamma	2.00	0.50
η_n	Inverse Labor Supply Elasticity	Gamma	2.00	0.50
θ_n	Calvo Price Stickiness	Beta	0.70	0.10
ρ_n	Taylor Rule: Persistence	Beta	0.50	0.20
$r_{y,n}$	Taylor Rule: Output Coefficient	Gamma	0.50	0.25
$r_{\pi,n}$	Taylor Rule: Inflation Coefficient	Gamma	1.50	0.25
$\tilde{\rho}_{\mathsf{x},n}$	Idiosyncratic Shock Persistence	Beta	0.50	0.20
$100\tilde{\sigma}_{\mathsf{x},n}$	Idiosyncratic Shock Standard Deviation	Inverse Gamma	0.50	4.00
$\lambda_{\mathsf{x},n}$	Factor Loading	Normal	0.15	0.50

described in Section 4.2. Recall that in applying the approach developed by Dekle et al. (2007) to the steady state of our dynamic setting, we first solve for a variant of the model whose steady state matches observed average trade imbalances over our entire sample, then use the implied set of parameters to calculate counterfactual steady-state allocations and prices corresponding to zero trade imbalances. The latter step uses an iterative procedure that comprises up to half the time needed to compute the model's likelihood for a given set of parameters. This step, therefore, potentially doubles the time taken to compute the likelihood relative to the typical steady-state calculations in prototypical New Keynesian models.

To address these computational challenges, we estimate the model using the adaptive sequential Monte Carlo (SMC) algorithm described in Cai et al. (2021). Relative to standard Markov Chain Monte Carlo methods, SMC is more robust to irregular posteriors and improves computational efficiency by allowing for parallelization.[24] The SMC algorithm constructs a sequence of intermediate *bridge distributions* which begin with the prior and end with the posterior. In some cases, Cai et al. (2021)'s adaptive approach to constructing the bridge distributions are key to obtaining accurate results. The procedure allows us to begin with particle draws from the prior and gradually modify those particles until they approximate the posterior.[25] Details of the algorithm can be found in

[24]These advantages have increasingly led to the use of SMC methods to estimate DSGE models (e.g., Creal (2007); Herbst and Schorfheide (2014)). In our case, the estimation takes 17 hours on an Intel 1.8 GHz workstation with 16 cores using MATLAB.

[25]At each stage, the distribution is represented by a set of particles where each particle consists of a parameter value and a weight. With a large number of particles, the weighted average of particle values converges to expectations of the bridge distribution. To generate particles from the stage n distribution, we modify the stage $n-1$ particle values and weights appropriately. See Herbst and Schorfheide (2015) for a textbook treatment on SMC methods.

Table 3: Posterior Means for Selected Parameters

Parameter		**Mean Estimate**				
Global Parameters						
φ	Fréchet Parameter			6.01		
τ	Financial Frictions			1.3×10^{-3}		
Country-Specific Parameters		Canada	China	E.U.	Japan	U.S.
σ_n	Inverse Intertemporal Elasticity of Substitution	2.47	2.54	1.75	2.01	2.89
η_n	Inverse Labor Supply Elasticity	2.52	1.83	3.01	2.72	1.88
θ_n	Calvo Price Stickiness	0.83	0.73	0.85	0.90	0.74

Section G of the Appendix.

7.4 Posterior Parameter Estimates

Table 3 presents posterior means for selected parameters of interest. We plot the prior and posterior distributions for all model parameters in Section H of the Appendix. Overall, posterior estimates align with values used in both the New Keynesian and trade literatures.

We find a price elasticity of trade, φ, of 6, close to estimates typically used in trade though away from the largest values in that literature. The posterior mean of τ is 0.001, which implies low endogenous portfolio costs. In particular, the model at its posterior mean implies that UIP deviations in equation (21) between the U.S. and Canada, China, the E.U., and Japan are primarily driven by (differences in) exogenous financial shocks, $d\xi_t - d\xi_{1,t}$, with standard deviations that are, respectively, 9.4, 1.8, 14.7, and 10.9 times as large as that of the endogenous portfolio costs, $\beta\tau E_t\left(\frac{1}{e}ds_{t+1} - ds_{1,t+1}\right)$. The estimate is far in the left tail of our prior, with a *c.d.f.* of less than 0.001, suggesting that the data heavily favor such a small value.

The mean estimates of the inverse intertemporal elasticity of substitution, σ_n, lie between 1.75 and 2.89, which is consistent with existing estimates in the DSGE literature (e.g., Smets and Wouters (2007) and Herbst and Schorfheide (2015) report posterior means of 1.38 and 2.83, respectively, for the U.S.; Iiboshi, Shintani, and Ueda (2018) report a posterior mean of 1.55 for Japan). Similarly, estimates of the inverse labor supply elasticity, η_n, lie between 1.83 and 3.01, similar to existing estimates (e.g., Smets and Wouters (2007) and Justiniano et al. (2010) find point estimates of 1.83 and 3.79, respectively, for the U.S.; Iiboshi et al. (2018) find a posterior mean of 3.92 for Japan). The posterior mean estimates for the Calvo price setting parameter, θ_n, exhibit substantial heterogeneity across countries. The shorter average duration of prices in the U.S. and China, less than 4 quarters, is similar to typical estimates in the New Keynesian literature. In contrast, the average duration of prices in the E.U. and Japan is considerably longer, at more than 8 quarters, similar to those found within closed-economy settings for those countries (e.g., Sugo, Ueda, et al. (2007) find a mean estimate of 0.88 for θ_n using data from Japan; Galí, Gertler, and Lopez-Salido (2001) reports estimates of between 0.67 and 0.92 for Europe).

Table 4: Mean Cross-Country Correlations Under Different Counterfactual Models

	Model Details					**Mean Cross-Country Correlation**			
	Global Factors	Trade	φ	τ	θ	GDP Growth	Inflation	GDP Growth–Inflation	Consump Growth
Data	-	-	-	-	-	0.39	0.45	0.25	0.18
1.	Y	Y	$\hat{\varphi}$	$\hat{\tau}$	$\hat{\theta}$	0.39	0.45	0.25	-0.05
2.	Y	N	$\hat{\varphi}$	$\hat{\tau}$	$\hat{\theta}$	0.15	0.15	0.15	0.15
3.	N	Y	$\hat{\varphi}$	$\hat{\tau}$	$\hat{\theta}$	0.35	0.09	0.14	-0.02
4.	N	Y	0.03	$\hat{\tau}$	$\hat{\theta}$	0.28	-0.13	0.07	0.17
5.	N	Y	$\hat{\varphi}$	10^6	$0.01\hat{\theta}$	0.18	-0.14	0.07	0.19

Notes: 'Data' line presents correlations in the data; Line 1 presents correlations in full model with parameters set at posterior mean; line 2 corresponds to model without trade; line 3 corresponds to model without global factors; line 4 corresponds to model with low trade elasticity, φ; line 5 corresponds to model with high portfolio costs, τ and low price rigidity, θ. Notation $\hat{\mathsf{z}}$ denotes posterior mean of parameter z.

7.5 Multilateral Trade and Comovement Between Countries

We now consider a series of counterfactual exercises that highlight the quantitative role of different model features. We use the Kalman smoother to extract the shocks implied by our model under the posterior mean, then feed the same shocks into versions of the model with alternative assumptions regarding trade in goods and financial assets, global factors, and price stickiness.

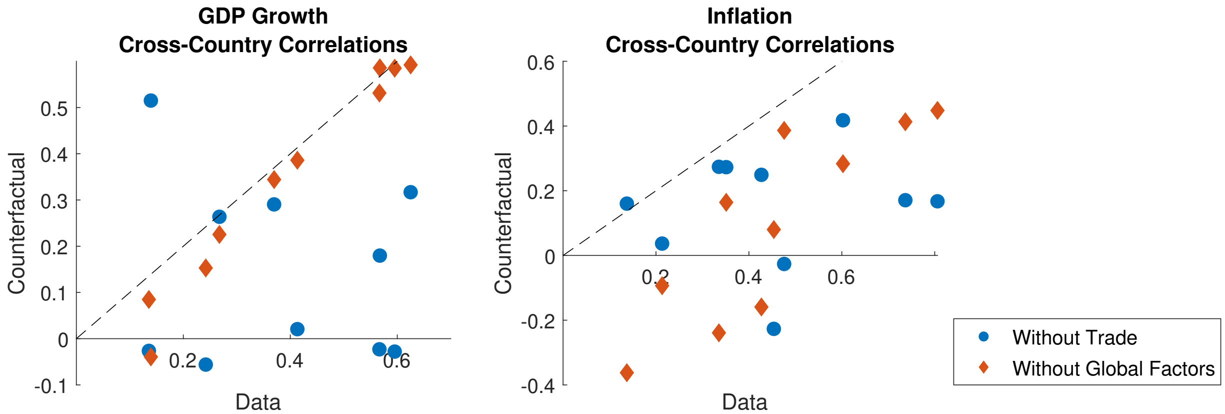
Cross-Country Correlations in GDP Growth and Inflation: Our estimated model indicates that absent trade, cross-country correlations in output growth and inflation are substantially lower. This finding emerges despite the presence of global factors, is shared broadly across all countries, and results primarily from third-country network effects.

The average correlation in GDP growth across countries is 0.39 in the data, but falls by about 60 percent to 0.15 without trade (line 2 of Table 4). Shutting down countries' exposure to global shocks rather than trade, the average cross-country correlation in output growth only falls to 0.35 (line 3 of Table 4). In other words, nearly 90 percent of the cross-country comovement in GDP growth arises through country-specific shocks propagating through the trade network.

The left panel of Figure 2 further highlights the striking difference in the role of trade and global factors in driving GDP growth cross-country correlations. In the counterfactual without trade, GDP growth correlations fall in almost all cross-country pairs, with four pairs even yielding negative correlations. In contrast, the markers for the counterfactual without global factors all lie close to the 45° line, indicating that shutting down global factors has little effect on any of the cross-country correlations. For example, the correlation in output growth between the U.S. and the E.U. is 0.57 in the data, falls to -0.02 without trade, but remains as high as 0.53 with trade but without global factors.

The average cross-country correlation in inflation is similarly driven to a large degree by trade. The average correlation falls from 0.45 in the data by about two-thirds to 0.15 in the counterfactual

Figure 2: Cross-Country Correlations in Data and Counterfactuals



Notes: Each marker indicates the correlation between a country pair's GDP growth (left panel) or inflation (right panel). Horizontal axis indicates correlation in the data; vertical axis indicates correlation in a counterfactual. Blue circles correspond to the counterfactual without trade; red diamonds correspond to the counterfactual without global factors.

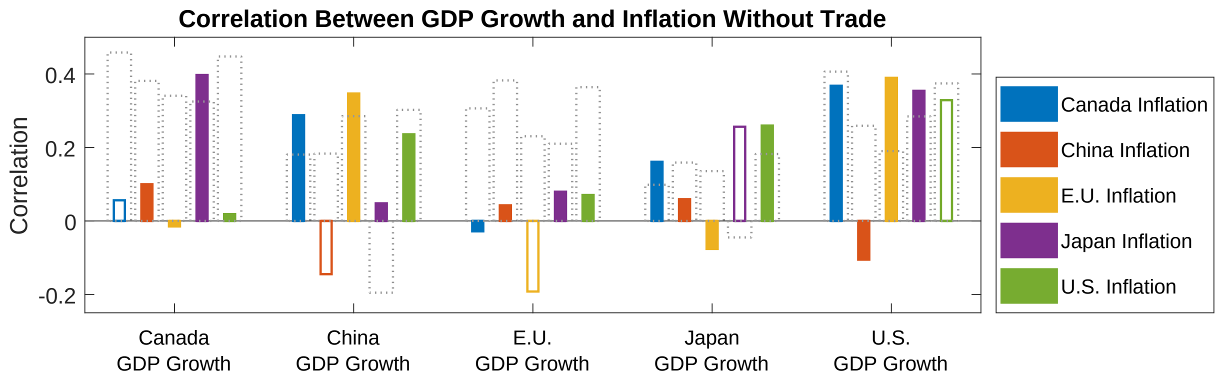
without trade (line 2 of Table 4). However, here global factors play an equally important role, as the mean correlation without global shocks falls to just 0.09 (line 3 of Table 4).

As indicated in the right panel of Figure 2, trade and global factors have varying relative importance depending on the specific country pair. For instance, the correlation in inflation between the U.S. and Canada, two major trading partners, falls from 0.81 in the data all the way down to 0.17 without trade. Absent global factors but with trade, this correlation falls much less to 0.45. In contrast, global factors appear more important in driving the correlation in inflation between the U.S. and Japan, who are not as tightly connected by trade. The correlation observed in the data, 0.33, falls only to 0.27 without trade, but becomes negative at -0.24 without global factors. Intuitively, equation (23) indicates that while inflation inherits the comovement properties of output (i.e., through the global Phillips curve), it is also driven in each country by inflationary supply shocks specific to that country (i.e., the markup shocks).

Finally, the comovement in GDP growth and inflation between countries also falls substantially without trade. This finding is evident in Figure 3, which compares the pairwise correlations between inflation and output across countries in the model without multilateral trade linkages (but with global factors) to the observed correlations reported in Figure 1. These correlations fall by about 40 percent on average, from 0.25 in the data to 0.15 in the counterfactual without trade.

These findings indicate that trade plays a fundamental role in driving international comovement despite countries trading mostly with themselves in Table 1 - a little trade goes a long way. The intuition follows mainly from two features of multilateral trade and nominal rigidities. First, while countries trade mostly with themselves, the multilateral nature of trade, and its implied network of backward and forward international linkages, means that the comovement in output between countries is substantially enhanced by indirect third-country effects reflected in the Leontief in-

Figure 3: Counterfactual Correlation Between GDP Growth and Inflation Without Trade



Notes: Each bar corresponds to correlation between GDP growth in country i with inflation in country j. Filled bars correspond to cross-country correlations (i.e., $i \neq j$) without trade, empty bars correspond to own-country correlations (i.e., $i = j$) without trade, gray dotted bars correspond to correlations in the data. Each group of bars corresponds to a country i's GDP growth and each color corresponds to a country j's inflation.

verse, $(1/a)\sum_1^\infty b^m(\psi^T\varpi)^m$, in equation (24), capturing the different rounds of trade generated by disturbances to the economic environment. Second, the global Phillips curve described by equation (20) makes clear that every country's inflation rate potentially comoves with real activity in every other country in a way that is proportional to their trade shares, reflected here in the elements of ϖ. This relationship in turn implies that the cross-country comovement between output and inflation depends in large part on the comovement in output between countries.

In line 4 of Table 4, we illustrate the quantitative importance of third-country indirect effects by setting the price elasticity of trade, $\varphi = 0.03$, or 0.5 percent of its benchmark value, thereby reducing the indirect effects from trade. Our analytical example in Section 6.2 provides intuition for how decreasing φ diminishes the role of indirect effects. Observe in equation (24) that indirect effects vanish in that example when $b = 0$ or $\varphi = -1$ (in which case $a = 1$). However, the parameterization of the Fréchet distribution imposes that $\varphi > 0$, with the general model becoming non-invertible near $\varphi = 0$. Thus, some indirect effects remain under the parameterization in line 4 of Table 4, yet we already find a notable fall in the comovement in output, the comovement in inflation becoming counterfactually negative, and the cross-country comovement between inflation and output declining close to 1/4 of its counterpart in the data, underscoring the material role of third-country effects.[26]

The Consumption Correlation Puzzle: The last column of line 1 in Table 4 indicates that our model 'overperforms' in resolving the Backus et al. (1992) cross-country consumption correlation puzzle. In a standard IRBC model, Backus et al. (1992) underscore that consumption growth should be more correlated across countries than output growth. However, the opposite is true in the data.

[26]These results suggest that in two-country models, as in Justiniano and Preston (2010), the ability of trade to generate cross-country comovement is a lower bound given the absence of third-country effects.

For our sample of countries, the corresponding mean correlations are 0.39 for output growth and 0.18 for consumption growth. Our model matches observed GDP growth cross-country correlations by construction, and yields an implied average consumption growth cross-country correlation that is even lower than that observed in the data at -0.05.

Proposition 1 provides some intuition for why output growth in our model is substantially more correlated across countries than is consumption growth. In particular, note that consumption is uncorrelated across countries in that example, while output growth in each country comoves positively with every other country. As indicated in equation (17), the reason is that even with an idiosyncratic shock that affects consumption in only one country, trade nevertheless dictates that the composition of production will be affected in the same direction in every country.

International Comovement, Flexible Prices and Financial Autarky: Lastly, we compare our findings to those obtained under a neoclassical benchmark defined by flexible prices and financial autarky. This version of the model is in the vein, for example, of recent work by Huo et al. (2019), who focus on the role of sectoral linkages and multilateral trade in explaining the comovement in real activity across multiple countries. Thus, line 5 of Table 4 sets the Calvo price stickiness parameter, θ, to 1 percent of the posterior mean (for each country) in the estimated model, and increases the financial frictions parameter, τ, to 10^6.

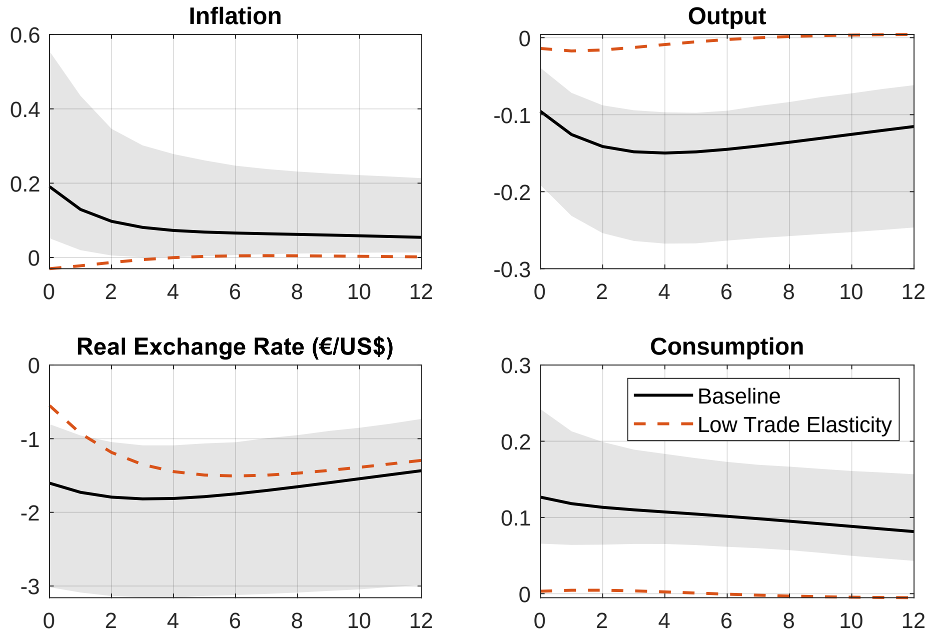
As in Huo et al. (2019), absent global factors, this combination of parameters results in considerably lower comovement in real activity than in the data, with an average cross-country correlation in output growth that is about 1/2 its observed value, 0.18 instead of 0.39. In addition to Huo et al. (2019), however, this benchmark case also results in counterfactually negative cross-country comovement in inflation, and a reduction in the cross-country comovement between output and inflation of more than 70 percent.

In essence, two changes take place in this case. First, as prices become more flexible, demand shocks cannot spread as easily across countries. In that case, demand shocks in individual countries lead to higher inflation and a stronger interest rate response under the maintained monetary policy rules. These higher rates then dampen the effects of the initial shocks. Second, under financial autarky, countries no longer run trade surpluses or deficits in response to shocks. As a result, as countries trade mostly with themselves, trade patterns remain largely unaffected by changes in the terms of trade, which lowers the comovement in output growth across countries. In this case, therefore, both the cross-country inflation correlations, and the cross-country correlations between output and inflation, no longer reflect whatever comovement in output remains from trade.

7.6 The Domestic Effects of Foreign Disturbances

We conclude our set of exercises by exploring two scenarios pertinent to the post-pandemic world. One concerns the effects of inflationary supply shocks in Europe, exacerbated by the war in Ukraine, on U.S. variables. The other considers the effects of a tightening of U.S. monetary policy on

Figure 4: Response of U.S. Variables to an E.U. Markup Shock



Notes: Impulse responses scaled to generate a 1 percent increase in E.U. inflation on impact. Solid black lines correspond to estimated impulse response at posterior mean; gray shaded regions correspond to 68 percent posterior credible intervals; red dashed lines correspond to impulse response at posterior mean with $\varphi = 0.03$. Horizontal axis is in quarters; vertical axis is in percent. Inflation is annualized; real exchange rate normalized so that a decrease corresponds to a depreciation in the U.S. dollar.

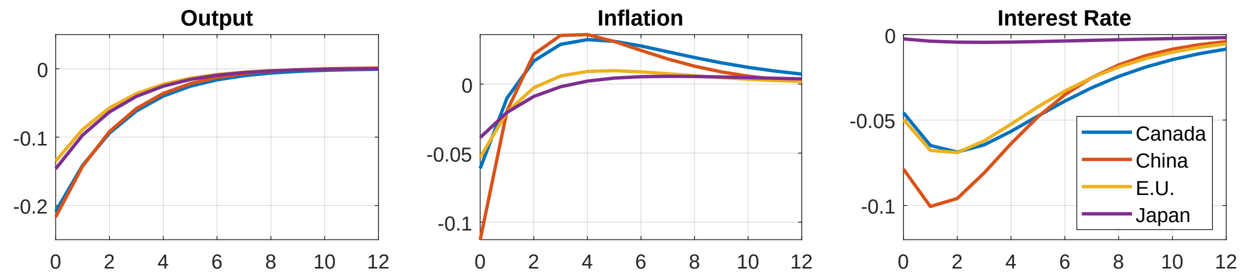
countries abroad.

Implications of European Inflationary Shocks for the U.S.: In the context of New Keynesian models, inflationary shocks driven by supply disturbances, such as shortages stemming from factory lockdowns or disruptions in supply chains, arise through changes in the markup.[27] In the New Keynesian block of equations, (12) through (15), markup shocks directly affect inflation but do not otherwise appear in the other equations. In other words, markup shocks essentially drive an exogenous change in inflation that then pervades to the rest of the economy. Figure 4 illustrates the effect of one such 1 percent increase in European inflation on key U.S. variables.

As supply disturbances increase inflation in the E.U., the European monetary policy rule prescribes raising interest rates, which lowers European consumption (shown in Section H of the Appendix). This fall in consumption lowers the demand for foreign goods, including from the U.S., which induces a decline in U.S. output in Figure 4. However, lower consumption in Europe also means, in equilibrium, that the U.S. dollar depreciates relative to the euro. In addition, the U.S. central bank reacts to the decline in domestic output by inducing lower real rates which, in addition

[27] Ramirez-Cuellar and Espinosa-Torres (2022) indeed document marked increases in firm-level markups in Compustat during and following the COVID-19 crisis.

Figure 5: Response of Foreign Variables to a Tightening of U.S. Monetary Policy



Notes: Impulse responses correspond to a 25 basis point increase in U.S. nominal interest rates on impact. Horizontal axis is in quarters; vertical axis is in percent. Inflation and interest rates are annualized.

to the depreciation of the dollar (making U.S. goods less expensive relative to foreign goods), then raises U.S. consumption. In equation (20), the fall in output lowers inflation while the increase in consumption and the depreciation of the exchange rate stimulate it. The net effect is the U.S. inheriting roughly 20 percent of the initial increase in European inflation.

To place the quantitative effect on U.S. inflation in the context of the recent global inflationary episode, suppose that 3/4 of the 4.4 percent increase in European inflation from 2021Q4 to 2022Q1 stems from the supply shock associated with the Ukraine war and its effect on energy prices (here modeled through an exogenous change in the markup that directly raises European inflation). Our estimates in Figure 4 then suggest that such a shock would account for around 1/2 of the increase in U.S. inflation over the same period.

We highlight two additional features of Figure 4. First, we estimate European markup shocks to be relatively persistent, so that any inflation inherited from these shocks in the U.S. also tends to persist. Second, the dashed red lines in the figure illustrate the response of U.S. variables under attenuated third-country effects as in line 4 of Table 4, i.e., where φ is set to 0.5 percent of its benchmark value. With attenuated third-country effects, the decline in U.S. output stemming from lower demand from abroad is considerably smaller. This smaller decline in output in turn mitigates the need from the U.S. central bank to lower real rates. The net effect is that U.S. inflation hardly responds.

The Effects of a Tightening of U.S. Monetary Policy Abroad: Following the steady rise in post-pandemic U.S. inflation, the Federal Reserve has reacted with a tightening of its monetary policy. In the New Keynesian world with multilateral trade relationships, this tightening of U.S. monetary policy has effects abroad not only because the U.S. bilaterally trades with other countries but also because its trading partners trade among themselves (i.e., through third-country effects). Figure 5 then illustrates the global effects of a 25-basis-point increase in U.S. interest rates.

As the U.S. central bank raises domestic interest rates, U.S. consumption falls, which translates into lower demand for foreign goods. Consequently, all of its trading partners see their output decline in Figure 5, and the percentage decline in foreign output can reach 70 percent of the domestic response (shown in Section H of the Appendix). These declines of between 0.14 to 0.22 percent, in response to a 25-basis-point U.S. interest rate increase, are especially notable given the recent sequence interest rate hikes that have cumulatively totalled 3 percent over the first three quarters of 2022.

In response to the fall in output globally, foreign central banks lower real rates so that, despite inflation initially falling everywhere abroad, after two quarters Canada and China find themselves with both higher inflation and lower output. This exercise, of course, takes as given our estimated monetary policy rules for each country. In practice foreign central banks might choose to deviate from these rules to avoid the outcomes depicted in Figure 5. Nevertheless, the figure establishes that higher inflation and lower output abroad are not only feasible responses to tighter U.S. monetary policy, but also represent the likely outcomes based on the historical behavior of the data. Thus, key questions for future research are: what is the optimal monetary policy response at home to disturbances abroad in a New Keynesian world featuring multiple large countries? How do central banks of countries linked by multilateral trade internalize other banks' actions in choosing their monetary policy?

8 Concluding Remarks

This paper addresses the comovement between countries in output growth, in inflation, and between output growth and inflation. The model we develop brings nominal rigidities in the tradition of New Keynesian models together with multilateral trade. The result is an environment where variables in different countries reflect the effects of foreign disturbances not only directly, through pairwise trade between countries, but also indirectly through third-country effects arising from the network structure of trade. The combined propagation mechanisms mean that despite relatively low observed trade shares across countries, country-specific shocks alone generate sizeable comovement between countries.

International linkages in a world with nominal rigidities are associated with important changes to the standard closed-economy New Keynesian Phillips curve. These changes imply, outside autarky, a global system of Phillips curves explicitly connected by multilateral trade relationships. In this system, inflation in any one country is related to variations in output growth and exchange rates in all of its trading partners. Moreover, in equilibrium, variations in output growth and exchange rates in turn reflect the effects of foreign disturbances operating through adjustments in countries' trade balances and net asset positions along with global market clearing. Using Bayesian methods to quantify the role of global factors and international linkages, we find that country-specific shocks alone explain almost 90 percent of the observed average pairwise cross-country correlation in output growth. These idiosyncratic shocks also explain more than 1/2 the cross-country comovement in inflation, as well as between output and inflation. In contrast, as nominal rigidities vanish, multilateral trade generates only limited comovement in output between countries, counterfactually negative comovement in inflation, and almost no cross-country comovement between output and inflation. Finally, we estimate that a European inflationary shock results in significant U.S. inflation accompanied by lower output, and that these responses transpire almost entirely from the network effects of trade. We also find that a tightening of U.S. monetary policy generates a percentage decline in output globally that is comparable to 1/2 the domestic response.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007). Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics 72*(2), 481–511.

Aggarwal, R., A. Auclert, M. Rognlie, and L. Straub (2022). Excess savings and twin deficits: The transmission of fiscal stimulus in open economies. Working paper.

An, S. and F. Schorfheide (2007). Bayesian analysis of DSGE models. *Econometric Reviews 26*(2-4), 113–172.

Andrés, F., S. Schmitt-Grohé, and M. Uribe (2017). World shocks, world prices, and business cycles: An empirical investigation. *Journal of International Economics 108*, 2–14.

Aruoba, B. S., P. Cuba-Borda, and F. Schorfheide (2018). Macroeconomic dynamics near the ZLB: A tale of two countries. *Review of Economic Studies 85*(1), 87–118.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International real business cycles. *Journal of Political Economy 100*(4), 745–775.

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*, 297–316.

Baumeister, C. and J. D. Hamilton (2015). Sign restrictions, structural vector autoregressions, and useful prior information. *Econometrica 83*(5), 1963–1999.

Benigno, G. and C. Thoenissen (2008). Consumption and real exchange rates with incomplete markets and non-traded goods. *Journal of International Money and Finance 27*(6), 926–948.

Bianchi, F. and A. Civelli (2015). Globalization and inflation: Evidence from a time-varying VAR. *Review of Economic Dynamics 18*(2), 406–433.

Bianchi, F. and L. Melosi (2017). Escaping the Great Recession. *American Economic Review 107*(4), 1030–58.

Boeckelmann, L., J. Imbs, and L. Pauwels (2022). Shock propagation within and between countries. *Working Paper, Paris School of Economics.*

Borio, C. and A. Filardo (2007). Globalisation and inflation: New cross-country evidence on the global determinants of domestic inflation. Working paper 227, Bank for International Settlements.

Cai, M., M. Del Negro, E. Herbst, E. Matlin, R. Sarfati, and F. Schorfheide (2021). Online estimation of DSGE models. *Econometrics Journal 24*(1), C33–C58.

Caliendo, L. and F. Parro (2015). Estimates of the trade and welfare effects of NAFTA. *The Review of Economic Studies 82*(1), 1–44.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*, 383–398.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Ciccarelli, M. and B. Mojon (2010). Global inflation. *Review of Economics and Statistics 92*(3), 524–535.

Clarida, R., J. Gali, and M. Gertler (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics 49*(5), 879–904.

Corsetti, G., L. Dedola, and S. Leduc (2008). International risk sharing and the transmission of productivity shocks. *Review of Economic Studies 75*(2), 443–473.

Creal, D. (2007). Sequential Monte Carlo samplers for Bayesian DSGE models. Working paper, University of Chicago Booth School of Business.

Cúrdia, V., M. Del Negro, and D. L. Greenwald (2014). Rare shocks, great recessions. *Journal of Applied Econometrics 29*(7), 1031–1052.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced trade. *American Economic Review 97*, 351–355.

Del Negro, M., M. P. Giannoni, and F. Schorfheide (2015). Inflation in the Great Recession and New Keynesian models. *American Economic Journal: Macroeconomics 7*(1), 168–96.

Del Negro, M. and C. Otrok (2008). Dynamic factor models with time-varying parameters: measuring changes in international business cycles. *FRB of New York Staff Report* (326).

Eaton, J. and S. Kortum (2002). Technology, geography, and trade. *Econometrica 70*, 1741–1779.

Ferroni, F. and B. Klaus (2015). Euro area business cycles in turbulent times: convergence or decoupling? *Applied Economics 47*(34-35), 3791–3815.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework.*

Galí, J. and M. Gertler (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics 44*(2), 195–222.

Galí, J., M. Gertler, and J. D. Lopez-Salido (2001). European inflation dynamics. *European Economic Review 45*(7), 1237–1270.

Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies 72*, 707–734.

Guerron-Quintana, P. A. (2013). Common and idiosyncratic disturbances in developed small open economies. *Journal of International Economics 90*(1), 33–49.

Head, K. and T. Mayer (2014). Gravity equations: Workhorse, toolkit, and cookbook. In *Handbook of International Economics*, Volume 4, pp. 131–195. Elsevier.

Herbst, E. and F. Schorfheide (2014). Sequential Monte Carlo sampling for DSGE models. *Journal of Applied Econometrics 29*(7), 1073–1098.

Herbst, E. P. and F. Schorfheide (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press.

Huo, Z., A. A. Levchenko, and N. Pandalai-Nayar (2019). International comovement in the global production network. Working paper 25978, National Bureau of Economic Research.

Iiboshi, H., M. Shintani, and K. Ueda (2018). Estimating a nonlinear new Keynesian model with the zero lower bound for Japan. *Journal of Money, Credit and Banking*.

Imbs, J. (2006). The real effects of financial integration. *Journal of International Economics 68*(2), 296–324.

Imbs, J., H. Mumtaz, M. Ravn, and H. Rey (2005). PPP strikes back: Aggregation and the real exchange rate. *Quarterly Journal of Economics 120*(1), 1–43.

Itskhoki, O. and D. Mukhin (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy 129*.

Justiniano, A. and B. Preston (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics 81*(1), 61–74.

Justiniano, A. and G. E. Primiceri (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review 98*(3), 604–41.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Kehoe, T. J. and K. J. Ruhl (2008). Are shocks to the terms of trade shocks to productivity? *Review of Economic Dynamics 11*, 804–819.

Kollmann, R. (1995). Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance 14*(2), 191–211.

Kose, M. A., C. Otrok, and C. H. Whiteman (2003). International business cycles: World, region, and country-specific factors. *American Economic Review 93*(4), 1216–1239.

Leeper, E. M., N. Traum, and T. B. Walker (2017). Clearing up the fiscal multiplier morass. *American Economic Review 107*(8), 2409–54.

Lubik, T. and F. Schorfheide (2005). A Bayesian look at new open economy macroeconomics. *NBER Macroeconomics Annual 20*, 313–366.

Matthes, C. and F. Schwartzman (2021). The consumption origins of business cycles: Lessons from sectoral dynamics. Working paper.

Mavroeidis, S., M. Plagborg-Møller, and J. H. Stock (2014). Empirical evidence on inflation expectations in the New Keynesian Phillips curve. *Journal of Economic Literature 52*(1), 124–88.

Müller, U., J. Stock, and M. Watson (2020). An econometric model of international long-run growth dynamics for long-horizon forecasting. *Review of Economics and Statistics, forthcoming.*

Mumtaz, H., S. Simonelli, and P. Surico (2011). International comovements, business cycle and inflation: A historical perspective. *Review of Economic Dynamics 14*(1), 176–198.

Mumtaz, H. and P. Surico (2012). Evolving international inflation dynamics: World and country-specific factors. *Journal of the European Economic Association 10*, 716–734.

Neiss, K. S. and E. Nelson (2005). Inflation dynamics, marginal cost, and the output gap: Evidence from three countries. *Journal of Money, Credit and Banking*, 1019–1045.

Ramirez-Cuellar, J. and J. A. Espinosa-Torres (2022). The pandemic effects on market power and profitability. *Manuscript, University of Southern California.*

Roberts, J. M. (1995). New Keynesian economics and the Phillips curve. *Journal of Money, Credit and Banking 27*(4), 975–984.

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of Economic Literature 34*, 647–668.

Sbordone, A. M. (2005). Do expected future marginal costs drive inflation dynamics? *Journal of Monetary Economics 52*(6), 1183–1197.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*, 163–185.

Schmitt-Grohé, S. and M. Uribe (2012). What's news in business cycles. *Econometrica 80*(6), 2733–2764.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review 97*(3), 586–606.

Stock, J. H. and M. W. Watson (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association 3*(5), 968–1006.

Sugo, T., K. Ueda, et al. (2007). Estimating a DSGE model for Japan: Evaluating and modifying a CEE/SW/LOWW model. Bank of Japan Working Paper 07-E-2, Bank of Japan.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 39, pp. 195–214. Elsevier.

Taylor, J. B. (1999). A historical analysis of monetary policy rules. In *Monetary Policy Rules*, pp. 319–348. University of Chicago Press.