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Sovereign Debt and Credit Default Swaps

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Abstract

How do credit default swaps (CDS) affect sovereign debt markets? We analyze how liquidity, exposure to default risk, and regulation affect the answer to this question using a sovereign debt model where investors trade bonds and CDS over the counter via directed search. Restricting portfolios can improve bond prices and bond-market activity, but the net effect depends on relative frictions in bond and CDS markets, the exposure of investors, and how the sovereign responds to the policy. Our novel identification strategy exploits confidential microdata to quantify trading frictions and the exposure distribution. The calibrated model generates realistic CDS-bond basis deviations, bid-ask spreads, and CDS volumes and positions. Our baseline specification predicts trading frictions and an inability to short sell bonds significantly improves sovereign debt prices, but policies that restrict CDS trading have small effects.

Keywords: sovereign debt, CDS, directed search, over-the-counter

JEL codes: F34, G12

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1 Introduction

Credit default swaps (CDS) are financial derivatives created in the 1990s that provide insurance against default risk. The volume of transactions on CDS linked to sovereign government bonds, which we call sovereign CDS, has steadily increased since their inception.[1] However, we know little about the interactions between sovereign bond and sovereign CDS markets, and how quantitatively relevant these interactions are. This lack of knowledge is likely due to the fact that both of these tightly related assets are traded in opaque over-the-counter (OTC) markets, where transactions occur bilaterally between market participants.

We propose a new model of sovereign default in which bonds and CDS trade over the counter, and we use regulatory data to quantify the impact of the CDS market on economic outcomes in Argentina. The model uses directed search to capture key liquidity properties of the data, such as bid-ask spreads, dealer CDS positions and volume, and CDS-bond basis deviations (a measure of potential arbitrage opportunities between CDS and bonds). We obtain CDS positions for sovereign debt from regulatory data provided by the Depository Trust and Clearing Corporation (DTCC). We also obtain data on large dealers' exposure to sovereign default (CDS and bond holdings) from the FR Y-14Q regulatory filings, which are part of the Federal Reserve's Capital Assessments and Stress Testing information collection. We expand this data with bid and ask quotes from Bloomberg and CDS-bond basis deviation data from Gilchrist et al. (2022). These data allow us to determine how frictional bond and CDS markets are, how risk is shared among dealers and investors, and ultimately how CDS affect the sovereign debt market.

In our model, market participants are divided into dealers, investors, and a sovereign government. Dealers and investors have the same preferences, but investors differ in their ex-ante exposure to default risk, modeled as an exogeneous and normally distributed exposure to default risk. There are three assets: a risk-free asset with perfectly elastic supply, a sovereign bond with endogenously determined supply, and CDS on the sovereign bond in zero net supply. Dealers have access to competitive inter-dealer markets for bonds and CDS. Investors first choose whether to trade bonds or CDS, and then search for dealers to trade with. Search is directed, and investors choose one of a continuum of submarkets where dealers charge different intermediation fees. There is free entry of dealers into each submarket, and in equilibrium, investors can increase their matching probability by paying higher fees to dealers. If matched, investors purchase as many bonds (or CDS) as they want at inter-dealer prices. Dealers and investors can freely access the risk-free asset market which, without loss of generality, opens after OTC trading.

[1] According to the BIS Quarterly Review of June, 2018, the volume traded of sovereign CDS by 2007 was around \$1.6 trillion and representing 3.4% of the total CDS market and had more than duplicated its size to \$3.3 trillion by mid-2013, accounting for 13.3% of the CDS market.

We embed this OTC market structure into a standard sovereign default model (Arellano, 2008). The state at the beginning of the period is a country's endowment and stock of debt. The government chooses to repay or default strategically, comparing the two alternatives. The value of repaying depends on the market value of bonds, which is determined by the OTC structure of the model. If the government repays its debt, it chooses an amount of new bonds to issue to maximize the lifetime utility of the representative consumer in its country.

We start our analysis with a theoretical exploration into the impact of trading frictions, the distribution of exogenous exposure, and portfolio restrictions on sovereign debt markets. We demonstrate that in an extreme case where entry costs diminish to zero, prices tend towards risk-neutrality if CDS are freely tradable. However, prices could persist above risk-neutral levels if CDS are absent or naked CDS—i.e., owning CDS protection without also owning the underlying bond—have been banned. Such policies subtly enhance the sovereign's welfare in situations characterized by minimal trading frictions. These findings align with the policies advocated in articles like Portes (2012) and Murdock (2012). It is crucial to underscore, however, that while in our model these conclusions are always true with minimal trading frictions, they are ambiguous otherwise. When trading frictions are significant, banning CDS or naked CDS can increase or decrease sovereign debt prices and welfare, depending on the model parameters. This is consistent with arguments made against naked-CDS bans, such as those in Duffie (2010) or made by the Internation Monetary Fund (2013), highlighting the importance of the quantitative analysis we present in this paper.

After exploring the limit case when entry costs are close to zero, we focus on the impact of portfolio restrictions and search frictions on market activity. In particular, we demonstrate that portfolio restrictions and search frictions interact in ways that can completely shut down investor activity in bond or CDS markets. Intuitively, investors tend to trade in the market with lower search frictions (i.e., the more liquid market). However, if portfolio restrictions, such as limits on bond shorting or bans on naked CDS, prevent investors from obtaining the desired exposure to sovereign credit risk with one asset, they may opt to trade in the more frictional market.

Next, we examine the primary metric of arbitrage breakdown between bond and CDS markets: CDS-bond basis deviations. While the basis for Argentina varies from close to zero to significantly positive in periods of credit crisis, other countries, such as some European countries during the debt crisis, experienced negative basis deviations. We show that the model can generate both positive and negative deviations, and provide sufficient conditions in terms of the policies in place and trading frictions for the basis to be strictly positive or strictly negative.

Finally, we derive two closed-form solutions in a special case with investor homogeneity. These solutions highlight how trading frictions, investor exposure, default risk, and debt issuance affect sovereign debt prices, bid-ask spreads, and the CDS-bond basis. Increases in default risk generate additional gains from trade, driving up bid-ask spreads and amplify existing risk premia and CDS-bond basis deviations. More trading frictions increase bid-ask spreads, risk-premia, and tend to increase the CDS-bond basis in absolute value.

Trading frictions and portfolio restrictions determine equilibrium bond prices through several mechanisms. Three of these are *direct effects*. First is a classical Walrasian *demand effect*. For example, eliminating CDS induces substitution from CDS to bonds, changing aggregate bond demand. This force would be present absent search frictions. Second is an *intermediation effect*. Portfolio restrictions change dealer profits and, through free entry, trading fees. Different fees change the probability of matching and consequently aggregate demand. Third is an *entry effect*. As investors' incentives to trade change, so do the fees they are willing to pay. This induces more or less entry by dealers, and this measure matters when dealers share risk. These three direct effects are complemented by an *indirect effect*—the *default risk effect*. When one of the direct effects changes bond prices, the sovereign's borrowing and default decisions change as well, resulting in a general equilibrium effect on bond prices.

How much these effects matter for the impact of any policy change depends crucially on trading frictions. Our first quantitative contribution is in identifying these frictions using data and showing the model delivers key empirical patterns. In the data, as in the model, we divide market participants into three categories: sovereign governments; large banks active in CDS markets (dealers); and other market participants (investors). Trading volumes in the CDS market and dealer bond and CDS holdings identify exposure heterogeneity, both between dealers and investors and among investors. The average bid-ask spreads of bonds and CDS identify trading fees. The elasticities of bid-ask spreads to changes in default risk identify the elasticities of the matching technology in OTC markets. The CDS-bond basis deviation identifies the relative efficiency of the matching technologies.

Identified in this way, the model replicates three stylized facts we document. First, bid-ask spreads increase in default risk, both for bonds and CDS. This indicates a common measure of liquidity breaks down in times of stress. Second, CDS-bond basis deviations, while normally close to zero, sharply increase in crises. Hence, arbitrage opportunties seem to become more prevalent in periods of elevated risk. Last, CDS volume increases in default risk, but dealer CDS positions are anchored close to zero. Thus risk-shifting occurs, but dealers do not take on substantial amounts of risk themselves. The model also reproduces a number of untargeted patterns in OTC markets and the standard sovereign debt statistics.

Our second quantitative contribution is to assess the impact of three key counterfactuals on the sovereign bond market: (1) perfectly liquid bond and CDS markets, (2) allowing bond shorting, and (3) banning naked CDS positions.

We first assess the quantitative impact of frictional markets relative to perfectly liquid markets. Let us consider the direct and indirect effect discussed above. The direct effect is positive, meaning that frictional markets improve bond prices. The direct effect of frictional markets is positive because they change who the marginal investor is. Our benchmark, frictional calibration finds that the CDS market is much more frictional than the bond market, leading to a much lower probability of trading CDS. This produces a crucial sorting pattern: moderately- or positively-exposed investors are likely to purchase CDS because bond shorting is not allowed, while negatively-exposed investors are likely to purchase bonds. This generates bond prices in the benchmark that are *better* than risk neutral, the friction-less price. When the direct effect is positive, defaulting becomes less attractive for a given level of debt, further improving bond prices. This makes the indirect effect coming from default risk also positive. We also show that, while bond prices improve for a given amount of debt, the sovereign tends to issue more debt when facing better prices, leading to higher leverage and bond spreads on average. We find that the direct effect accounts for a decrease in bond spreads of about 4%, while the indirect effect combined with higher debt issuance increases bond spreads by 3.1%.

We then analyze the impact of allowing short positions in sovereign bonds. We find that allowing short positions has a large adverse impact on bond prices. To better understand this result, we decompose the impact of allowing for bond shorting into the four effects we highlighted before: a *demand effect*, an *entry effect*, an *intermediation effect*, and a *default risk effect*.

The decomposition reveals a large negative demand effect, consistent with the Walrasian predictions. The magnitude of the response, however, lies in the large disparity between the matching efficiency of the bond and CDS markets. The data suggest that the bond market is orders of magnitude more efficient. However, for investors with high exogenous exposure to default, bonds are not a great substitute for CDS. CDS can be used to reduce total exposure, which is impossible in the bond market since shorting is not allowed. Once bond-shorting is allowed, investors who wanted to reduce exogenous exposure, but find it difficult to do in the CDS market due to trading frictions, now can do it in the less frictional bond market. This causes a shift of high-exposure investors from the CDS market to the bond market, increasing the bond risk premium and reducing the bond price.

Finally, we turn our attention to a ban on naked CDS, which is motivated by the regulatory change implemented in the European Union in 2011-2012. The benchmark calibration of the model predicts that a naked CDS ban will have almost no impact on bond prices.

To understand why the total effect is small for the benchmark calibration, consider first the direct effects of the policy: the demand effect, the entry effect, and the intermediation effect. The demand effect is positive, because a naked CDS ban reduces the ability of highly-exposed agents to bid up the price of CDS protection (and thereby bidding down the price of the bond). But note that this mechanism operates by preventing investors with high exogeneous exposure from buying CDS, and in our model investors already have a hard time doing that because the CDS market is frictional. So the demand effect is small. The entry and intermediation effects are almost zero for the following reason. The CDS-bond basis deviation is positive, indicating that bonds are expensive relative to synthetic bonds. As a result, dealers prefer to sell CDS protection to very exposed investors rather than buy bonds to obtain exposure. Because dealers' bond positions are zero, their entry and exit decisions have no effect on bond demand. A naked CDS ban would prevent dealers from selling CDS protection, which could potentially reduce their profits and, as a result, intermediation. However, the CDS market is very frictional, so the amount of CDS that dealers sell is small and does not noticeably impact intermediation. Finally, the indirect or default risk effect, caused by the change in the sovereign's default policy in response to bond pricing, moves in the same direction of the demand effect. Since the demand effect is small, the default risk effect is also small.

Relation to the Literature. Our paper contributes to two strands of the literature. First, it contributes to the sovereign default literature that followed the seminal work of Eaton and Gersovitz (1981) and the quantitative literature arising after the influential work of Arellano (2008) and Aguiar and Gopinath (2007). Our contribution is to bring into consideration how the CDS market affects sovereign bond markets and consequently a sovereign's ability to issue debt. The closest paper to ours is Salomao (2017)—to our knowledge the first paper in this literature incorporating CDS. Her work investigates how CDS affect debt restructuring outcomes and the corresponding implications for government decisions. She finds that CDS can generate uncertainty on the recovery value of defaulted bonds and such uncertainty make investors be more aggressive in debt restructuring negotiations. Our work is complementary, highlighting the risk-sharing role of CDS while taking into account the frictional nature of bond and CDS markets. In this respect, our paper is also closely related to the emergent literature on illiquidity in sovereign debt markets with random search (Passadore and Xu, 2022) and directed search (Chaumont, 2022). We extend those analyses by incorporating CDS, identifying trading frictions in a novel way, and assessing the impact of the naked CDS ban implemented during the European sovereign debt crisis.

Second, our paper contributes to the large finance literature that followed Duffie et al. (2005) and studies search frictions in OTC markets. Our model is closer to those in Lagos and Rocheteau (2009) and Lester, Rocheteau, and Weill (2015) since we allow for assets holdings to be traded in perfectly divisible quantities using directed search. Most of this literature focuses on outcomes of a single asset market, but some recent work investigates interactions between multiple assets. This is the case for recent work by Oehmke and Zawadowski (2015) and even more closely related work by Sambalaibat (2022).[2] Oehmke and Zawadowski (2015) characterize the interactions between

[2]Additionally, Sambalaibat (2018) examines empirically the effects of the European naked CDS ban finding the permanent ban decreased bond market liquidity.

corporate bonds and CDS and propose a theory where corporate CDS are not redundant because they are cheaper to trade. The assumption that CDS are less costly to acquire than bonds does not necessarily carry over to the sovereign CDS market, and work by Calice et al. (2013) suggests it does not.[3]

Sambalaibat (2022) uses a theory to study the interactions between bonds and CDS in OTC markets. In her model, heterogeneous investors, some of whom would like to take short positions in bonds, trade through random search. The main finding is that allowing for CDS trade improves bond prices by reducing the bond illiquidity discount. There are two main forces at play. First the participation of naked CDS buyers reduces the price of the bonds because bond buyers can instead sell CDS to get exposed to default risk, which reduces the demand for bonds. The second and dominant force is that naked CDS buyers attract more investors who want to take exposure to default risk into the market. This is because those investors can meet a CDS buyer if they cannot find a match with a bond seller. Therefore, more bond buyers enter the market, which increases the likelihood that bond sellers find a match, reduces the illiquidity discount and increases the price of the bonds.

While our work focuses on similar topics to Sambalaibat, our model differs along four key dimensions. First and foremost, our model matches the data, which is something Sambalaibat does not look at. Second, search in her model is random while ours is directed. This means matching rates in her model are limited by market participation of other agents, so investors can be trapped in a severely inefficient allocation. In our model, investors have a mechanism to escape these inefficient allocations by paying larger fees. Third, Sambalaibat only allows agents to trade one indivisible unit of an asset, while we allow agents to trade divisible quantities, allowing an intensive margin. This generates demand effects that are absent from her model. Finally, we allow the government to respond to the OTC market by changing its default decision and adjusting bond issuance. This is captured by our default risk effect, which we find is quantitatively important, but this effect is totally absent in Sambalaibat as default rates and issuance are fixed in her model. Overall, her model assumes a number of frictions and artificial restrictions that our model relaxes.

The paper is organized as follows. We present our model in Section 2. Section 3 establishes theoretical properties of the model. Section 4 describes measurement, stylized facts, and our identification strategy. Section 5 analyzes the benchmark results. Section 6 studies the effects of different regulations on bond prices and welfare. Section 7 concludes.

[3]Table 6 in Calice et al. (2013) shows the bid-ask spreads for CDS are larger those for the underlying sovereign bonds for Austria, Belgium, France, Greece, Ireland, Italy, Netherlands, Portugal and Spain for sovereign bonds/CDS with 5 and 10 years maturity between 2001-2010.

2 The model

Our model is comprised of an OTC block and a sovereign block. The OTC block takes the issuance of bonds and default probability as given, and determines the market clearing bond and CDS prices. The sovereign block takes the price schedule for the bond as a function of the bond issuance, and determines the bond issuance and default probability.

2.1 The OTC block

We begin with the OTC block since it is the novel component of our sovereign debt model.

2.1.1 Agents, preferences, and endowments

At any given moment, there are two types of agents in action: a finite measure I of investors and an infinite measure of dealers. To ensure that prior histories do not affect current outcomes, we assume investors and dealers permanently disappear from the market after closing their trades and consuming (and are replaced with fresh ones).

Investors and dealers have quasi-linear utility functions that value consumption g this period and g' next period as $g + \beta \mathbb{E}_\delta u(g')$, where $\beta \in (0,1)$ and $u(\cdot)$ satisfies the usual regularity conditions. These preferences enable portfolio decisions to be swayed by risk while simultaneously keeping endowments from influencing anything other than the risk-free asset, which is not relevant for our purposes. Thus, we normalize dealer and investor endowments to zero in the current period. In the next period, the endowment is potentially correlated with default, paying out ω in repayment and 0 in default. Hence, investors have *exogenous exposure* to default in the amount ω.[4] Exogenous exposure ω is distributed across investors according to a normal with mean μ_ω and variance σ_ω^2. Investors will also have *endogenous exposure* equal to their bonds position less any CDS protection.

Heterogeneity in ω serves two purposes. First is that previous financial investments, from which our model abstracts, have ongoing implications. Second is that investors are heterogeneous in default-cost incidence. E.g., investors with local-currency debt and USD-denominated assets may gain from a default if it produces rampant inflation. Or, investors may be hedged across multiple countries, reducing idiosyncratic risk.

2.1.2 Financial markets and technology

There are three assets: a risk-free asset with a perfectly elastic supply, a sovereign bond b in fixed supply $B' > 0$, and CDS contracts c in zero net supply. To simplify the language, we refer to the

[4] This formulation of the exogenous exposure is equivalent, due to quasi-linearity, to one with payments 0 in no-default states and $-\omega$ in default states.

sovereign bond as just the bond when not ambiguous. One unit of the risk-free asset a pays one unit of consumption next period and costs $q_f = \beta$. The supply of the risk-free asset is perfectly elastic. That is, investors and dealers can buy or sell any quantity of the risk-free asset at the price q_f.

One unit of the sovereign bond (CDS) pays one unit of consumption next period in states where the sovereign repays (defaults) and zero in states where it defaults (repays).[5] As there will be essentially only two states of uncertainty for investors and dealers, the bond is like an Arrow security paying out in repayment and the CDS an Arrow security paying out in default. We use $\delta = 1$ to denote the state of the world in which the bond defaults, and $\delta = 0$ to denote the state of the world in which it does not default. We denote the default probability $\bar{\delta}$. We allow for a technological constraint on shorting bonds in the form of $b \geq \underline{b}$, where $\underline{b} \leq 0$ and $\underline{b} = -\infty$ means there is no constraint. Similarly, we allow for a constraint on *endogenous exposure* $x \equiv b - c \geq \underline{x}(\omega)$ where $\underline{x} = -\infty$ means there is no constraint. We implement a naked CDS ban as $\underline{x}(\omega) = -\theta_\omega \max\{0, \omega\}$, which for $\omega \leq 0$ (un- or negatively-exposed) agents must have $x \geq 0$ while $\omega \geq 0$ (positively-exposed) agents are allowed to protect themselves using $x < 0$ up to a fraction θ_ω. preventing an agent from benefiting financially from default. Agents may take any short or long position in CDS as long as it satisfies the previous constraints on endogenous exposure (i.e., c is chosen from $\mathbb{R}$).

For investors to trade bonds or CDS, they must match with dealers in frictional markets. Specifically, if they wish to purchase bonds in the amount b, they choose a submarket characterized by dealer fee $f_b \in \mathbb{R}^+$. If matched, they pay an inter-dealer unit price q for a total of qb plus the fee f_b. In choosing f_b, they take as given the market tightness $\theta_b(f_b)$ in that submarket, which is the measure d of dealers active in submarket f_b relative to the measure of investors n active in that submarket. The constant returns-to-scale matching technology $M_b(n, d)$ determines investors' matching probability $\alpha_b(\theta_b(f_b))) \equiv M_b(1, \theta_b(f_b)) = M_b(n, d)/n$.[6] Likewise, to purchase or sell CDS, c, they must pay f_c plus the inter-dealer cost pc if they match, which occurs with probability $\alpha_c(\theta_c(f_c))$.

Active dealers trade bonds or CDS with investors. To do so, they must pay an entry cost to enter the respective market. Active dealers in the bond market have to pay an entry cost $\gamma_b > 0$, while active dealers in the CDS market have to pay an entry cost $\gamma_c > 0$. After entering *either* market, they can purchase any desired amount of bonds b and CDS c at inter-dealer prices in a frictionless inter-dealer market. An active dealer in the bond market chooses a submarket f_b to visit, matching at rate $\rho_b(\theta_b(f_b)) \equiv M_b(1/\theta_b(f_b), 1) = M_b(n, d)/d$.[7] Similarly, an active dealer in

[5] We abstract from counterparty risk. In the data, CDS contracts pay an exogenous coupon when there is no default. The CDS contract here is identical to a CDS contract with a coupon κ paired with a short-position in the risk-free asset equal to κ. Since the risk-free asset is liquid, setting the coupon to zero is a normalization.

[6] We assume $M_b(0, \cdot) = M_b(\cdot, 0) = 0$.

[7] The trading rate ρ may be greater than unity, which can be interpreted as dealers executing more than one bilateral

the CDS market chooses a submarket f_c and matches at rate $\rho_c(\theta_c(f_c))$. A useful property of the matching technology is that $\alpha_i(\theta) = \rho_i(\theta)\theta$ for $i = b, c$. The quantitative model will employ the parameterization

$$M_i(n, d) = \bar{\alpha}_i n \frac{n^{-\xi_i}}{n^{-\xi_i} + d^{-\xi_i}} \implies \alpha_i(\theta) = \frac{\bar{\alpha}_i}{1 + \theta^{-\xi_i}}, \tag{1}$$

for $\xi_i \in (0, 1), \bar{\alpha}_i \in (0, 1]$and $i \in \{b, c\}$. The elasticity of α_i is controlled by ξ_i, and $\bar{\alpha}_i$ controls matching efficiency.

2.1.3 Timing

There are two sequential sub-periods, s_1 and s_2. In s_1, dealers decide to become active in either bond or CDS markets. Investors decide whether to enter the bond or CDS market, how many bonds/CDS to purchase and the submarket they wish to enter.[8] Bond and CDS market matching realizations occur at the end of s_1. In s_2, investors choose their risk-free bond position. Dealers choose their bonds, CDS, and risk-free positions, and settle all their outstanding obligations (delivering b or c to investors as promised) simultaneously. At the beginning of the next period, the default shock is realized, payments are settled, and consumption occurs.[9]

2.1.4 The dealer's problem

For use in characterizing both the dealer and investor problems, consider the value of choosing a risk-free position conditional on already having a bond position b, CDS position c, and exogenous exposure ω (though for dealers, ω will be zero):

$$L(b, c, \omega) \equiv \max_a -q_f a + \beta \mathbb{E}_\delta \left[u(a + (1 - \delta)(b + \omega) + \delta c)\right]. \tag{2}$$

Quasi-linear utility eliminates wealth effects, leaving only the overall exposure $b+\omega-c$ as relevant in choosing a and giving $L(b, c, \omega) = q_f c + L(b + \omega - c, 0, 0)$.[10] So we can define

$$X(x) \equiv L(x, 0, 0) = \max_a -q_f a + \beta \mathbb{E}_\delta[u(a + x(1 - \delta))], \tag{3}$$

transaction in a given period. The more transactions dealers make (in expectation) in a given submarket, the higher expected profits are. Thus, more dealers enter such submarkets to satisfy the zero expected profit condition that follows from free entry of dealers into each submarket. In submarkets where transaction fees are relatively low, the zero profit condition may imply that each dealer entering the submarket makes more than one transaction, in expectation, in order to cover the fixed entry cost.

[8] If they do not wish to trade, they can choose a zero-fee submarket where trade will occur with zero probability.

[9] In an early version of the paper, the bond market would open before the CDS market, which is potentially important in driving the results. We now have both markets opening simultaneously but find similar quantitative results.

[10] The proof is

$$\begin{aligned} L(b, c, \omega) &= \max_a -q_f a + \beta \mathbb{E}_\delta[u(a + c + (b + \omega - c)(1 - \delta))] \\ &= \max_{\tilde{a}} -q_f(\tilde{a} - c) + \beta \mathbb{E}_\delta[u(\tilde{a} + (b + \omega - c)(1 - \delta))] \\ &= q_f c + L(b + \omega - c, 0, 0). \end{aligned}$$

which implies that

$$L(b,c,\omega) = q_f c + X(b+\omega-c). \tag{4}$$

In the quantitative work, we assume flow-utility is $u(c) = c^{1-\sigma}/(1-\sigma)$ and is the same across dealers, investors, and the sovereign.

For an active dealer, the demand for risk-free asset, sovereign bonds and CDS is independent of which markets she intermediates. This is because dealers have access to a perfectly competitive inter-dealer market. We also know that a dealer payoff from trading is the same as an investor with $\omega = 0$. So the gain from becoming an active dealer's trading portfolio is

$$\pi = \max_{\substack{b\geq \underline{b} \\ c\leq b-\underline{x}(0)}} \underbrace{-qb + (q_f - p)c + X(b-c)}_{\text{Value of acquiring } (b,c)} - \underbrace{X(0)}_{\substack{\text{Value of} \\ \text{no trade}}}. \tag{5}$$

where we used above equation (4). The first order conditions are

$$q \geq X'(b-c), \text{ with equality if } b > \underline{b}, \text{ and} \tag{6}$$

$$q_f - p \geq X'(b-c), \text{ with equality if } b-c > \underline{x}(0). \tag{7}$$

Equations (6) and (7) have two interesting implications. First, the problem only has a solution if $q \geq q_f - p$. This says the bond price must exceed the price of a synthetic bond (one unit of the risk-free less one unit of CDS, which has the same yield structure as one unit of the bond). When $q < q_f - p$, there is an arbitrage opportunity that generates infinite profits, precluding a solution of the dealer problem. A dealer can buy an additional unit of bonds and one additional unit of CDS for a total cost of $q + p$. This operation generates a risk-free asset, which the dealer can then short sell for the price q_f. Since $q_f > q + p$, this operation increases the dealer's profit by $q_f - q - p > 0$ without violating any constraints, since b is increased while keeping the total exposure $b - c$ constant. Second, when $q = q_f - p$, a solution exists, but the demand for bonds and CDS is not determined. This is because for any solution pair (b, c), every other pair $(\tilde{b}, \tilde{c})$ with $\tilde{b} \geq \underline{b}$ and $\tilde{b} - \tilde{c} = b - c$ will also attain the same profit.

The value from becoming an active bond $i = b$ or CDS $i = c$ dealer is given by the value of being active, π, minus the entry cost to become active, γ_i, plus the expected benefits from trading with an investor in their preferred submarket.[11] That is,

$$\Pi_i = \pi - \gamma_i + \max_{f_i} \rho_i(\theta_i(f_i))f_i, \quad i = b, c.$$

We assume free-entry of dealers, so in equilibrium $\Pi_b, \Pi_c \leq 0$.

In order to characterize active submarkets (those with $\theta > 0$), we consider the choice of which submarket to enter. In an active bond submarket f_b, the dealer must choose f_b to maximize Π_b and,

[11] Because of free entry, dealers would never become active in both bonds and CDS because they pay the entry cost twice but get the benefit π once.

due to free entry, must have $\Pi_b = 0$. The situation is identical for active CDS submarkets. Defining net entry costs as $\tilde{\gamma}_b \equiv \gamma_b - \pi$ and $\tilde{\gamma}_c \equiv \gamma_c - \pi$, this requires $\rho_i(\theta_i) f_i = \tilde{\gamma}_i$ for $i = b, c$. Since the matching technology implies that $\alpha_i(\theta) = \rho_i(\theta)\theta$ for $i = \{b, c\}$, we obtain the following equation:

$$\underbrace{\alpha_i(\theta_i) f_i}_{\substack{\text{Investor expected} \\ \text{fees in submarket } f \\ \text{of market } i}} = \underbrace{\tilde{\gamma}_i}_{\substack{\text{Net entry} \\ \text{costs in} \\ \text{market } i}} \times \underbrace{\theta_i,}_{\substack{\text{Tightness in} \\ \text{submarket } f_i \\ \text{of market } i}} \qquad i = b, c. \tag{8}$$

We use this mapping from market tightness to fees in the investor problem.

2.1.5 The investor's problem

An investor must choose whether to be active in the bond or CDS market, a submarket to enter and a demand for bond/CDS in case of matching with a dealer. When making these decisions, an investor anticipates they will choose their risk-free position optimally afterwards, resulting in a value $L(b, c, \omega)$.

Conditional on being active in the market for bonds, the investor solves

$$V_b(\omega) = \underbrace{X(\omega)}_{\substack{L(0,0,\omega) \\ \text{Value of not matching}}} + \max_{\substack{\theta_b \geq 0 \\ b \geq \underline{b}}} - \underbrace{\tilde{\gamma}_b \theta_b}_{\substack{\alpha_b(\theta_b) f_b \\ \text{Exp. fees}}} + \alpha_b(\theta_b) \underbrace{[X(b+\omega) - X(\omega) - qb]}_{\substack{L(b,0,\omega) - L(0,0,\omega) - qb \\ \text{Gains from trade}}}. \tag{9}$$

The fee in the bond market, given implicitly by $\tilde{\gamma}_b \theta_b / \alpha(\theta_b)$, is paid conditional on matching, resulting in an expected fee of $\tilde{\gamma}_b \theta_b$. The first order conditions of this problem are

$$q \geq X'(b + \omega), \text{ with equality if } b > \underline{b}, \text{ and} \tag{10}$$

$$\tilde{\gamma}_b \geq \alpha_b'(\theta_b) \left[X(b+\omega) - X(\omega) - qb \right], \text{ with equality if } \theta_b > 0. \tag{11}$$

Let an optimal bond choice and optimal market tightness be denoted $b_i(\omega)$ and $\theta_b(\omega)$.

Similarly, we can write the investor's problem if he is active in the CDS market as

$$V_c(\omega) = X(\omega) + \max_{\substack{\theta_c \geq 0 \\ c \leq -\underline{x}(\omega)}} -\tilde{\gamma}_c \theta_c + \alpha_c(\theta_c)[X(\omega - c) - X(\omega) + (q_f - p)c] \tag{12}$$

The first order conditions are

$$q_f - p \geq X'(\omega - c), \text{ with equality if } c < -\underline{x}(\omega), \text{ and} \tag{13}$$

$$\tilde{\gamma}_c \geq \alpha_c'(\theta_c) \left[(q_f - p)c + X(\omega - c) - X(\omega) \right], \text{ with equality if } \theta_c > 0. \tag{14}$$

Let an optimal CDS choice and optimal market tightness be denoted $c_i(\omega)$ and $\theta_c(\omega)$, respectively.

As in the dealers problem, the first-order conditions given by equations (10)–(14) have an interesting implication. We discussed that when the CDS-bond basis holds in the inter-dealer market, that is $q = q_f - p$, the dealer problem has not a unique solution. For the same reason, when $q = q_f - p$, the investors' gains from trading bond or CDS are the same if the constraints $b \geq \underline{b}$ and $c \leq -\underline{x}(\omega)$ do not bind. To see that, note that choices of $c = -b$ generates the same gains from trade since

$$X(\omega - c) - X(\omega) + (q_f - p)c = X(\omega + b) - X(\omega) - (q_f - p)b = X(\omega + b) - X(\omega) - qb.$$

As a result, the investors choice between bond or CDS trading will depend solely on which market has lower trading frictions. For example, if $\alpha_b = \alpha_c$ then the market with lower dealer entry cost, γ_i for $i = b, c$, will generate higher value. In this respect, trading costs are a first-order consideration for evaluating how CDS markets affect bond markets.

More generally, the choice between bonds and CDS is defined in the following way. When investors choose between being active in bonds or CDS, we assume investors are influenced by a small taste shock that makes choice probabilities continuous, facilitating computation. Formally, an investor's problem is

$$V(\omega) = \mathbb{E}_{\epsilon_b, \epsilon_c} \max\{V_b(\omega) + \epsilon_b/\sigma_m, V_c(\omega) + \epsilon_c/\sigma_m\}. \tag{15}$$

Assuming ϵ_b and ϵ_c are distributed (i.i.d.) Type-1 extreme value, the ex-ante probability of choosing to trade in the bond market is given by[12]

$$m_b(\omega) = \frac{1}{1 + \exp(\sigma_m(V_c(\omega) - V_b(\omega)))}. \tag{16}$$

Define $m_c(\omega) = 1 - m_b(\omega)$. For all propositions we consider the case where σ_m is arbitrarily large.

2.1.6 Market clearing

There is a perfectly elastic supply of the risk-free asset at the price q_f, the market clearing price. We next provide market clearing conditions for the sovereign bond and CDS markets.

Let us introduce some notation. Define $d_b(\omega)$ the measure of dealers active in the bond market in sub-period s_1 trading with investors with exposure ω, $d_c(\omega)$ the measure of dealers active in the CDS market serving investors with exposure shock ω in sub-period s_1, and $D = \bar{d} + \int[d_b(\omega) + d_c(\omega)]d\omega$ is the total measure of dealers. Let F represent the distribution of exposure shocks.[13] The exogenous mass of dealers, $\bar{d}$, is always active in the inter-dealer market.

In s_1, denote $n_b(\omega) = m_b(\omega)f(\omega)$ the mass of investors with exposure ω active in the bond market, and $n_c(\omega) = m_c(\omega)f(\omega)$ the mass of investors with exposure ω active in the CDS market. The bond market clears if

$$B' = \int_\omega M_b(n_b(\omega), d_b(\omega))b_i(\omega)dF(\omega) + Db_d. \tag{17}$$

The CDS market clears if

$$0 = \int_\omega M_c\left(n_c(\omega), d_c(\omega)\right) c_i(\omega)dF(\omega) + Dc_d. \tag{18}$$

[12]This was first pointed out by McFadden (1974). For additional details, see Gordon (2019). Since taste shocks are taken to be small, we compute $V(\omega)$ as $V(\omega) = m_b(\omega)V_b(\omega) + (1 - m_b(\omega))V_c(\omega)$.

[13]Explicitly, $d_i = I\alpha_i(\theta_i(\omega))\theta_i(\omega)$.

2.1.7 Definition of equilibrium in OTC markets

We define *equilibrium in OTC markets given* $\bar{\delta}$ *and* B' as follows:

Definition 1. *Value functions* $\{V, V_b, V_c, \pi, L, X\}$ *with associated policy functions and prices* $\{p, q\}$ *constitute an equilibrium in OTC markets if the values and policies solve their respective problems taking prices as given and the policy functions imply market clearing.*

2.2 The sovereign block

We now describe the sovereign block of the model, which endogenizes the supply of bonds, B', and default decisions, δ. For any given default probability and bond supply, equilibrium in the OTC markets determines a bond price q.

2.2.1 Agents, preferences, and endowments

There is a sovereign government who has a stochastic, Markov "potential" output stream Y. The log of potential output follows a Gaussian AR(1) process,

$$\log Y = \rho_Y \log Y_{-1} + \sigma_Y \epsilon_Y,$$

with ϵ_Y innovations drawn from a standard normal distribution. If the sovereign does not default and is not in autarky, this potential output stream is actual output. If the sovereign does default or is in autarky, this output stream is reduced to $h(y) \equiv y - \max\{0, d_0 y + d_1 y^2\} \leq Y$. The sovereign values stochastic consumption streams $\{C_t\}$ according to $\mathbb{E} \sum_t \beta_g^t u(C_t)$.

2.2.2 Financial markets

For tractability, we assume that bonds mature in one period. With one-period bonds, investors hold bonds and CDS contracts at most for one period. Consequently, the distribution of bond holdings is re-started every period. Thus, the distribution of investor types and bond holdings is not part of the aggregate state of the economy, which greatly simplifies the solution of the model.[14]

At the beginning of each period, the sovereign has some amount of existing debt obligations B. It then chooses to honor those obligations, $\delta = 0$, or default on them $\delta = 1$. If it defaults, then the sovereign enters autarky, i.e., is unable to save or borrow, from which it exits with probability ϕ. In autarky, we say the sovereign's debt is zero.

If the sovereign is not in autarky and does not default, then it chooses an amount of debt B' to issue, taking the price schedule $q(Y, B')$ as given.

[14]Assuming long-term debt would require tracking distributions of bond and CDS holdings or assuming that investors who leave the economy after one-period somehow can re-allocate all bonds and CDS positions to new investors without being subject to trading frictions in OTC markets. This is an interesting extension with even more effects to consider, but it is worth understanding the many mechanisms present in this simpler formulation, first.

2.2.3 Timing and state of the economy

The timing of the sovereign block is as follows. Shocks determining the level of potential output Y and whether the sovereign leaves autarky (if applicable) are realized. The sovereign makes its default decision (if not in autarky). If the sovereign was not in autarky and repays maturing debt, the sovereign can issue new debt B'.

The pair (Y, B) is the relevant state of the economy because the endowment follows a standard AR(1) process and bonds mature in one period, so the distribution of bond holdings is reset to zero every period. These two variables determine government's optimal policy for new debt issues, $B'(Y, B)$, and next period probability of default, $\mathbb{E}_{Y'|Y}\delta(Y', B'(Y, B))$, where $\delta(Y, B)$ is government's default policy. This is all the information that investors and dealers need to price bonds and CDS in the OTC block. Because (1) investors and dealers portfolios mature in one period and new investors and dealers enter the economy each period and (2) all shocks in the OTC block are idiosyncratic, no state variables are carried from the OTC block to the government's problem.

2.2.4 Government's problem

At the beginning of each period, the government has outstanding level of debt that it needs to repay, B, and observes the new realization of the endowment, Y. After observing the state of the economy, the government decides whether to repay the outstanding level of debt or to default.

The optimal decision is determined by weighting the costs and benefits of repaying the outstanding level of debt. The benefit of default is debt service costs can instead be used to boost current period consumption. The costs are lost output and a temporary exclusion from international credit markets. The temporary exclusion from international credit markets reduces the ability of the government to use credit as a source for consumption smoothing. The length of the exclusion is captured by an exogenous probability of regaining access to credit markets, $\phi \in (0, 1)$. When the government re-gains access to credit markets, it starts next period with no debt. The output cost is an endowment loss while the government is in default and it is given by the function $h(y) \leq y$, for all y.

The recursive formulation of government's problem is then given by

$$\begin{aligned} W(Y, B) &= \max_{\delta \in \{0,1\}} \{\delta W^d(Y) + (1-\delta) W^r(Y, B)\}, \\ W^d(Y) &= u(h(Y)) + \beta_g \mathbb{E}_{Y'|Y}\{\phi W(Y', 0) + (1-\phi) W^d(Y')\}, \\ W^r(Y, B) &= \max_{C, B' \geq 0} u(C) + \beta_g \mathbb{E}_{Y'|Y}\{W(Y', B')\}, \\ s.t.: C &= Y + q(Y, B')B' - B \end{aligned} \tag{19}$$

where W is the option value of repaying the debt, W^d is the value of defaulting, and W^r is the value of repaying. Whenever the government decides to repay its debt, it is allowed to choose the new debt issuance, B', taking as given the price scheduled that it faces in the market, $q(Y, B')$.

The frictions in secondary markets for bonds and CDS play an important role in the default decision of the government. They enter the problem of the government by affecting the market price of newly issued bonds, q, and thus directly affecting the value of repaying debt, W^r.

2.2.5 Definition of equilibrium in the sovereign block

We define partial equilibrium in the sovereign block given q as follows:

Definition 2. *A partial equilibrium in the sovereign block given a price schedule q is a family $\{W, W^r, W^d, B', \delta\}$ that is a solution to the sovereign's problem.*

2.3 Combining the model blocks

To combine the two blocks, we need to impose consistency between the price schedule arising from the OTC block, and the bond issuance and default probability optimally chosen by the government. We say the price schedule $q(Y, B')$ is *consistent with OTC equilibrium* if, for every $Y, B' > 0$, there exists an equilibrium in the OTC markets, given the default probability $\bar{\delta} = \mathbb{E}_{Y'|Y}\delta(Y', B'(Y, B))$ and debt issuance B', that results in price $q(Y, B')$. We are now ready to state equilibrium.

Definition 3. *An equilibrium is a set of functions $\{q, W, W^r, W^d, B', \delta\}$ such that $q(Y, B')$ is consistent with OTC equilibrium, and $\{W, W^r, W^d, B', \delta\}$ solves the sovereign's problem given q.*

3 Theoretical results

In this section, we provide theoretical insights into how trading frictions, the distribution of exogenous exposure, and CDS and sovereign debt policy affect sovereign bond markets. We first consider limit cases where entry costs go to zero. We show that prices converge to risk-neutral if CDS are freely tradable, but can remain above risk-neutral without CDS or with a naked CDS ban in place. We then show how regulation, trading technology, matching frictions, and entry costs can interact to shut down either the bond or CDS investor market. Next, we turn to the primary metric of arbitrage breakdown between bond and CDS markets, the CDS-bond basis deviation, and show that the model can generate both positive and negative deviations theoretically. Finally, we establish two closed-form solutions in a special case with investor homogeneity. These solutions reveal how trading frictions, investor exposure, default risk, and debt issuance affect sovereign debt prices, intermediation fees, and the CDS-bond basis. All results refer to the over-the-counter (OTC) block and its equilibrium for given $B' > 0$ and $\bar{\delta} \in (0, 1)$.

3.1 Frictionless limit: $\gamma_b, \gamma_c \to 0$

In the introduction, we discussed how the CDS market allows investors to create synthetic bonds—portfolios that replicate the payoff of a bond. Investors can also sell these synthetic bonds, which increases the effective supply of bonds in the market. Many researchers and policymakers have asserted that this has a negative effect on bond prices, and therefore, a ban on such instruments would increase prices.[15] While these conclusions ultimately depend on the trading frictions in our model, we do get this result when trading frictions are small. Formally, in this section, we show theoretically that as trading frictions vanish, meaning $\gamma_b, \gamma_c \to 0$, either a CDS ban, or a ban on naked CDS, have a positive impact on prices. All proofs are relegated to Appendix B.

Proposition 1. *Assume that* $\lim_{\theta\to\infty} \alpha_b(\theta) = \alpha_c(\theta) = \bar{\alpha} \in (0, 1]$*, and* $\underline{x}(\cdot) = -\infty$*; further assume* u *is unbounded above. Then, in equilibrium, as the entry costs go to zero (*$\gamma_b, \gamma_c \to 0$*), the bond and the CDS prices converge to the risk neutral equilibrium prices. That is,* $q \to X'(0) = \beta(1-\bar{\delta})$ *and* $p \to q_f - X'(0) = \beta\bar{\delta}$*.*

Proposition 1 shows that as the market becomes more liquid, the model converges to a risk-neutral limit. The proof works by establishing that as markets become more liquid, the measure of dealers grows infinite.[16] Since dealers are not exposed—and CDS allows risk to be shared—the risk born by each agent tends to zero. Investors and dealers are risk-neutral at the margin, and so prices converge to risk-neutral. This result indirectly shows the role of CDS in risk sharing: The proof does not go through without CDS as there is no mechanism for agents to share risk, as Proposition 2 shows.

Proposition 2. *Assume that* $\lim_{\theta\to\infty} \alpha_b(\theta) = \bar{\alpha} \in (0, 1]$*,* $\underline{b} = 0$ *and that either* $\alpha_c(\theta) = 0$ *for all* θ*, or there is a naked-CDS ban of the form,* $b - c \geq 0$*; further assume* u *is unbounded above. If*

$$B + \bar{\alpha} I \int_{\omega \leq 0} \omega dF(\omega) < 0, \tag{20}$$

in equilibrium, as the entry costs go to zero ($\gamma_b, \gamma_c \to 0$*), the bond price is bounded from below by some* $\bar{q}$ *strictly higher than the risk neutral equilibrium price. That is,* $q \geq \bar{q}$ *for some* $\bar{q} > \beta(1-\bar{\delta})$*.*

Propositions 1 and 2 combined show the standard intuition from a Walrasian equilibrium: Preventing bond shorting or naked CDS positions can only improve sovereign debt prices. The reason is that it prevents relatively-more exposed investors from endogenously creating more financial exposure. For instance, if dealers go from a zero bond-position to $b_d < 0$, it is as if the bond supply goes from B' to $B' + D|b_d|$. Naturally this depresses prices. But if bond shorting and naked CDS are impossible, this additional supply of exposure cannot be created.

[15] See, for example, Portes (2012) and Murdock (2012).

[16] Dealer entry, a difference of our model from Sambalaibat (2022), plays an important role here.

3.2 Market activity

Because of the tight no arbitrage relationships between bonds and CDS, there are a number of conditions under which either the CDS, bond, or both markets could be inactive in equilibrium. Investors have four main considerations. First, investors are concerned about what consumption allocations are possible using either bonds or CDS, as dictated by $\underline{b}$ and $\underline{x}$. Second, investors care about the price of bonds q and synthetic bonds $q_f - p$. Bond and synthetic bond buyers prefer smaller prices else equal. Third, investors desire lower fees, which leads them to markets with lower entry costs else equal. Last, higher matching probabilities for a given θ are better ceteris paribus. In some cases, these four forces align, and a market can completely dominate the other market. Proposition 3 establishes these cases.

Proposition 3. *Investors are surely inactive in bond and CDS markets, depending on trading technology and policy, as indicated in the following table:*

(A) $\underline{b} = -\infty, \underline{x} = -\infty$

	$M_b > M_c$	$M_b < M_c$
$\gamma_b < \gamma_c$	*CDS inactive*	
$\gamma_b > \gamma_c$		*Bonds inactive*

(B) $\underline{b} = 0, \underline{x} = -\infty$

	$M_b > M_c$	$M_b < M_c$
$\gamma_b < \gamma_c$		
$\gamma_b > \gamma_c$		*Bonds inactive*

(C) $\underline{b} = 0, \underline{x} = 0$

	$M_b > M_c$	$M_b < M_c$
$\gamma_b < \gamma_c$	*CDS inactive*	
$\gamma_b > \gamma_c$		*Bonds inactive*

In the proposition, the top left entry of each panel is where bonds are a superior technology both in terms of matching ($M_b > M_c$) and fees ($\gamma_b < \gamma_c$). So when a CDS allocation can be replicated using bonds and the risk-free asset and cost weakly less in inter-dealer prices, one should expect the CDS to not be used. This is what happens in panel A: There are no portfolio restrictions and $q = q_f - p$, so the portfolios cost the same in the inter-dealer market. In panel B (the benchmark restrictions), CDS offer some choices that a bond cannot achieve, so it may be active despite inferior trading technology. In panel C (no short selling with a naked CDS ban), the argument to show CDS are inactive is like in panel A when the basis holds in inter-dealer prices; but an additional step is required to show the basis holds in that case.

The bottom right entry of each panel is where CDS are a super technology both in terms of matching and fees. In each panel, a bond portfolio can be replicated using a corresponding long risk-free, short CDS portfolio. In panel A, these portfolios cost the same in inter-dealer terms, but CDS have smaller OTC frictions. In panels B and C, the basis might not hold, but when it doesn't bonds are more expensive than synthetic bonds at weakly smaller cost. So CDS are also dominant in this respect.

This result shows policy can completely shutdown markets and investor demand for bonds (or CDS). But the overall effects depend on entry fees and matching technology. Carefully identifying these OTC frictions will be important for an accurate assessment of how trade affects bond and CDS markets.

3.3 CDS-bond basis

The CDS-bond basis is a no-arbitrage relationship between bonds, CDS, and the risk-free asset. As we will discuss in Section 4.1, the CDS-bond basis is typically measured as the CDS running spread minus the bond Z-spread, which involves complicated and nonlinear transformations of q and p. That measure only *approximately* captures the no-arbitrage relationship, and it can be nonzero even if no-arbitrage holds. So in this section we establish theoretical results about the true, underlying arbitrage deviations rather than the approximate nonlinear empirical formulas. We focus on the CDS-bond basis deviation in effective prices, which we define as $\tilde{\psi} \equiv \mathbb{E}[\tilde{q} + \tilde{p} - q_f]$. Here, $\mathbb{E}[\tilde{q}]$ and $\mathbb{E}[\tilde{p}]$ are the average transacted prices, including the fees paid by investors. The effective bond price for an investor is the total cost per unit of bond $(qb + \tilde{\gamma}_b f_b)/b$ or $q + \tilde{\gamma}_b \frac{\theta_b}{\alpha_b(\theta_b)b}$ using (8). Aggregating, the volume-weighted average transacted bond price is given by

$$\mathbb{E}[\tilde{q}] = \frac{\int_\omega M_b(n_b, d_b)|b_i| \left[q + \frac{\tilde{\gamma}_b \theta_b}{\alpha_b(\theta_b)b_i}\right] dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)} = q + \frac{\int_\omega \operatorname{sgn}(b_i) d_b \tilde{\gamma}_b dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)}, \tag{21}$$

where we omitted the argument ω from the functions n_b, d_b and b_i to keep the notation short, and sgn is the sign function, $\operatorname{sgn}(x) = -\mathbf{1}_{[x<0]} + \mathbf{1}_{[x>0]}$. Similarly, the volume-weighted average transacted CDS price is given by

$$\mathbb{E}[\tilde{p}] = \frac{\int_\omega M_c(n_c, d_c)|c_i| \left[p + \frac{\tilde{\gamma}_c \theta_c}{\alpha_c(\theta_c)c_i}\right] dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)} = p + \frac{\int_\omega \operatorname{sgn}(c_i) d_c \tilde{\gamma}_c dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)}, \tag{22}$$

where again we omitted the argument ω from the functions n_c, d_c and c_i to keep the notation short.

Combining equations (21) and (22) the basis deviation in effective prices is

$$\tilde{\psi} = \underbrace{q + p - q_f}_{\substack{\text{Basis deviation in} \\ \text{inter-dealer prices}}} + \underbrace{\frac{\int_\omega \operatorname{sgn}(b_i) d_b \tilde{\gamma}_b dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)}}_{\substack{\text{Bond market} \\ \text{Intermediation costs}}} + \underbrace{\frac{\int_\omega \operatorname{sgn}(c_i) d_c \tilde{\gamma}_c dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)}}_{\substack{\text{CDS market} \\ \text{Intermediation costs}}}. \tag{23}$$

Equation (23) decomposes the CDS-bond basis in three components: the inter-dealer CDS-bond basis and bond and CDS market intermediation costs. The CDS-bond basis in inter-dealer prices must be weakly positive.[17] While this pushes the basis if anything positive, the effect of

[17]This appeared already in the discussion of equations (6) and (7) in Section 2.1.4, but we also establish this

intermediation costs on the basis is in general ambiguous. We are able to sign the effect if (1) OTC frictions in market i are less than in j (2) market j does not completely shutdown (like the cases in Section 3.2) because market j offers some allocation that is not feasible in market i. Propositions 4 and 5 sign the basis with this approach, establishing $\tilde{\psi} > 0$ in one case and $\tilde{\psi} < 0$ in another.

Proposition 4. *If bond shorting is not allowed, $\underline{b} = 0$, and the bond search technology is better than the CDS search technology, that is $\gamma_b < \gamma_c$ and $M_b(n,d) \geq M_c(n,d)$ for all $(n,d) \in \mathbb{R}^2$, then in any OTC equilibrium where $q + p - q_f = 0$ the CDS-bond basis in effective prices is strictly positive.*

Proposition 5. *If bond shorting is allowed, $\underline{b} = -\infty$, and there is a naked CDS ban, $c \leq -\underline{x} = 0$, and the CDS search technology is better than the bond search technology, that is $\gamma_b > \gamma_c$ and $M_b(n,d) \leq M_c(n,d)$ for all $(n,d) \in \mathbb{R}^2$, then the CDS-bond basis in effective prices is strictly negative.*

While allowing for bond shorting and having a naked CDS ban are part of the sufficient conditions for Proposition 5, they are not necessary, though we have only been able to demonstrate this numerically. Specifically, in the appendix Section C.1, we use a predictive prior exercise to investigate the model's ability to generate negative and positives bases and determine the parameters determining the sign. The model can readily generate both positive and negative bases. The basis tend to be negative when the dealers entry cost into the CDS market is not too high, the distribution of exposures ω is fairly concentrated in negative values (negative average and low dispersion), CDS matching is reasonably efficient, and the CDS matching elasticity is not too low.

3.4 Closed-form solutions

In this subsection we make specific assumptions that allow us to write closed-form expressions for equilibrium objects. In particular, we make the following assumptions:

Assumption 1. *The distribution of investors' exogenous exposure is degenerate at $\omega = \mu_\omega$*

Assumption 2. *Investors and dealers have a quadratic utility function given by*

$$u(z) = -\sigma(\bar{z} - z)^2/2,$$

Assumption 3. *The matching technology is a Cobb-Douglas technology with constant elasticity such investors' probability of trading is given by*

$$\alpha_i(\theta) = \bar{\alpha}_i \theta_i^{\xi_i}$$

when less than one.

formally in the appendix).

Let $\lambda_b \equiv 1 - \frac{q}{\beta(1-\bar{\delta})}$ and $\lambda_c \equiv 1 - \frac{q_f - p}{\beta(1-\bar{\delta})}$ be the premium implied in assets prices over their corresponding risk neutral pricing; let $\mathbb{V}[\delta] \equiv \bar{\delta}(1-\bar{\delta})$ be the variance of default risk; and denote Δ_i, for $i \in \{b, c\}$, the investor's gains from trading in asset market i.

The first special case we construct is an equilibrium where $q + p - q_f > 0$ and $p = \beta\bar{\delta}$. The positive basis deviation makes $b_d = 0$. The risk-neutral price for CDS makes dealers demand zero CDS. So $b_d = c_d = \pi = 0$. The risk-neutral price for CDS would generally be attractive to investors, but we assume CDS markets are sufficiently frictional so that investors do not wish to go to them.

Proposition 6. *Under assumptions 1-3, if the entry cost parameters, γ_i, are such that the investors prefer to achieve their desired exposure trading in the bond market; and $\mu_\omega < 0$ is low enough that investors are allocated all the bonds, i.e. $b = \frac{B'}{\alpha_b I}$, we obtain the following equilibrium expressions in closed form:*

a. Investors' gain from trade are given by $\Delta_b = \left[\left(\frac{B'}{I}\right)^{2(1-\xi_b)}\left(\frac{\sigma\beta\mathbb{V}[\delta]}{2}\right)^{1-\xi_b}\left(\frac{\gamma_b}{\bar{\alpha}_b^{1/\xi_b}\xi_b}\right)^{2\xi_b}\right]^{1/(1+\xi_b)}$.

b. The intermediation fees per unit of bond are $\frac{f_b}{b} = \left[\frac{1}{2}\left(\frac{B'}{I}\right)^{1-\xi_b}\gamma_b^{\xi_b}\bar{\alpha}_b^{-1}\xi_b\sigma\beta\mathbb{V}[\delta]\right]^{1/(1+\xi_b)}$

c. Individual investor bond positions are $b = \left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{1-2\xi_b/(1+\xi_b)}$

d. Risk premium in asset prices are $\lambda_b = \sigma\bar{\delta}\left(\mu_\omega + b\right) < 0$ *and* $\lambda_c = 0$

e. The CDS-bond basis in inter-dealer prices, $-\lambda_b\beta(1-\bar{\delta})$*, is positive and increasing in* $\bar{\delta}, \forall\bar{\delta} \in (0, 1/2)$.

These equilibrium conditions provide us with several comparative statics results. For simplicity, restrict attention to $\bar{\delta} \in (0, 1/2)$, which is where $\mathbb{V}[\delta]$ is increasing in $\bar{\delta}$. In this case, as $\bar{\delta}$ increases, the utility gap between being matched and unmatched increases, driving up gains from trade ceteris paribus. With larger gains from trade, investors have more incentive to match, which shows up as increased fees per unit of bond; and we will show in Section 4.1 this measure corresponds to bid-ask spreads. This induces greater matching else equal, a positive extensive margin. At the same time, investors demand smaller bond positions on the intensive margin. In equilibrium, these effects must cancel, as $\alpha_b b I = B'$. So the bond price reveals the effect of $\bar{\delta}$ on total demand $\alpha_b b$ (which must only equal B'/I at the equilibrium price). Since the bond risk premium, which is negative in this equilibrium, becomes more negative, q increases in $\bar{\delta}$. The larger q means bond demand $\alpha_b b$ must have *risen*, necessitating a larger q to offset that. Therefore in this example the extensive margin of demand adjustment α_b dominates the intensive margin of demand b, making

$\alpha_b b$ increase in $\bar{\delta}$ else equal. With the larger equilibrium q, the CDS-bond basis in inter-dealer prices rises.

Also of note is the effect of matching frictions and exposure on prices. As $\bar{\alpha}_b$ declines, investors must hold more bonds (b must be larger) for markets to clear. This means the risk premium must rise (q must fall). With investors holding more bonds, the implied gains from trade increase, since matching entails a larger change in exposure. Entry costs play a similar role. As entry costs increase, f_b/b (the bid-ask bond spread) must rise: Dealers demand larger fees. In response, investors reduce their probability of matching, and for markets to clear bonds must rise on the intensive margin, which they do. This is incentivized by a decline in bond prices, reflected in a larger risk premium and smaller basis. Interestingly, the only place exposure μ_ω shows up with quadratic utility is in the risk-premium.[18] With more exogenous exposure, investors demand less endogenous exposure b else equal, and so for markets to clear the price must rise.

The second special case we construct is an equilibrium with a naked CDS ban where $q+p-q_f = 0$. Investor exposure μ_ω is sufficiently negative that q and $q_f - p$ are large, driving the dealer to want zero exposure, $b_d - c_d = 0$, which implies $\pi = 0$. And we suppose bond markets are frictional enough that investors do not wish to search there.

Proposition 7. *Under assumptions 1-3 if (i) there is a ban in Naked CDS, $b - c \geq 0$; (ii) the entry cost parameters γ_i are such that the investors prefer to achieve their desired exposure trading in the CDS market; (iii) $\mu_\omega < 0$; and (iv) dealers hold all bonds; then we obtain the following equilibrium expressions in closed form:*

a. Investors' gain from trade are given by $\Delta_c = \left[\left(\frac{B'}{I}\right)^{2(1-\xi_c)}\left(\frac{\sigma\beta\mathbb{V}[\delta]}{2}\right)^{1-\xi_c}\left(\frac{\gamma_c}{\bar{\alpha}_c^{1/\xi_c}\xi_c}\right)^{2\xi_c}\right]^{1/(1+\xi_c)}$.

b. Intermediation fees per unit of CDS are $\frac{f_c}{c} = -\left[\frac{1}{2}\left(\frac{B'}{I}\right)^{(1-\xi_c)}\gamma_c^{\xi_c}\bar{\alpha}_c^{-1}\xi_c\sigma\beta\mathbb{V}[\delta]\right]^{1/(1+\xi_c)} < 0$

c. $q + p = q_f$*, and* $\lambda_c = \lambda_b \neq 0$

d. $\frac{f_c}{c}$ *is negative and decreasing in* $\bar{\delta}, \forall\bar{\delta} \in (0, 1/2)$.

Much of the intuition from Proposition 6 carries over to Proposition 7 as well. Restricting attention to $\bar{\delta} < 1/2$, larger $\bar{\delta}$ increases gains from trade. This translates into more negative fees per unit of CDS. The bid-ask spread in this case is the absolute value of that, so those increase. In this example, the CDS-bond basis holds in inter-dealer prices, which is necessitated by dealers holding bonds in equilibrium. However, what is highly suggestive of the CDS-bond basis in effective prices, $\tilde{\psi}$, is the term f_c/c. The CDS-bond basis deviation in effective prices amounted to the inter-dealer

[18] As is well-known, precautionary savings behavior is connected to the third-derivative of the utility function. Here, that is zero, which simplifies incentives and the problem significantly.

basis deviation plus volume-weighted fees per unit of bond plus volume-weighted fees per unit of CDS (see equation (23)). Unfortunately, here the investor bond market shuts down, so the bond fees cannot be volume-weighted. However, f_c/c being negative and decreasing in $\bar{\delta}$ suggests $\tilde{\psi}$ would decline, if it were well-defined.

4 Measurement, stylized facts, and identification

In this section, we discuss measurement, stylized facts, and the identification of our model. We begin by describing how we compute data variables in our model, such as spreads and the CDS-bond basis, that we use for calibration. We then discuss the stylized facts of the Argentine bond and CDS markets that we aim to match with the model. Finally, we discuss our identification strategy. The model is calibrated at a quarterly frequency to match the Argentinean bond and CDS markets.

4.1 Measurement

To fit the model to the data, we need to define a few key concepts:

- **Running spread:** A *running spread* is an endogenous coupon payment such that the expected, discounted, net present value of a CDS contract is zero. Our CDS payments f_c are upfront, but they can also be quoted in terms of a running spread.

- **Z-spread:** A *Z-spread* is the internal rate of return less the risk-free rate (the usual metric). Our bond payments f_b are also upfront, but they can be quoted in terms of a Z-spread.

- **CDS-bond basis deviations in spreads**: The running spread minus the Z-spread, which is the measure used in our data and the typical measure used in the literature.

Appendix A.5 describes in detail how these measures work. We distinguish basis deviations in spreads from basis deviations in effective prices, $\mathbb{E}[\tilde{q}] + \mathbb{E}[\tilde{p}] - q_f$, and inter-dealer prices $q + p - q_f$. Qualitatively, a positive value for any of these deviations means obtaining exposure through bonds is expensive relative to CDS, but the measures can diverge due to economic reasons (such as wedges between $\mathbb{E}[\tilde{q}]$ and q) or, less interestingly, nonlinearities in the formula for deviations in spreads. When we do not specify whether the basis deviations are in spreads or prices, we mean spreads.

We define a bid-ask spread in the model as follows. The bid-ask spreads for bonds in our data are quoted in yield to maturity (YTM), which is equivalent to being quoted in Z-spreads. Note that bond dealers' expected profit is $\rho_b(\theta_b(f_b))f_b$, where ρ_b is matching probability and θ_b is a function that maps from fees to tightness. To be willing to transact with at least some positive probability, this must be positive. As any bid less than qb will never be transacted ($f_b \leq 0 \Rightarrow \rho = 0$), we think of the "bid" price as being qb. On the other hand, $f_b > 0$ will sometimes transact and sometimes not. Hence, we say dealers ask for $qb + f_b$, in which case if that is met, they will transact for sure. (As the f_b are heterogeneous, there will also be heterogeneity here, but we will aggregate to a single number.) Consequently, a bid-ask spread in prices per unit bond is $(qb + f_b - qb)/b$ or f_b/b, which can be volume weighted to derive an aggregate number. However, for a bid-ask spread in terms of the YTM, we take the volume-weighted price $(qb + f_b)/b$ and inter-dealer price q, convert those both to YTM, and take their difference.

The bid-ask spreads for CDS are likewise quoted in the data in running spreads, and its calculation is similar to the calculation of bond bid-ask spreads. However, we need to construct separate (volume-weighted) bid-ask spreads for buying ($c > 0$) and selling ($c < 0$) because CDS protection can be bought or sold by investors. We then express these buying-or-selling-specific bid-ask spreads in terms of running spreads and volume-weight them by buying-versus-selling activity. This approach ensures that the premiums investors pay to buy or sell do not cancel each other when averaging. Section A.6 of the appendix gives the details.

A final key measurement issue is that our bonds and CDS are both short-term contracts, while in the data they are five-year contracts. For bonds, we handle this by focusing on the debt service targets rather than debt stocks. This is a standard approach for modeling short-term bonds. For CDS, we do something similar, reducing our position and volume measures to one quarter's worth (5%) of a five-year CDS contract.

4.2 Stylized facts on bonds and CDS

This section presents stylized facts on bonds and CDS markets, focusing on three key areas: Bid-ask behavior, CDS-bond basis deviations, and Dealer behavior. The goal is to provide a comprehensive overview of the key features of these markets, and to highlight how it relates to our model.

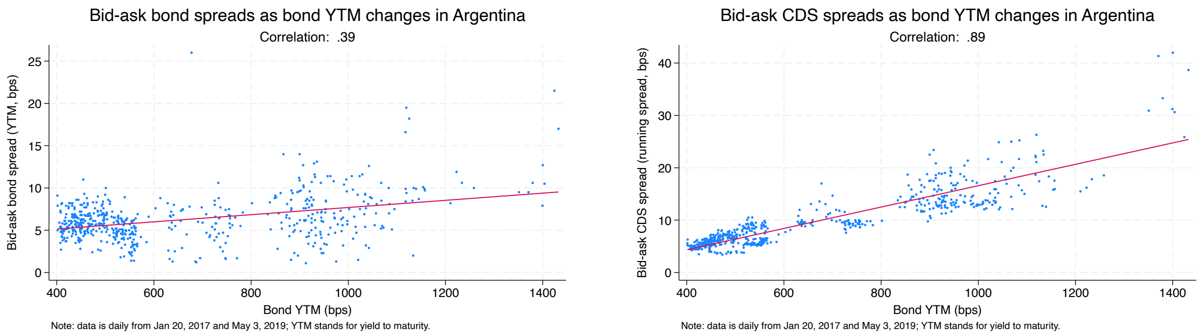
4.2.1 Stylized fact #1: Bid-ask behavior

Our first stylized fact is the tight relationship between default risk and bid-ask spreads, indicating that a common measure of liquidity breaks down in times of stress.

Figure 1 shows the relationship between bond yield-to-maturity (YTM), which here we interpret as a proxy for default risk, and bid-ask spreads for bonds and CDS in the case of Argentina. The literature has found similar results in the Italian sovereign bond market (Pelizzon et al., 2016) and in the context of the Greek crisis (Chaumont, 2022).[19]

[19] Pelizzon et al. (2016) finds that a 10% change in the credit default swap (CDS) spread leads to a 13% change in the bid-ask spread for bonds.

Figure 1: Bid-ask spreads and yield-to-maturity for Argentina



(a) Bid-ask spreads for bond

(b) Bid-ask spreads for CDS

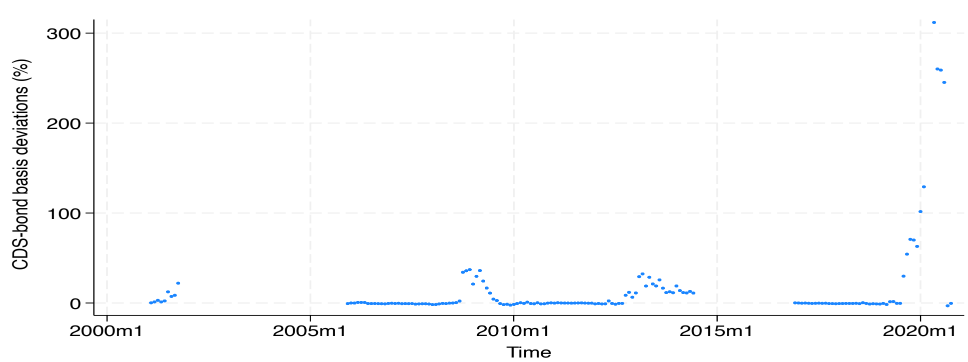
Note: Data is daily from Jan 20, 2017 to May 3, 2019; YTM stands for yield to maturity.

4.2.2 Stylized fact #2: CDS-bond basis deviations

Our second stylized fact is that CDS-bond basis deviations, while normally close to zero, tend to explode positive in times of stress. That is, the bond price q is elevated relative to the synthetic bond price $q_f - p$.

This is evident in Figure 2, where the basis soars above zero during each of Argentina's high default risk episodes in 2001, 2009, 2014, and 2020. As discussed in Appendix A.5.1, a positive CDS-bond basis is not unique to Argentina for two reasons. First, many countries, both developing and developed, have experienced significantly positive CDS-bond basis deviations. Second, Gilchrist et al. (2022) find that in response to times of stress, the basis trends upwards. However, the magnitude of the movement in Argentina is unusual.

Figure 2: CDS-bond basis deviations for Argentina



Note: Data are from Gilchrist, Wei, Yue, and Zakrajsek (2022), averaged over observations within each month; the restructuring period of July 2014 to November 2016 has been excluded.

There are other countries, such as those highlighted in Salomao (2017), where the CDS-bond basis has either trended down or flat in times of stress. As we have already seen in Section 3.3, the model can generate a negative CDS-bond basis deviations depending on parameter values. Additionally, one can incorporate trigger- or counterparty-risk in the model to generate arbitrarily negative CDS-bond basis deviations for given parameters.[20]

4.2.3 Stylized fact #3: Dealer intermediation

Our last stylized fact is that dealers primarily intermediate, and this intermediation increases in default risk. Consequently, risk-shifting increases as default risk increases, but dealers do not take on more risk themselves.

Figure 3 shows that dealers tend to remain mostly neutral in terms of protection, although they do buy a modest amount of CDS protection as risk increases. Complementing this, we report later in Table 3 the unified endogenous exposure ($b-c$) position of dealers, and it is almost zero. Lastly, CDS volume increases as risk increases. Specifically, Table 3 reports a positive correlation between volume and CDS implied default probabilities (IDPs).

4.3 Identification strategy

We now discuss our identification strategy. Table 1 provides the parameters that we set exogenously, along with a rationale for each value. Most of these values are standard, but a few deserve further explanation.

As discussed at the end of Section 2.1.1, exogenous exposure in part captures the impact of previous financial investments. To ensure market clearing, the exogenous measure $\bar{d}$ of dealers is set to 0.001 (which will be tiny relative to the estimated measure of investors), ensuring that the bond market can still clear even if there is no entry due to a lack of gains from trade. The taste shock scaling parameter σ_m makes taste shocks small but still large enough to ensure that the projection method we use does not have noticeable oscillations.[21]

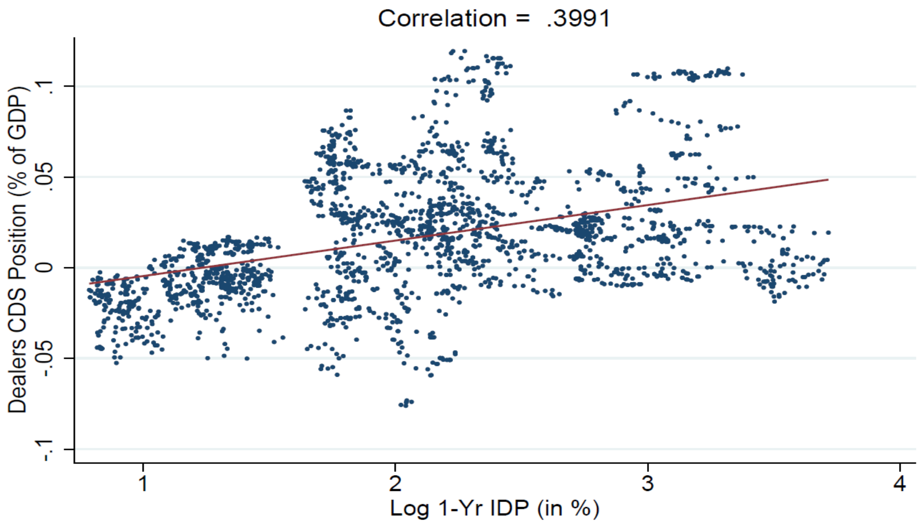
[20] The Salomao (2017) mechanism for generating a negative basis is a risk that default will occur without the CDS "trigger" being activated. This was an issue with the Greek default, where Salomao cites concerns that the Greek debt would be restructured without missed payments. This and counterparty risk can be incorporated in the model. One easy way to do it is to assume that (1) when $\delta = 1$ the CDS only pays out with probability τ and (2) investors and dealers must insure that ex-ante by engaging with risk-neutral intermediaries (at a cost $q_f\bar{\delta}\tau$), then an equilibrium with prices (q, p) for $\tau = 0$ is an equilibrium with prices $(q, p - q_f\bar{\delta}\tau)$ for $\tau > 0$. This can be seen easily from the dealer's problem with prices $(q, \hat{p})$, which we can write

$$\max_{a,b,c} -qb - (\hat{p} + q_f\bar{\delta}\tau)c - q_f a + \beta\bar{\delta}u(a+c) + \beta(1-\bar{\delta})u(a+b) - X(0).$$

The allocations for $\tau > 0$ will be the same as in the equilibrium with $\tau = 0$ (and therefore clear markets) if the after-insurance price $\hat{p} + q_f\bar{\delta}\tau$ corresponds to p, i.e., $\hat{p} = p - q_f\bar{\delta}\tau$. As τ increases the CDS-bond basis in prices $q + \hat{p} - q_f = q + p - q_f\bar{\delta}\tau - q_f$ can easily go negative. E.g., if q and p were within ϵ of the risk-neutral prices, then the price basis is bounded above by $2\epsilon - q_f\bar{\delta}\tau$.

[21] We represent value and policy functions from the OTC block using thousands of Chebyshev polynomials (see Appendix D for details). Chebyshev polynomials can arbitrarily well approximate any Lipschitz continuous function, and taste shocks help ensure this property.

Figure 3: Dealers net CDS position and implied default probability for Argentina



Note: data is daily, from January 1, 2010 to December 31, 2019 excluding defaulted periods. Correlation is for daily observations (which is not identical to the quarterly-based correlation in table 3); IDP stands for the implied default probability, which is a transformation of the running spread; dealers' CDS positions are aggregated.

It is difficult to separately identify $(I, \bar{\alpha}_b, \bar{\alpha}_c)$ (the match efficiency parameters $\bar{\alpha}_b, \bar{\alpha}_c$ correspond to the matching function in (1)). If one halves $\bar{\alpha}_b$ and $\bar{\alpha}_c$ but doubles I, overall demand for bonds and CDS will be roughly the same. In calibrating the model, we found that the data clearly preferred $\bar{\alpha}_b > \bar{\alpha}_c$. Therefore, to solve this normalization/identification problem, we impose $\bar{\alpha}_b = 1$. The relative match efficiency of CDS $\bar{\alpha}_c/\bar{\alpha}_b$ will be cleanly identified by the CDS-bond basis deviations, which is one of our targeted moments.

This leaves eleven parameters to be calibrated: three for the sovereign block (β_g, d_0, d_1) and eight for the OTC block $(\gamma_b, \gamma_c, \xi_b, \xi_c, \bar{\alpha}_c, \sigma_\omega, \mu_\omega, I)$. For the sovereign block, we identify the discount factor and the two default cost parameters using standard moments in the literature. Specifically, we target the mean spread, the debt-service output ratio, and the correlation between spreads and (log) output.

In the OTC block, we need to identify the bond and CDS entry costs, the matching function elasticities for bonds and CDS, the mean and variance of the exogenous exposure distribution, and the measure of investors. We exploit the model's tight relationship between entry costs, matching elasticities, and bid-ask spreads to identify the first four parameters. For example, the bid-ask spread for bonds quoted in upfront payments, f_b/b, is related to entry cost γ_b by the free entry condition (8):

$$\frac{f_b}{b} = \frac{\gamma_b - \pi}{b} \frac{\theta_b}{\alpha_b(\theta_b)}. \tag{24}$$

A similar equation connects the bid-ask spread for CDS quoted in upfront payments, f_c/c and the entry cost γ_c. Hence, we can use the entry cost parameters γ_b and γ_c to match bid-ask spreads for

Table 1: Exogenously fixed parameter values with explanations

Parameter	Description	Value	Reason
Sovereign			
σ	Risk aversion	2	Standard value
ρ_Y	GDP persistence	0.949	Chatterjee and Eyigungor (2012)
σ_Y	GDP innovation std.	0.027	Chatterjee and Eyigungor (2012)
ϕ	Autarky escape prob.	0.0385	Chatterjee and Eyigungor (2012)
Dealers			
β	Discount factor	0.99	Standard value
σ	Risk aversion	2	Standard value
$\bar{d}$	Exogenous measure	0.001	Market clearing regularity
Investors			
β	Discount factor	0.99	Standard value
σ	Risk aversion	2	Standard value
σ_m	Taste shock scaling	10^6	Smallest shocks with good polynomial fit
Markets			
$\bar{\alpha}_b$	Bond matching efficiency	1	Normalization/identification
$\underline{b}$	Bond shorting limit	0	Benchmark technology
q_f	Risk-free price	0.99	Standard value

bonds and CDS, respectively.

Combining equation (24) with the submarket optimality condition (11), we obtain

$$\frac{f_b}{b} = \frac{\alpha_b'(\theta_b)\theta_b}{\alpha_b(\theta_b)} \frac{1}{b} \frac{\gamma_b - \pi}{\alpha'(\theta_b)} = \epsilon_{\alpha_b,\theta_b}(\theta_b) \frac{X(\omega+b) - X(\omega) - qb}{b}, \tag{25}$$

where $\epsilon_{f,x}(x)$ denotes the elasticity of f with respect to x evaluated at x. The term $X(\omega+b) - X(\omega) - qb$ represents the gains from trade when accessing the bond market. The elasticity of the bond market matching function controls how much changes in gains from trade translate into increases in bid-ask spreads (the left-hand side). Since default risk drives changes in the gains from trade, the slope in a regression of bid-ask bond spreads on default risk, which can be measured using yield-to-maturity (YTM), identifies the elasticity of α_b, and similarly for CDS and α_c.

We therefore choose the matching function elasticities for bonds and CDS to match the slopes of the lines in Figure 1. This tight link between bid-ask spreads and risk in the data and model provides a novel and robust way to identify these parameters. We believe this approach to identifying matching elasticities is a significant advance, with the potential to become the standard identification strategy for sovereign default models that incorporate financial markets frictions.

For reasons we will elaborate on later, the relative efficiency of CDS vs bond matching $\bar{\alpha}_c/\bar{\alpha}_b$ is identified by the CDS-bond basis deviations. As $\bar{\alpha}_c$ shrinks else equal, the sorting of investors into bond and CDS markets shifts. Highly- and moderately-exposed investors sort themselves into CDS markets while less-exposed investors go to the bond market. This drives up (down) risk premia in CDS (bond) markets, causing the CDS-bond basis deviation to increase.

The remaining parameters are the measure of investors and the mean and standard deviation of exogenous exposure ω. Since exposure ω is orthogonal to default risk, the first-order effect of increases in the variance of ω is to increase the desire for risk reallocation across investors. This allows the standard deviation σ_ω to be cleanly identified by aggregate CDS volume. Due to risk-sharing, increases in the mean level of exposure μ_ω passthrough both to investors and dealers, and this is reflected (for investors who match in CDS markets) in a higher CDS position for investors and, by market clearing, a lower CDS position for dealers.

The measure of investors I is the last parameter. Note that the total amount of exposure in the economy—how many resources stand to be lost in the case of a default—is the bond supply B plus $I\mu_\omega$. Increasing the measure I of investors changes total risk in the economy due to μ_ω, but also changes how much exposure needs to be allocated per agent, which is $(B + I\mu_\omega)/(I + D)$—at least ignoring the endogenous response of the dealer measure D, which we will find is small. So as I goes infinite, the amount of exposure borne by each agent tends toward zero and with it the amount of debt. Conversely, as I goes to zero, exposure per agent converges tends to $B/D \gg 0$. So the exposure of dealers and bond holdings of dealers identifies the measure of investors.

5 Quantitative results

5.1 Fit of targeted moments

Our identification strategy and estimated fit for the OTC block are summarized in Table 2. The model closely reproduces the targeted moments. The bid-ask spread for CDS is too high on average. We cannot exactly pin down the average level of both bid-ask spreads, in part because of strong theoretical linkages between the bonds and CDS market. The bond position of dealers is small, like in the data, but also is zero. This is a material miss in that whenever dealers hold strictly positive bonds, $q + p - q_f$ must equal zero (the CDS-bond basis holds in prices). We view this as an acceptable miss on account of the position being small in the data and dealers potentially having reasons outside the model to hold a small amount of bonds.[22] However, the fit is quite good overall, and the key matching elasticity identification strategy works as expected.

While the entry costs and elasticities differ a fair bit between bonds and CDS, the largest difference lies in the matching efficiency $\bar{\alpha}_c = 0.01$ vs. $\bar{\alpha}_b = 1$. This makes bonds a far superior matching technology (since $\alpha_c(\cdot) \ll \alpha_b(\cdot)$) but an inferior portfolio technology (since $b \geq 0$ in the benchmark).[23] This creates an interesting sorting pattern that we will discuss in Section 5.4. In our environment, the CDS must be more frictional than bonds to rationalize the large, positive

[22] For instance, investors may wish for dealers who resell bonds to keep some exposure on their books due to a moral hazard problem.

[23] In the corporate bond market, CDS are generally perceived as less frictional than bonds, an assumption that Oehmke and Zawadowski (2015) make from the outset.

Table 2: Targeted moments from the calibration

Moment	Model	Data	Parameter	Value
Bond spread mean (%)	8.15	8.15	Sovereign discount factor (β_g)	0.87
Debt-service to output ratio (%)	5.53	5.53	Default cost level (d_0)	-0.09
Cyclicality of spreads	-0.88	-0.88	Default cost slope (d_1)	0.10
Bid-ask spread for bonds mean (%)	0.04	0.06	Bond entry cost (γ_b)	13.19
Bid-ask spread for CDS mean (%)	0.16	0.10	CDS entry cost (γ_c)	3.33
CDS-bond basis deviation				
Mean (%)	5.97	6.04	CDS match efficiency ($\bar{\alpha}_c$) $\times$ 100	1.00
Aggregate dealer				
CDS volume (%)	0.19	0.19	Investor exposure s.d. (σ_ω)	0.29
CDS position (%)	-0.01	0.00	Investor exposure mean (μ_ω)	-0.03
Net exposure (%)	0.01	0.02	Investor measure (I)	3.67
Bond position (%)	0.00	0.02		
Reg. coef. of YTM on bid-ask bond spreads (%)	0.43	0.42	Bond match elasticity (ξ_b) $\times$ 100	4.12
Reg. coef. of YTM on bid-ask CDS spreads (%)	1.97	2.04	CDS match elasticity (ξ_c) $\times$ 100	5.44

Note: volume and position variables have been deflated by Argentina mean GDP (GDP exhibited little to no growth in this time range); cyclicality of spreads means the correlation between the series and log GDP; the targets for debt-service to output and cyclicality of spreads are from Arellano (2008); the target for the mean bond spread is from Chatterjee and Eyigungor (2012).

CDS-bond basis deviations.

Investor exposure is estimated to be close to zero on average but disperse, with almost 50% of the mass on $\omega < 0$. This heterogeneity provides scope for significant sorting across markets. And in the benchmark this sorting is so extreme that it drives the sovereign bond risk premium *negative*, as will be seen in Table 3.

Another important facet of the calibration is that investors outnumber dealers by several orders of magnitude, with $I = 3.67$ and $D \approx 0.001$.[24] Consequently, dealer entry tends to have small effects, as dealers in aggregate cannot bear much risk (helping match the stylized fact that dealers are intermediaries, not risk-bearers).

5.2 Fit of untargeted moments

The model's fit for some untargeted moments is reported in Table 3. The model reproduces well some typical moments from the sovereign debt literature, such as the volatility of bond spreads and the current account, and qualitatively captures the current account cyclicality.

The correlations between volumes and bond spreads all have the correct sign, though the magnitudes are overstated. As there are forces in the data not captured in the model, it is natural for the model to exaggerate these correlations.

The model has a large standard deviation and maximum for CDS-bond basis deviations, which

[24]Dealer measures are reported in Table C.1.

Table 3: Untargeted moments from the calibration

Moment	Model	Data
Bond spread s.d. (%)	3.81	4.43
Cyclicality of CA/GDP	-0.14	-0.64
CA/GDP s.d./GDP s.d. (%)	0.18	0.22
Bid-ask spread for bonds s.d. (%)	0.02	0.03
Bid-ask spread for CDS s.d. (%)	0.08	0.06
CDS-bond basis deviation		
Std. (%)	3.79	13.58
Min (%)	1.92	-2.37
Max (%)	57.12	70.77
Interdealer mean (%)	5.84	-
Aggregate dealer		
CDS buy volume (%)	0.09	0.10
CDS sell volume (%)	0.10	0.10
CDS position s.d. (%)	0.00	0.00
Time spent in default (%, full sample)	1.79	-
Default probability (%, one-period ahead, ann.)	11.77	-
Risk premium (%)	-3.62	-
Correlation of		
log IDP and basis deviations	0.94	-
log IDP and interdealer basis deviations	0.94	-
log IDP and agg. dealer CDS position	0.91	0.51
log IDP and CDS volume	0.95	0.09
log IDP and CDS buy volume	0.95	0.09
log IDP and CDS sell volume	0.95	0.09
YTM spreads and bond bid-ask spreads	0.99	0.39
YTM spreads and CDS bid-ask spreads	0.99	0.89
Pred. bid-ask bond spreads at YTM target (%)	0.04	0.07
Pred. bid-ask CDS spreads at YTM target (%)	0.16	0.13

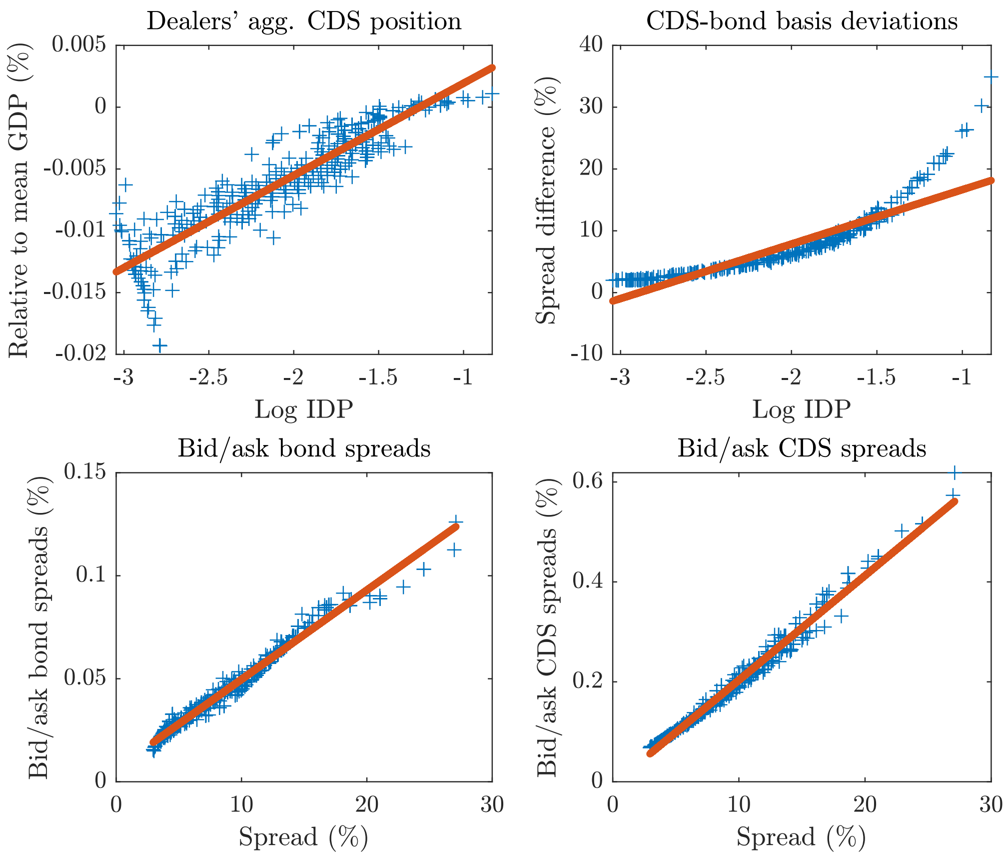
Note: correlations are for CDS position and dealer volume are based on quarterly values; volume and position variables have been deflated by Argentina mean GDP (GDP exhibited little to no growth in this time range); a hyphen indicates a missing value; CA stands for current account, which in the model is $Y - C$; the risk premium is measured as the bond spread minus the one-period ahead default rate; the data moments pertaining to the CA are from Arellano (2008); the bond spread standard deviation is from Chatterjee and Eyigungor (2012).

are similar to those in the data. In fact, the model's CDS-bond basis deviations are increasing in risk (our second stylized fact), which we will show in the next section.

5.3 Reproduction of the stylized facts

The model reproduces the stylized facts given in Section 4.2. This can be seen in Figure 4, and we will begin with the bottom panels. They show that as bond spreads increase, so do CDS and bond bid-ask spreads (Stylized Fact #1). Bid-ask spreads for both bonds and CDS increase because more default generates larger gains from trade and commensurately larger fees.

Figure 4: Model counterparts of the stylized facts



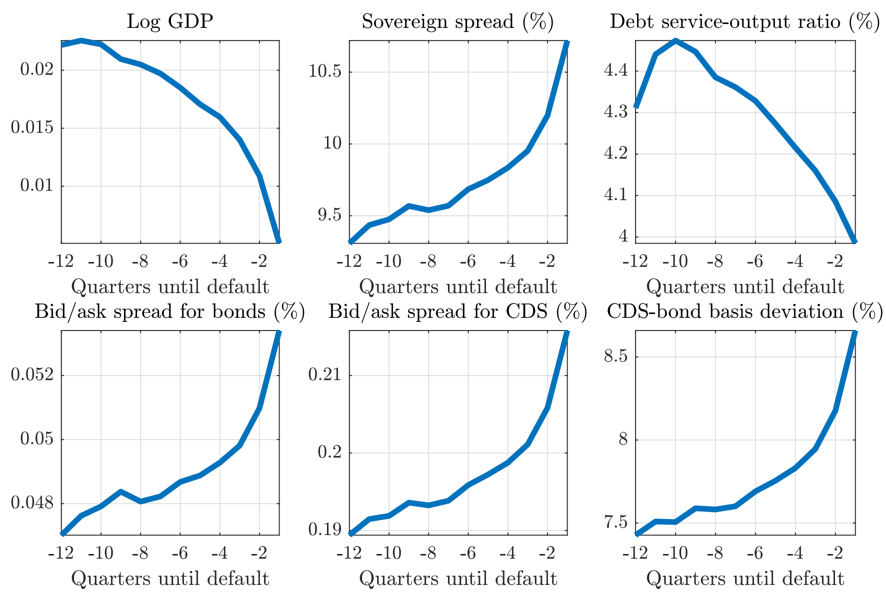
Note: the figure gives a model-based simulation of four key series indicated by the blue plus symbols and a best fit line in orange.

Second, the top left panel shows that as yields and default probabilities increase, dealers tend to remain more or less neutral in terms of protection (part of Stylized Fact #3). This result is partly due to having orders of magnitude more investors than dealers, which limits the ability of dealers to take large positions in the aggregate. Table 3 shows the other component of Stylized Fact #3, which is that CDS volume increases in default risk.

The last stylized fact, that CDS-bond basis deviations tend to be close to zero but blow out in crises (Stylized Fact #2), is evident in the top right panel. As indicated in Propositions 6, increases in default risk amplify whatever the underlying risk premia are, driving up the CDS-bond basis in this case. We will further investigate the mechanism for why this occurs in the next section.

The documented patterns between default risk, bid-ask spreads, and CDS-bond basis breakdowns all show up when considering default events, as is done in Figure 5. As output declines, default risk and spreads increase. With larger default risk, the consumption gap between matched and unmatched investors is larger and gains from trade increase, inducing investors to pay higher intermediation fees to achieve their optimal level of exposure to default risk. Larger intermediation fees are mapped into larger bid-ask spreads in the bond and CDS markets. The increases in risk and reduction in debt both generate a larger deviation of the CDS-bond basis (this will be evident in Figure 6 and Figure C.3 in the appendix). Figure C.4 in the appendix provides a last view on the benchmark model behavior, showing the simulated path for one particular default episode.

Figure 5: Default events

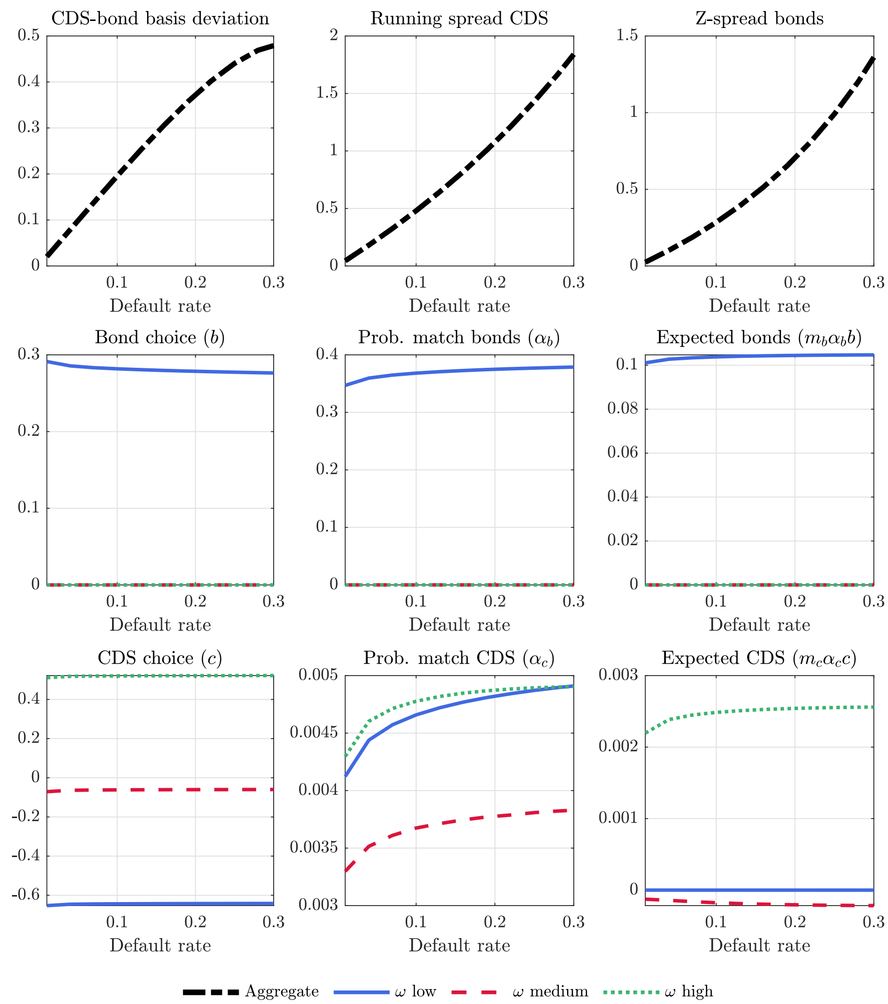


Note: the figure gives the simulation average in the quarters leading up to default.

5.4 Examining the benchmark mechanisms

Figure 6 sheds light on why the model generates these patterns. It plots for a fixed debt supply some key OTC variables as the expected default rate varies. Note that only investors with low (in fact, negative) exogenous exposure ω are substantially active in the bond market (the more exposed investors are allowed to take a small short position). As default risk increases, they reduce their risk by reducing individual bond demand. At the same time, the gains from trade increase, meaning investors are willing to pay higher fees to access the market and consequently match at higher rates. Because of the model's tight connection between fees and bid-ask spreads, this generates an increase in bid-ask spreads (not pictured). It also generates an increase in matching probabilities, which means aggregate demand is decoupled from individual demand.

Figure 6: The OTC block as default rates vary



Note: plotted for a bond supply of 0.03; the bond choice and probability of matching bonds are conditional on selecting the bond market, and similarly for CDS; this graph applies to both the benchmark and the case with liquid sovereign policies but frictional OTC markets since the bond issuance is fixed and default risk is on the horizontal axis.

The dealers' small and stable CDS position is reflected in how investor demand for CDS and bonds hinges on the exogenous exposure level. Dealers, who have zero exogenous exposure, look like the moderate ω investor type, demanding almost no bonds and having little CDS demand. This behavior of dealers is targeted and so must hold in the simulation on average because dealers have a small CDS position on average, which gives buy and sell volume roughly equal—so investors are selling and buying protection from each other.

The figure also reveals why CDS-bond basis tend to increase monotonically in default risk.

Note in the "Expected bonds" and "Expected CDS" panels—which take into account the market choice decision m_b and $m_c := 1 - m_b$—low ω investors go to the bond market while moderate and high ω investors go to the CDS market.

The reason for this sorting is fairly straightforward. Low-ω investors definitely want $b - c > 0$. Since $\alpha_c \ll \alpha_b$, the sure way to obtain that is by buying bonds. Conversely, high-ω investors want $b - c < 0$. Because there is no bond short selling, the only way to achieve that is by searching in the CDS market. But from whom do the high-ω investors buy protection? We already established low ω prefer the bond market. So it is the medium-ω investors—who are relatively indifferent between not matching and matching (as they are already close to their ideal exposure) but want a little more exposure on the margin—brave the low matching probabilities in the CDS market to sell protection.

With this sorting pattern, high- and medium-exposed investors trade CDS, while low-exposed investors trade bonds. Naturally, then, the risk premium is lower in the bond market than the CDS market.

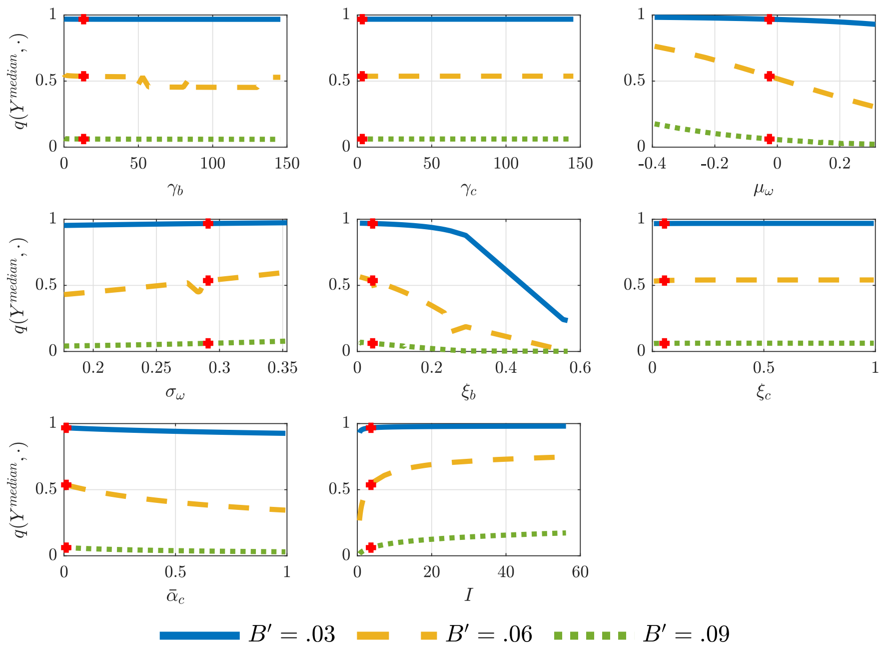
The reason the basis increases in risk, or equivalently decreases as default risk shrinks, is fairly straightforward as well. As default risk goes to zero, investors become increasingly homogeneous—they still differ by ω, but the differences in expected losses $\bar{\delta}\omega$ go to zero. The sorting still occurs, but with less at stake, risk premium for both bonds and CDS converge to each other, implying the basis goes towards zero.

5.5 The role of trading frictions and exposure

We begin our analysis by using comparative statics to examine how the price of bonds, q, is affected by trading frictions, the distribution of risk and the measure of investors. This is done in Figure 7 which shows how the price responds to changes in the parameter for the median level of endowment, Y, and three different values for debt issued, B'. As a reference, the red dots show the benchmark parameter values. The parameters governing the distribution of exposure and measure of investors ($\mu_\omega, \sigma_\omega, I$) significantly move bond prices. Of note, more dispersion σ_ω drives up bond prices, reflecting the sorting that goes in the sovereign's favor. Similarly, more investors I means more agents to bear risk, increasing q. Finally, increases in the average exogenous exposure, μ_ω, reduce the price of bonds as more aggregate risk is borne by investors and dealers.

The trading frictions ($\gamma_b, \gamma_c, \xi_b, \xi_c, \bar{\alpha}_c$) have disparate effects. At the benchmark, $\bar{\alpha}_c$ is close to 0, which makes γ_c and ξ_c not very important for bond pricing. But, $\bar{\alpha}_c$ has a large negative effect on bond prices, as it reduces sorting. In contrast, ξ_b sharply decreases bond prices by attracting more investors (with larger ω) to participate in bonds. The entry cost for bonds, γ_b, has a noticeable but nonlinear effect. Overall, trading frictions and the distribution of risk significantly affect sovereign bond pricing.

Figure 7: Comparative statics: Response of q at median GDP and select debt levels



Note: The figure shows the price schedule q for various B' values and Y^{median} the midpoint of the discretized Y process; the red plus signs indicate the benchmark parameter values; values only plotted if the market clearing error was sufficiently small; this figure is constructed using a lower precision solution to reduce run-times, see the appendix for details.

6 Counterfactuals

In this section we analyze a series of counterfactuals. We first assess the quantitative importance of trading frictions on bond prices and the response of the government by comparing our baseline model to an alternative version of the model in which bonds and CDS are liquid and can be traded in competitive markets—that is, when dealers face zero entry cost as discussed in Proposition 1. We then study the equilibrium responses to policy changes that modify the constraints of trade in CDS and bonds. We consider the following policies: allowing bond shorting; eliminating trading in CDS; and banning naked CDS. And finally, we consider the welfare gains or losses associated with these policies for the sovereign government and investors.

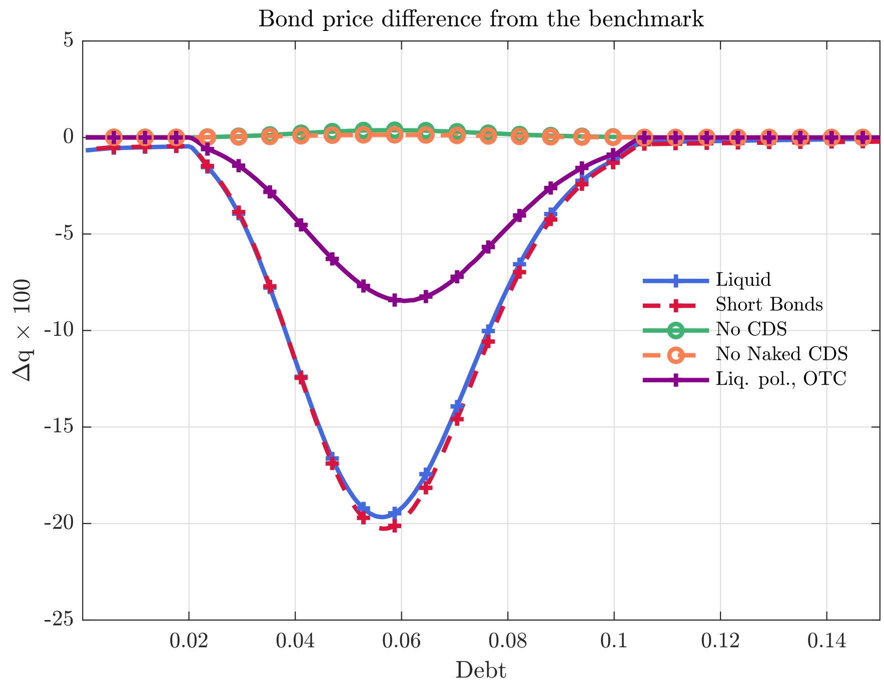
Table 4 reports key simulation statistics for the various cases. These simulations combine the effects on prices and the optimal debt issuance response. We can isolate the effects of the changes at the same debt issuance by looking at the price schedule, which we do by exampling $q(Y^{median}, B')$ for differing levels of B' and the median GDP level. These, plotted as differences from the benchmark price schedule, are displayed in Figure 8. Even these changes have multiple effects, including not only direct effects on investors but feedback from default decisions into prices. We decompose these forces in Sections 6.1–6.3.

Table 4: Model comparison

Statistic	(1) Benchmark	(2) No Naked CDS	(3) No CDS	(4) Short Bonds	(5) Liquid	(6) Liq. pol., OTC
Bond spread mean (%)	8.15	8.12	8.12	9.03	9.04	5.04
Bond spread s.d. (%)	3.81	3.78	3.76	4.88	5.15	1.51
Debt-service to output ratio (%)	5.53	5.54	5.54	4.28	4.26	4.26
Cyclicality of spreads	-0.88	-0.88	-0.87	-0.79	-0.79	-0.70
Cyclicality of CA/GDP	-0.14	-0.14	-0.14	-0.11	-0.11	-0.11
CA/GDP s.d./GDP s.d. (%)	0.18	0.18	0.18	0.16	0.15	0.15
Bid-ask spread for bonds mean (%)	0.04	0.04	0.04	0.11	-	0.03
Bid-ask spread for bonds s.d. (%)	0.02	0.02	0.01	0.06	-	0.05
Bid-ask spread for CDS mean (%)	0.16	0.22	-	0.01	-	0.11
Bid-ask spread for CDS s.d. (%)	0.08	0.12	-	0.00	-	0.04
CDS-bond basis deviation						
Mean (%)	5.97	4.49	-	-0.83	-	4.22
Std. (%)	3.79	2.41	-	0.83	-	2.55
Min (%)	1.92	1.69	-	-37.52	-	1.93
Max (%)	57.12	34.20	-	-0.25	-	57.95
Interdealer mean (%)	5.84	4.29	-	-0.79	-	4.13
Aggregate dealer						
CDS volume (%)	0.19	0.13	-	0.00	-	0.21
CDS position (%)	-0.01	0.00	-	0.00	-	-0.01
Net exposure (%)	0.01	0.00	0.00	0.00	-	0.01
Bond position (%)	0.00	0.00	0.00	0.00	-	0.00
CDS buy volume (%)	0.09	0.06	-	0.00	-	0.10
CDS sell volume (%)	0.10	0.07	-	0.00	-	0.11
CDS position s.d. (%)	0.00	0.00	-	0.00	-	0.00
Time spent in default (%, full sample)	1.79	1.79	1.79	1.37	1.35	1.35
Default probability (%, one-period ahead, ann.)	11.77	11.76	11.77	7.64	-	7.61
Risk premium (%)	-3.62	-3.64	-3.65	1.39	-	-2.57
Correlation of						
log IDP and basis deviations	0.94	0.96	-	-0.74	-	0.89
log IDP and interdealer basis deviations	0.94	0.96	-	-0.73	-	0.89
log IDP and agg. dealer CDS position	0.91	0.87	-	-0.99	-	0.81
log IDP and CDS volume	0.95	0.92	-	-0.91	-	0.87
log IDP and CDS buy volume	0.95	0.91	-	-0.92	-	0.87
log IDP and CDS sell volume	0.95	0.93	-	-0.90	-	0.88
YTM spreads and bond bid-ask spreads	0.99	0.99	0.98	1.00	-	0.51
YTM spreads and CDS bid-ask spreads	0.99	0.98	-	0.98	-	0.97
Sovereign welfare gain (CEV, bps)	-	0.07	0.16	-5.36	-5.27	-
Investor welfare gain (agg., money metric, bps)	-	-0.04	-0.81	47.75	86.49	-

Note: all CDS position and volume measures are deflated by mean GDP; s.d. stands for standard deviation; agg. stands for aggregate; IDP stands for implied default probability; YTM stands for yield to maturity; welfare gains are based on ex-ante utility $\mathbb{E}_Y[W(Y,0)]$ and $\mathbb{E}_{\omega,\bar{\delta},B'}[V(\omega;\bar{\delta},B')]$ relative to the benchmark; CEV stands for consumption equivalent variation; the "Liq. pol., OTC" case uses the policy functions from the liquid version of the model with benchmark OTC frictions; welfare for the "Liq. pol., OTC" case is not reported because prices and policies are inconsistent.

Figure 8: Price schedule differences under alternative policies



Note: Each line represents $q(Y^{median}, B')$ for various B' values and Y^{median} the midpoint of the discretized Y process minus the benchmark model's value; positive values indicate better pricing for the sovereign.

6.1 Comparing the liquid and frictional markets

We assess the quantitative importance of frictional markets by taking the limit as entry costs γ_b, γ_c go to zero, recovering a version of the Arellano (2008) model. The results for a few key variables are displayed in Table 4 in columns (1) and (5). OTC frictions have two types of effects on bond prices. There is a direct effect, which changes bond prices as investor and dealer demand changes, and an indirect effect, which is how q changes as the sovereign reoptimizes default and debt issuance. To delineate these, we consider an intermediate case that uses the sovereign policies from the liquid model with pricing from the benchmark model. The results for this experiment are displayed in the column (6). The direct effect is captured in moving from (5) to (6), while the indirect effect is found in moving from (6) to the benchmark (1).

The direct effect of frictional trading is a positive impact on prices, reflected in the 4pp decrease in average bond spreads from 9.04% to 5.04% (Table 4). Looking beyond averages and to specific debt levels in Figure 8, the direct effect (which is the negative of the "Liq. pol., OTC" curve) improves prices for most to all debt issuance levels.

Why is the direct effect of trading frictions to improve bond prices? Frictions change who the marginal investor is. As discussed in Section 5.4, only negatively-exposed investors are willing to purchase bonds in the benchmark (Figure 6), while moderately- or positively-exposed investors trade in the CDS market. Section 5.5 builds on this, using Figure 7 to show a lower probability of accessing the CDS market ($\bar{\alpha}_c \downarrow$) amplifies this sorting and thereby improves bond prices. This is why the frictional model generates a negative risk premium, as column (6) of Table 4 reports, improving bond prices relative to the zero-risk-premium liquid case.

The indirect effect of frictional trading has some nuance. Consider first the case of fixed B', in which case the direct effect is the large positive gap between line "Liq. pol. OTC" minus "Liquid" in Figure 8. In this and the other experiments, whenever the direct effect is positive, the indirect effect is as well conditional on debt issuance: A positive direct effect boosts W^r, reducing default risk, $\bar{\delta}$, and improving q. (And similarly in reverse when the direct effect is negative.)

The indirect effect along the simulated path, summarized in moving from column (6) to (1) of Table 4, combines this effect for fixed B' with the endogenous response of debt issuance $B'(Y, B)$. In response to lower default rates and lower bond spreads, the sovereign borrows more. This increases the average cost of debt service in the simulation and brings the average spreads closer to the liquid case. This behavior is driven by the sovereign's Euler equation. Loosely speaking, the sovereign will always borrow up to the point where the cost of borrowing more is equal to its discount factor.[25] When prices improve, this incentivizes borrowing, which drives the price back down. When prices worsen, the reverse happens. This means that, whatever a policy does to q for fixed B', the sovereign will tend to undo in the simulation. And this is why the spreads in columns (1) to (5) of Table 4 are similar, even though the debt service can vary substantially. (In column (6), policies are not optimal and so the Euler equation does not hold.)

6.2 The role of bond shorting

Figure 8 shows that allowing investors and dealers to take negative bond positions (i.e., to short sovereign bonds) has a large adverse impact on bond prices. This is consistent with the classic Walrasian perspective that allowing bond-shorting should always hurt bond prices. The reason for this is that bond demand must equal bond supply. Allowing agents to take short positions can increase bond supply, coming from those shorting it, at any given price. This increase in bond supply will drive down bond prices. We label this canonical Walrasian force as the *demand effect*.

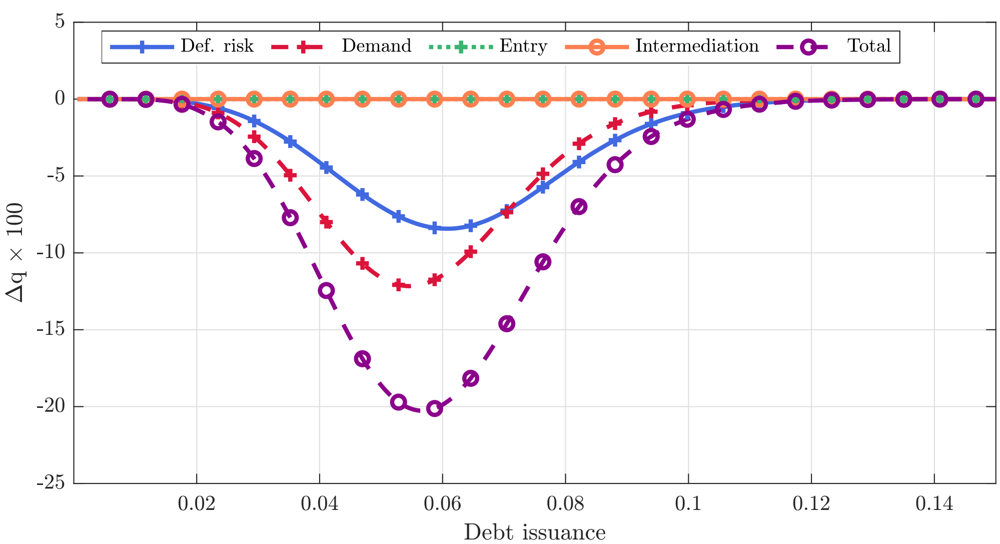
In addition to the demand effect, there are three other forces that can affect bond prices in our model: an *entry effect*, an *intermediation effect*, and a *default risk effect*. The entry effect is caused by the change in the number of dealers in the market. The intermediation effect is caused by the change in dealers' profits (when bond shorting is allowed, dealers may earn higher profits because they can take advantage of high bond prices). The default risk effect is caused by the change in the

[25] If we assume differentiability and that consumption growth is almost zero or that the sovereign is risk-neutral, the Euler equation is approximately $q/(1-\bar{\delta}') + \frac{\partial q}{\partial B'} B'/(1-\bar{\delta}') = \beta$. From this, the sovereign will borrow until either risk-premia (reflected in $q/(1-\bar{\delta}'))$ dissuade him or prices move sufficiently against him ($\partial q/\partial B'$ small enough).

sovereign's default risk in response to bond pricing.

We quantify these effects by allowing only one channel to operate at a time. We first maintain the benchmark policy environment but change $\mathbb{E}_{Y'|Y}[\delta(Y', B')]$ to its value in the new policy environment (bond shorting in this case). The incremental change, from $q_0(Y, B')$ to $q^{def}(Y, B')$ is our default risk effect. We then hold dealer profit, π, and the measure of dealers, D, fixed but otherwise solve for a "general equilibrium" (with misspecified dealer profit and measure of dealers) allowing dealer and investor behavior to change. The incremental change in bond prices, $q^{demand}(Y, B') - q^{def}(Y, B')$, is our demand effect. We then allow the measure of dealers, D, to change and recompute equilibrium (with misspecified dealer profit π). Our entry effect is the change in bond prices, $q^{entry}(Y, B') - q^{demand}(Y, B')$. Last, we allow dealer profit, π, to change, resulting in the new equilibrium prices $q_1(Y, B')$. The difference $q_1(Y, B') - q^{entry}(Y, B')$ is our intermediation effect. By construction, the individual effects sum to the total effect.

Figure 9: Bond-shorting decomposition



Note: Each line represents a decomposition of $q(Y^{median}, B')$ for various B' values and Y^{median} the midpoint of the discretized Y process; positive values indicate better pricing for the sovereign.

The decomposition in Figure 9 reveals a large negative demand effect, consistent with the Walrasian predictions. The magnitude of the response, however, lies in the large disparity between the matching efficiency parameters $\bar{\alpha}_b$ and $\bar{\alpha}_c$. To rationalize the data, $\bar{\alpha}_c$ is two orders of magnitude smaller than $\bar{\alpha}_b$. Although trading bonds is significantly easier in the benchmark calibration, for investors with high ω, bonds are not a great substitute for CDS. CDS can be used to reduce total exposure $b - c + \omega$, which is impossible in the bond market in the benchmark. Once bond-shorting is allowed, investors who wanted to reduce exogenous exposure, but find it difficult to do in the CDS market due to low $\bar{\alpha}_c$, now find it attractive to trade bonds. This causes a shift of high-ω investors from the CDS market to the bond market, making the bond risk premium positive and driving the CDS-bond basis deviation negative, as alluded to in Table 4.

The large demand effect is amplified by a large default risk effect. Since the combined direct effects push prices lower, that drives down W^r and increases default rates, further depressing prices. In contrast, both the entry and intermediation effect are essentially zero. Since bond-shorting does not directly change the span of dealers' portfolios, bond-shorting has no direct effect on π (but could still alter it through general equilibrium forces). So it is natural for the intermediation effect (the effect of π changes) to be close to zero.

That the entry effect is close to zero is a numerical result. Bond shorting does change trading behavior, submarket tightnesses, and equilibrium fees, which implies the measure of dealers changes. However, the data demand that the measure of dealers be small relative to the measure of investors (contrast Table 2 and Table C.1 in the appendix); and when dealers enter they do not bring any exogenous exposure ω with them nor do they generally have much demand for endogenous exposure, as dictated by the calibration. Consequently, the entry effect is small, which will also be true in the other counterfactuals.

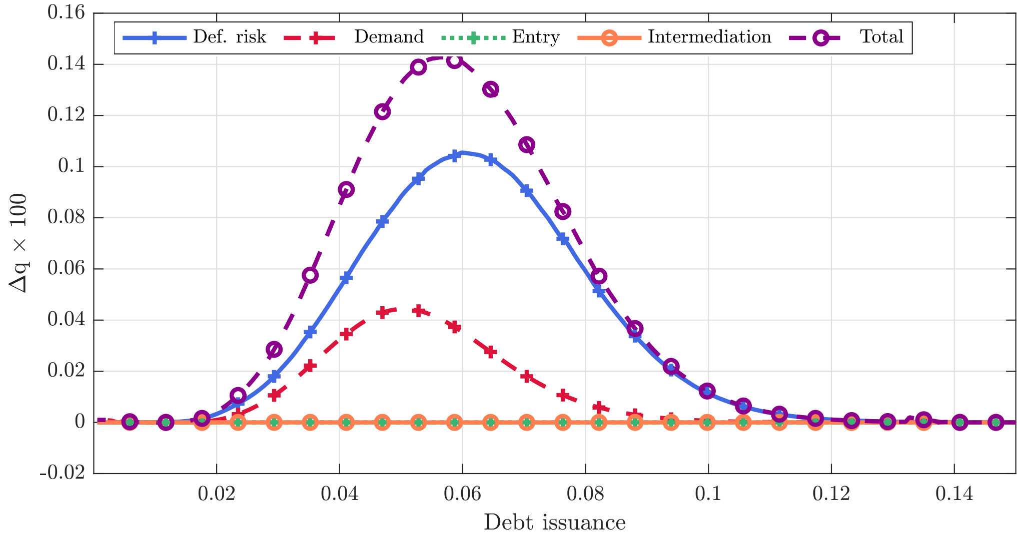
6.3 A naked CDS ban

We now turn our attention to the CDS market. Motivated by the regulatory change implemented in the European Union in 2011-2012, we use the model to investigate the consequences of a naked CDS ban—defined as a ban on the purchase of CDS protection in excess of the amount of bond exposure. Formally, we take $\underline{x}(\omega) = -\theta_\omega \max\{0, \omega\}$ and set $\theta_\omega = 1/2$. This formulation allows investors and dealers to be have $b - c < 0$ (negative endogenous exposure) as long as they have some exogenous exposure ω to hedge against. This captures the actual regulation, because the naked CDS ban allows CDS protection against "assets or liabilities [that are not necessarily sovereign bonds] whose value is correlated to the sovereign debt" (European Commission, 2011, p. 7). Appendix C.6 gives robustness to θ_ω values, varying it from 0 to 1. As we found the results and rationale from closing the CDS market were similar to a naked CDS ban, we have relegated those findings to appendix C.5.

The benchmark calibration predicts there is almost zero response in terms of prices from this policy (Figure 10), but this is not a generic result. We show in the appendix Section C.7 that a naked CDS ban can significantly improve, worsen, or leave prices unchanged, depending on parameters.

To understand why the total effect is small here, we apply our decomposition strategy to create Figure 10. The demand effect again is positive. From theory, we expect the demand effect to be positive for the following reason. If we think about a unified asset that is just endogenous exposure (i.e., $b - c$ but excluding ω), then the total supply of exposure is the bond plus the supply synthetic bonds. By eliminating the ability to have negative endogenous exposure, the total supply of exposure decreases which should drive up the exposure price, q. The model captures that.

Figure 10: Naked-CDS ban decomposition



Note: Each line represents a decomposition of $q(Y^{median}, B')$ for various B' values and Y^{median} the midpoint of the discretized Y process; positive values indicate better pricing for the sovereign.

Like with all the counterfactuals, the entry effect is small and the default risk effect moves in line with the total effect for the same reasons as before. One should generally expect the intermediation effect to be negative because the portfolio choice of the dealers is being restricted, which lowers π else equal and raises net entry costs else equal. However, it is zero here for an interesting reason. The CDS-bond basis deviation is positive as targeted in the calibration (Table 4), which indicates bonds are expensive relative to synthetic bonds ($c < 0$). As a result, dealers have incentives to obtain the exposure they want by selling CDS protection to the very exposed investors rather than obtain exposure by buying bonds.[26] Because selling CDS protection is allowed in a naked CDS ban, dealers positions' are not directly affected by the ban.

Although the effects of a naked CDS ban are quantitatively small in our benchmark calibration, it is worth noticing that the bulk of the effects of the ban come from the demand and the default risk effects. While the two models are not perfectly comparable, we find that these two effects are absent in Sambalaibat (2022) because in her model assets are indivisible, investors cannot adjust at the margin their trading probability, and default risk is exogenous. In particular, these effects are significantly larger than the extensive margin intermediation effect of bringing more dealers into the market and affecting matching probabilities, which is similar to Sambalaibat's entry effect.

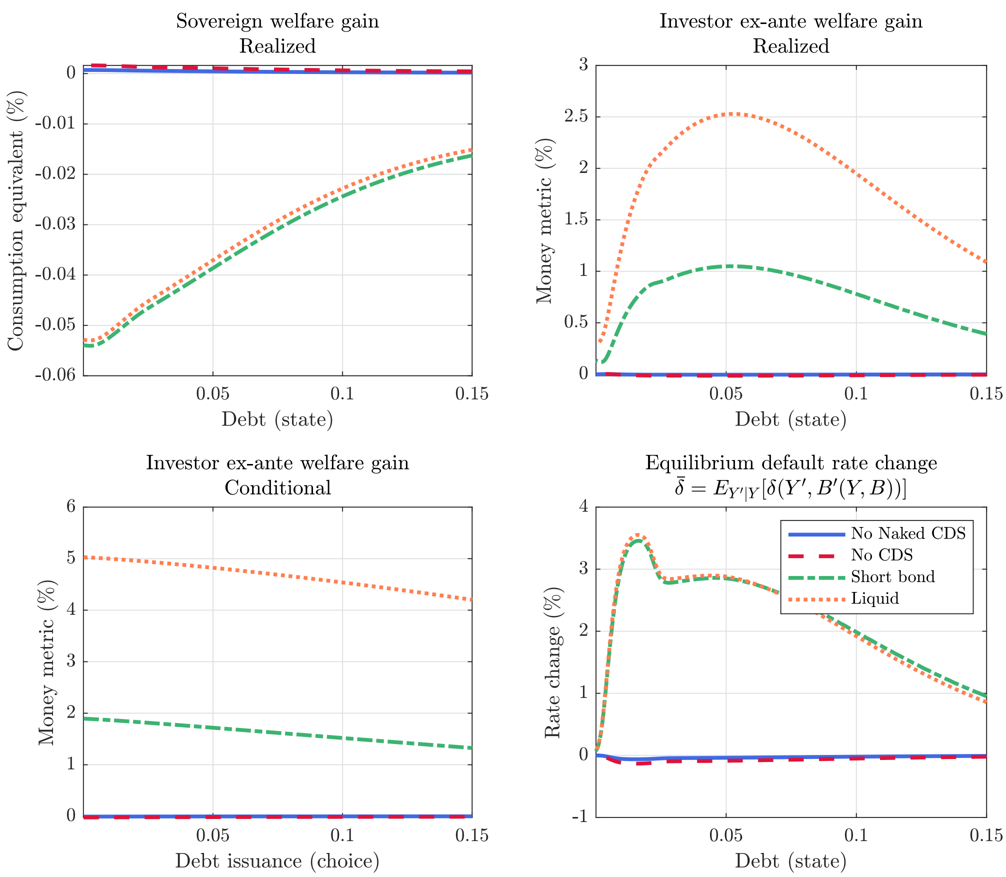
6.4 Welfare

In this section, we consider the welfare impact of each of the counterfactual policies.

[26]This behavior is perhaps most clearly seen in Table C.1 in the appendix.

We measure welfare for the sovereign using the standard consumption equivalent variation (CEV) measure. This measure scales the consumption policy in the benchmark up or down until the value associated with that policy equals the value under the new regime. Dealers' welfare is always zero by free entry. To measure welfare for investors, we use a monetary metric. Specifically, given indirect utility U_1 from a new regime and U_0 from the benchmark, the welfare measure is simply $U_1 - U_0$ times the measure of investors I. Because of quasi-linear utility, this gain is effectively measured in terms of the consumption good. Additionally, since GDP is close to one, the monetary metric gain can be thought of as a share of GDP. Similarly, because GDP is close to one and the sovereign's consumption roughly equals GDP, the CEV measure is also in terms of a share of GDP (approximately). Therefore, the measures are roughly comparable.

Figure 11: Welfare analysis



Note: The top panels and the bottom right panel are functions of (Y, B) and have been averaged using the invariant distribution of Y; "Realized" indicates the usual general equilibrium welfare measures where the sovereign's bond policies and default rates are allowed to vary; the panel labeled "Conditional" is a partial equilibrium concept that holds $(B', \bar{\delta})$ fixed but takes a numerical average across the $\bar{\delta}$ grid.

Figure 11 depicts the changes in welfare relative to the benchmark for different levels of debt issuance. Consistent with the previous findings that the ban on naked CDS and eliminating the CDS market had little effect on prices, we find virtually no impact of such policies on welfare.

Figure C.6 in the appendix plots only those two policies, revealing that the sovereign benefits from them while investors end up worse off, though both are very small changes in welfare.

We observe, however, that introducing sovereign bond shorting or perfect liquidity has a significant welfare effect on investors and sovereign governments. In both cases the sovereign government welfare decreases while investors welfare increases. The sovereign loses about 2 to 5 bps of GDP (top left panel), while investors gain around 50 to 100 bps of GDP in the short bond case, and 50 to 250 bps of GDP in the liquid case (top right panel). How much investors gain is actually a general equilibrium effect also reflecting higher default rates (bottom right panel). In partial equilibrium with the default rate fixed, welfare gain for investors is even higher, around 200 bps in the short bond and 500 bps in the liquid case (bottom left panel). These gains are counterbalanced by a higher default rate of the sovereign in response to lower bond prices.

7 Conclusions

The market for sovereign debt CDS is relatively new and not well understood. At the same time, policy is being implemented regulating it. To understand the role of CDS, it is essential to incorporate and understand issues of liquidity, risk-sharing, violations of arbitrage, and relationships between primary and secondary markets. In this paper, we have proposed a model that addresses many of these issues in an attempt to understand and quantify the market as it is and as it could be. We characterized how trading and matching frictions interact with policy to determine sovereign bond prices, investor activity in markets, and the CDS-bond basis. Closed-form solutions in a special case allowed clear insights into the model mechanisms, including the effects of increased default risk, debt supply, entry costs, matching efficiencies, and matching elasticities.

Turning to the data, we established stylized facts, showing bid-ask spreads, the CDS-bond basis, and dealer intermediation increase in default risk. We then showed how to identify the model's trading frictions, and show the model reproduces the stylized facts and other features of the data. We analyzed the benchmark model's mechanisms, finding a key sorting pattern induced by frictional CDS matching: Highly- and moderatly-exposed investors traded with each other in CDS markets, while lowly-exposed investors bought bonds. We then investigated the role of trading frictions and exposure distribution, finding large effects on sovereign debt pricing.

In the counterfactuals where we eliminate CDS or implement a naked CDS ban, we find few spillover effects on the Argentinean bond price. In contrast, in the counterfactuals where we make markets perfectly liquid or allow bond shorting, bond prices significantly fall, imposing a tighter financial constraint on the government and reducing its welfare. As highlighted in the paper, these conclusions crucially depend on the identified OTC frictions and might not extend to other contexts. While there is much more to study and understand in these markets, our data-disciplined model provides novel insights into existing policy proposals, both for those that have been enacted and those that may be in the future.

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Online appendix for Sovereign Debt and Credit Default Swaps*

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A Data and measurement

We construct a database containing information about bond and CDS prices, bid-ask spreads for bonds and CDS, CDS and bond holdings of dealers, and standard aggregate macro variables for Argentina. In this appendix, we describe the data sources and some additional details.

A.1 Data sources

We obtain a given bank's CDS position on a sovereign's CDS via regulatory data from the Depository Trust and Clearing House Corporation (DTCC). The Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) requires real-time reporting of all swap contracts to a registered swap data repository (SDR), which the DTCC operates in the CDS market. The Dodd-Frank Act also requires SDRs to make all reported data available to appropriate prudential regulators.[1] As a prudential regulator, the Federal Reserve has access to the transactions and positions involving individual parties, counter-parties, or reference entities that are regulated by the Federal Reserve.

The DTCC data contains every US-regulated CDS trade. We drop any trades where the reference entity is not a country's government. This means that in addition to dropping any CDS trades where the reference entity is a non-governmental organization, we also remove CDS trades with city or state level reference entities.

In addition to CDS positions, we calculate the quarter-end net sovereign bond exposure (CDS and bond holdings) of U.S.-headquartered banks classified as dealers in the DTCC database. We obtain this information from the banks' FR Y-14Q regulatory filings as part of the Federal Reserve's Capital Assessments and Stress Testing information collection.

Our definition of dealers in the data consist of CDS dealers as classified by the DTCC and banks for whom we have data in Y14Q. Ultimately, this gives us a list of five banks. In our sample, every dealer actively trades CDS in every quarter.

We obtain bond prices and CDS prices from Bloomberg.

A.2 CDS positions

We create a CDS position for each bank at each point in time. We observe the initial position for each bank as of January 1st, 2010. Every time a bank buys (sells) protection in the entity during

*The views expressed are those of the authors and not necessarily those of the Federal Reserve Bank of Richmond or the Board of Governors.

[1]See Sections 727 and 728 of The Dodd-Frank Wall Street Reform and Consumer Protection Act.

the month, its position increases (decreases). We also subtract any expiring contracts from dealer's position in that reference entity. Thus, we assume a dealer d has CDS position at time t relative to the end of the previous period $(t-1)$ as follows:

$$Position_{dt} = Position_{d(t-1)} + CDSBought_{dt} - CDSSold_{dt} - ExpiredContracts_{dt}.$$

After calculating the position for each bank, we aggregate the positions of dealers. The volume measures likewise reflect aggregate volumes.

A.3 Bond and exposure position of dealers

In Y14Q, we have the notional quarter-end Sovereign Debt Securities and CDS net exposure to Argentinean sovereign debt as reported on the Securities Main and Hedging schedule and Trading Sovereign schedule. Combining this bonds-less-net-CDS protection information with the quarter-end net position on CDS we get from DTCC allows us to infer the quarter-end bond position of dealers.

A.4 CDS and bond prices

CDS and bond price data was collected from Bloomberg by downloading data for generic 5-year CDS and bond. The 5 year CDS was chosen because it is the most commonly traded CDS contract. We also collected generic 5-year bond yield data from Bloomberg.

A.5 Running, Z-spread, and CDS-bond basis deviation measurement

To measure the CDS-bond basis deviations, we use the Z-spread approach, consistent with our data that comes from Gilchrist et al. (2022).[2] This compares the CDS running spread with the usual spread from sovereign debt. We now describe the measurement of these in the model and the data.

The running spread is the endogenous coupon s^{cds} amount paid at predetermined intervals such that—assuming a constant Poisson arrival rate λ for default and some recovery rate in the case of default—the net present value of the CDS contract is zero. In the model, default intensity λ is given implicitly by the solution to

$$\underbrace{\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)/4})}_{\text{"Floating leg" value } \int_0^{1/4} e^{-\rho\tau}\lambda e^{-\lambda\tau} d\tau} = \underbrace{F}_{\text{"Fixed leg" value}}$$

where $e^{-\rho} = (1+r^*)^{-4}$ gives the discount rate ρ and F is the upfront payment per unit of notional. (The IDP associated with λ is $1-e^{-\lambda}$.) The running spread is then the s that solves

$$\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)/4}) = \underbrace{se^{-(\rho+\lambda)/4}}_{\text{Expected coupon value}}$$

$$\Leftrightarrow s = \frac{\lambda}{\rho+\lambda}(e^{(\rho+\lambda)/4}-1)$$

The annualized spread is $s^{cds} = 4s$. Small default rates and discount rates imply $s^{cds} \approx 1-(1-\mathbb{E}[d])^4$, i.e., the running spread is approximately the default rate. However, it should be kept in

[2]We thank the authors for providing us with their CDS-bond basis deviations data.

mind this is an approximation. In particular, as λ goes large, s^{cds} can become very large. We can consider this map, from a fixed up front payment F to s^{cds}, as a function $\mathcal{R}$. In the model, the total upfront payment an investor makes per unit of notional is $(pc+f_c)/c$, so the implied running spread is $\mathcal{R}(p+f_c/c)$. When we report either the IDP or the running spread, these use volume-weighted averages of $(p+f_c/c)$ as the upfront payment (unless we indicate inter-dealer, in which case we use p). That is, we use $\mathcal{R}(\mathbb{E}[\tilde{p}])$ for the running spread and similarly for the IDP.

The Z-spread is the usual spread in sovereign debt. Specifically, it is the constant Z such that the net present value of the bond, discounted by $1+r^*+Z$, is zero. Annualized then, this spread is

$$s^{bond} = (1/q)^4 - (1+r^*)^4.$$

For small default rates, $s^{bond} \approx (1+r^*)^4\mathbb{E}[d]^4$. We can consider this map from bond prices to a spread as a function $\mathcal{Z}$. In the model, the effective price an investor pays per unit of notional is $(qb+f_b)/b$, so the implied Z-spread is $\mathcal{Z}(q+f_b/b)$. When we report the sovereign spread, we use volume-weighted averages of $(q+f_b/b)$ as the effective price (unless we indicate inter-dealer, in which case we use q). That is, we use $\mathcal{Z}(\mathbb{E}[\tilde{q}])$.

The CDS bond basis deviation is defined as

$$s^{cds} - s^{bond}.$$

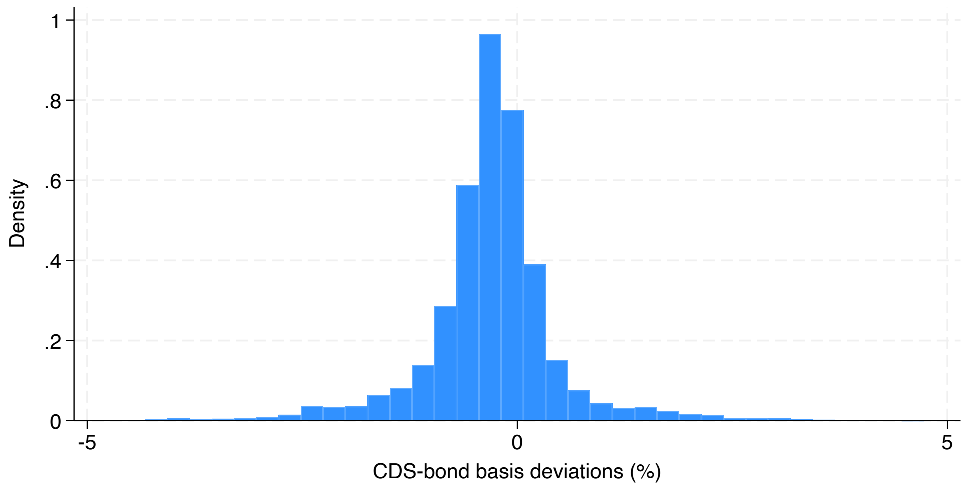
Consequently, for small default rates and small risk-free rates, the deviations should be roughly $4r^*$ times the annual default probability, or just a few basis points. In our model, the deviations do not occur simply because the approximations fail to hold but also because the fees investors pay in CDS and bond markets break the no-arbitrage relationship.

A.5.1 CDS-bond basis deviations in the data

Deviations from the CDS-bond basis in the data are of different magnitudes and signs. Taking CDS-bond basis deviations constructed by Gilchrist et al. (2022) (and then averaged within month, country pairs), Figure A.1 shows the distribution of these deviations in a pooled sample that includes 59 countries and 7,849 total month-country observations from 2001 to 2019. A substantial number of observations have positive CDS-bond basis deviations, although Argentina's is the largest in the sample. Figure 7 in their paper also shows that substantial upwards deviations in the CDS-bond basis are not exceptional in times of stress, with "investment-grade countries" experiencing large positive deviations from 2009 to 2014 and "speculative-grade countries" experiencing them from 2003 to 2004 and from 2013 to 2014. Gilchrist et al. Table 7 also shows that shocks to a measure of risk premia (the "excess bond premium") or the global financial crisis shock drive up or have no statistical impact on the CDS-bond basis deviations.[3]

[3] There is one exception: The response to the excess bond premium shock after 6 months for investment-grade countries has a slight negative impact at 90% confidence levels.

Figure A.1: CDS-bond basis deviations in a cross-section of countries



Note: Data are from Gilchrist, Wei, Yue, and Zakrajsek (2022), averaged over observations within each month; values above 5% in absolute value have been excluded for readability; 2020 and on excluded.

A.6 Bid-ask spread measurement

To measure bid-ask spreads in the model, we use volume-weighted measures. Let the volumes be denoted by

$$V^{B,buy} = I\int_{b(\omega)>0} m_b(\omega)\alpha^b(\theta^b(\omega))|b(\omega)|d\mu(\omega)$$

$$V^{B,sell} = I\int_{b(\omega)<0} m_b(\omega)\alpha^b(\theta^b(\omega))|b(\omega)|d\mu(\omega)$$

$$V^{C,buy} = I\int_{c(\omega)>0} m_c(\omega)\alpha^c(\theta^c(\omega))|c(\omega)|d\mu(\omega)$$

$$V^{C,sell} = I\int_{c(\omega)<0} m_c(\omega)\alpha^c(\theta^c(\omega))|c(\omega)|d\mu(\omega).$$

In the benchmark, $V^{B,buy}$ is the total bonds purchased by investors and $V^{B,sell} = 0$.

Define volume-weighted average transacted inclusive of fees analogously as

$$\tilde{q}^{buy} = I\int_{b(\omega)>0} m_b(\omega)\alpha^b(\theta^b(\omega))|b(\omega)|(qb(\omega)+f^b(\omega))/b(\omega)d\mu(\omega)\ /\ V^{B,buy}$$

$$\tilde{q}^{sell} = I\int_{b(\omega)<0} m_b(\omega)\alpha^b(\theta^b(\omega))|b(\omega)|(qb(\omega)+f^b(\omega))/b(\omega)d\mu(\omega)\ /\ V^{B,sell}$$

$$\tilde{p}^{buy} = I\int_{c(\omega)>0} m_c(\omega)\alpha^c(\theta^c(\omega))|c(\omega)|(pc(\omega)+f^c(\omega))/c(\omega)d\mu(\omega)\ /\ V^{C,buy}$$

$$\tilde{p}^{sell} = I\int_{c(\omega)<0} m_c(\omega)\alpha^c(\theta^c(\omega))|c(\omega)|(pc(\omega)+f^c(\omega))/c(\omega)d\mu(\omega)\ /\ V^{C,sell}.$$

Let $\mathcal{Z}$ denote the conversion from a bond price to a Z-spread. Let $\mathcal{R}$ denote the conversion from a CDS price to a running spread. Bid-ask spreads are defined as

$$bidask_b = \frac{V^{B,buy}|\mathcal{Z}(\tilde{q}^{buy}) - \mathcal{Z}(q)| + V^{B,sell}|\mathcal{Z}(\tilde{q}^{sell}) - \mathcal{Z}(q)|}{V^{B,buy} + V^{B,sell}}$$

$$bidask_c = \frac{V^{C,buy}|\mathcal{R}(\tilde{p}^{buy}) - \mathcal{R}(p)| + V^{C,sell}|\mathcal{R}(\tilde{p}^{sell}) - \mathcal{R}(p)|}{V^{C,buy} + V^{C,sell}},$$

respectively, whenever well-defined. When not well-defined because $V^{i,j} = 0$ for some $i \in \{B, C\}, j \in \{buy, sell\}$, we drop that term. For instance, when bond-shorting is not allowed (like in the benchmark) $bidask_b = |\mathcal{Z}(\tilde{q}^{buy}) - \mathcal{Z}(q)|$.

B Proofs

This section collects proofs and a few omitted theoretical results.

B.1 Proofs for Section 3.1

Proof of proposition 1. We show this proof in three steps.
Step 1

As γ_c goes to zero, dealer profits from trading bonds and CDS must converge to zero. To see this, note that if dealer trading profits do not converge to zero, then their demand of either bond or CDS cannot converge to zero, since

$$\pi = \max_{b \geq \underline{b}, c \leq b - \underline{x}} -qb + (q_f - p)c + X(b - c) - X(0)$$

equals zero if $b = c = 0$. But if trading profits are bounded away from zero and γ_c is converging to zero, there must be an infinite measure of dealers entering the market and demanding a non-zero amount of either bonds or CDS, which implies that one of these markets does not clear. This is a contradiction with equilibrium. Therefore, dealer profits from trading must converge to zero.
Step 2

Note that dealer profits converging to zero implies that the risk-neutral equilibrium price is the only possible equilibrium price for CDS. That is, the price of the CDS, p, must converge to the risk-neutral price $\beta\bar{\delta}$. To show this, first note that

$$\pi \geq \max_c (q_f - p)c + X(-c) - X(0) \geq 0.$$

Thus, since π converges to zero, the solution of the above problem must also converge to zero. The first-order condition of this problem is

$$q_f - p = X'(-c).$$

But the above FOC must be satisfied at $c = 0$, otherwise dealers would be able to make strictly positive profits. Moreover, the definition of the function $X(\cdot)$ gives us that

$$q_f = \beta\mathbb{E}[u'(a)] = \beta u'(a) \quad \text{and} \quad X'(0) = \beta\mathbb{E}[(1-\delta)u'(a)] = (1-\bar{\delta})\beta u'(a).$$

Using the above and that $q_f = \beta$, we must then have that $p = \beta - (1 - \bar{\delta})\beta = \beta\bar{\delta}$.
Step 3

Finally, we must show that q converges to $\beta(1 - \bar{\delta})$. Note that the FOC of the dealer problem implies that

$$q \geq X'(b - c) = q_f - p,$$

with equality if the bond-shorting constraint, $b \geq \underline{b}$, is not binding. Note that, if the constraint is not binding we have the result since

$$q = q_f - p = \beta - \beta\bar{\delta} = \beta(1 - \bar{\delta}).$$

Suppose by the way of contradiction that this is not the case and we have $q > q_f - p$. In this case the bond market cannot clear with a positive supply $B > 0$ as $\gamma_b, \gamma_c \to 0$. That is because the demand for bonds from dealers is non-positive, given by $\underline{b} \leq 0$. And the demand for bonds from investors is zero because they can acquire exposure by constructing synthetic bonds at the price $q_f - p < q$ in the CDS market, and, since $\lim_{\theta\to\infty} \alpha_b(\theta) = \alpha_c(\theta) = \bar{\alpha} \in (0, 1]$, the probability of trading in both markets can be made arbitrarily close as $\gamma_b, \gamma_c \to 0$. This concludes the proof. $\square$

Proof of proposition 2. Below we consider both cases separately.

Case 1: No CDS Market

Suppose that there is no CDS market. That is, $\alpha_c(\theta) = 0$ for all θ. Assume by contradiction that the price of the bond in equilibrium approaches $\beta(1 - \bar{\delta})$. We know that $\beta(1 - \bar{\delta})$ is equal to $X'(0)$. As a result, the demand for bonds for each investor with $\omega \leq 0$ satisfies $X'(b + \omega) \approx 0 \implies b(\omega) \approx -\omega$. This implies that the aggregate demand for bonds from investors approaches $\bar{\alpha} I \int_{\omega\leq 0} -\omega dF(\omega) > B$. Since the demand for bonds from dealers is bounded below by $\underline{b} = 0$, the bond market does not clear, which is a contradiction.

Case 2: Naked-CDS Ban

Suppose now that there is a naked-CDS ban. That is, $b - c \geq 0$ for all agents. Again, assume by contradiction that the price of the bond approaches $\beta(1 - \bar{\delta})$. Because there is a CDS market, investors with $\omega \leq 0$ may choose to trade CDS instead of buying bonds, which reduces the market demand for bonds and could allow it to clear. However, the first-order condition of dealers implies that $q \geq q_f - p$, so an investor with $\omega \leq 0$ that enters the CDS market will choose a CDS amount $c \leq \omega \leq 0$. For every unit of CDS sold by investors, there must be one unit of bond bought, because the naked-CDS ban implies that $b - c \geq 0$ for all agents. Therefore, the total demand for bonds is bounded below by approximately $\bar{\alpha} I \int_{\omega\leq 0} -\omega dF(\omega) > B$, and the bond market cannot clear. This concludes the proof. $\square$

B.2 Proofs for Section 3.2

Lemma 1. *Let $\Delta_i(\omega)$ denote the gains from trade for a given investor in market i. If $c = -b_i(\omega)$ is feasible and $q\,\mathrm{sgn}(b_i) \geq (q_f - p)\,\mathrm{sgn}(b_i)$, then $\Delta_c(\omega) \geq \Delta_b(\omega)$. If $b = -c_i(\omega)$ is feasible and $q\,\mathrm{sgn}(c_i) \geq (q_f - p)\,\mathrm{sgn}(c_i)$, then $\Delta_b(\omega) \geq \Delta_c(\omega)$.*

Proof of Lemma 1. Let feasible b (c) choices be $\mathcal{B}$ ($\mathcal{C}$). Consider first $c = -b_i(\omega)$ feasible with $q\,\mathrm{sgn}(b_i) \geq (q_f - p)\,\mathrm{sgn}(b_i)$. Then

$$\begin{aligned}
\Delta_b(\omega) &= \max_{b\in\mathcal{B}} -qb + X(b + \omega) - X(\omega) \\
&= -qb_i(\omega) + X(b_i(\omega) + \omega) - X(\omega) \\
&= qc + X(\omega - c) - X(\omega) \\
&\leq (q_f - p)c + X(\omega - c) - X(\omega) \\
&\leq \max_{c\in\mathcal{C}} (q_f - p)c + X(\omega - c) - X(\omega) \\
&= \Delta_c(\omega).
\end{aligned}$$

Now consider $b = -c_i(\omega)$ feasible with $q\,\text{sgn}(c_i) \geq (q_f - p)\,\text{sgn}(c_i)$. Then

$$\begin{aligned}
\Delta_c(\omega) &= \max_{c\in\mathcal{C}} (q_f - p)c + X(\omega - c) - X(\omega) \\
&= (q_f - p)c_i(\omega) + X(\omega - c_i(\omega)) - X(\omega) \\
&= -(q_f - p)b + X(b + \omega) - X(\omega) \\
&\leq -qb + X(b + \omega) - X(\omega) \\
&\leq \max_{b\in\mathcal{B}} -qb + X(b + \omega) - X(\omega) = \Delta_b(\omega).
\end{aligned}$$

□

Lemma 2. *Let $i, j \in \{b, c\}$ with $i \neq j$.*

If i has superior matching technology $M_i(\cdot) > M_j(\cdot)$ whenever $M_i, M_j \neq 0$, smaller entry costs $\gamma_i < \gamma_j$, and weakly better gains from trade $\Delta_i(\omega) \geq \Delta_j(\omega) > 0$, then $V_i(\omega) > V_j(\omega)$.

Proof of Lemma 2. By virtue of $\Delta_i > 0$, $\theta_i > 0$. The value of being active in submarket i is

$$\begin{aligned}
V_i(\omega) &= \max_{\theta_i\geq 0} -\tilde{\gamma}_i\theta_i + \alpha_i(\theta_i)\Delta_i(\omega) \\
&\leq \max_{\theta_i\geq 0} -\tilde{\gamma}_j\theta_i + M_i(1, \theta_i)\Delta_j(\omega) \\
&< \max_{\theta_j\geq 0} -\tilde{\gamma}_j\theta_j + M_j(1, \theta_j)\Delta_j(\omega) \\
&= \max_{\theta_j\geq 0} -\tilde{\gamma}_j\theta_j + \alpha_j(\theta_j)\Delta_j(\omega) = V_j(\omega).
\end{aligned}$$

□

Proof of Proposition 3. In all of these panels, the dealer problem can be used to show $q \geq q_f - p$. In panel A, one knows $q = q_f - p$.

Let $\mathcal{B}$ ($\mathcal{C}$) denote the set of feasible trades for bonds (CDS). In these examples the feasible trades do not depend on ω.

In the top left of each panel, bond technology dominates: both matching technology and entry costs are better. In the bottom right of each panel, CDS technology dominates: both matching technology and entry costs are better. In the off-diagonals, a trade-off exists between higher entry costs and greater success of matching.

Consider panel A. The CDS-bond basis in inter-dealer prices holds as dealers are unconstrained. This implies gains from trade are equated whenever the reverse trade is unconstrained, and it is always unconstrained. When $M_b > M_c$ ($M_b < M_c$), Lemma 2 gives $V_b > V_c$ ($V_c > V_b$). Thus the top left only has investor bond market activity, the bottom right only CDS activity.

Consider panel B, which uses the benchmark's financial constraints. As discussed in the text, $q \geq q_f - p$ and $\text{sgn}(b_i) \geq 0$. By Lemma 1, this gives $\Delta_c(\omega) \geq \Delta_b(\omega)$. For any investor who would strictly prefer bond trading to no trade $V_b(\omega) > X(\omega)$, one has $\Delta_b(\omega) > 0$. So Lemma 2 then gives $V_c > V_b$ for all such agents, implying the bond market shuts down.

Consider panel C. Then $q \geq q_f - p$ and $\text{sgn}(b_i) \geq 0$ (same as panel B; see the discussion around equations (6) and (7)). By Lemma 1, this gives $\Delta_c(\omega) \geq \Delta_b(\omega)$. For any investor who would strictly prefer bond trading to no trade $V_b(\omega) > X(\omega)$, one has $\Delta_b(\omega) > 0$. So Lemma 2 then gives $V_c > V_b$ for all such agents, implying the bond market shuts down.

In the top left panel, bonds are a dominant technology. Suppose first the CDS-bond basis holds in this case. Then CDS would not be chosen because it offers the same gains from trade. Now suppose $q > q_f - p$. In that case, some investors might wish to hold CDS because it offers a cheaper form of exposure. If that were the case, they would need $c < 0$. By market clearing, dealers would need $c_d > 0$. However, they cannot be naked, so they need to offset that with $b_d \geq c_d$. They are unwilling to do this however, because they could reduce b_d and c_d each by c_d while increasing the riskfree position by c_d. This results in the same next period allocations, but results in cost-savings in the first subperiod of $q + p - q_f > 0$. This contradicts that the CDS-bond basis would not hold in this case. Therefore, the CDS market cannot be active.

□

B.3 Proofs for Section 3.3

Lemma 3. *In any OTC equilibrium, $q + p \geq q_f$. If in equilibrium $b_d > \underline{b}$, then $q + p = q_f$—that is, the CDS-bond basis holds in the inter-dealer market.*

Proof of lemma 3. The dealer can always purchase one unit of bonds and one unit of CDS at a cost $q+p$ and sell one unit of the risk-free asset at a cost q_f. The resulting next period consumption allocation is unaltered. Consequently, for an optimal portfolio choice to exist, profits must be bounded requiring $q + p - q_f \geq 0$. Additionally, if in equilibrium $b_d > \underline{b}$, then at the margin the dealer can implement the reverse trade and chose not to, implying $q + p - q_f \leq 0$, and we can conclude that $q + p = q_f$. □

Proof of proposition 4. Note that if $\underline{b} = 0$ then $\mathrm{sgn}(b_i) \geq 0$. Now we can examine the investors problem in equations 9 and 12 to notice that the first order conditions of the two problems implies that $b_i(\omega) = -c_i(\omega)$ as long as the constraint $b_i(\omega) \geq \underline{b} = 0$ does not bind. And since $\gamma_b < \gamma_c$ and $M_b(n, d) \geq M_c(n, d)$ for all $(n, d) \in \mathbb{R}^2$, the value functions satisfy $V_b(\omega) \geq V_c(\omega)$. This implies that the term in the $\tilde{\psi}$ given by equation 23 that accounts for the bond intermediation costs is strictly positive, while the term accounting for the CDS intermediation costs in absolute value for all ω such that $b_i(\omega) = -c_i(\omega) \geq 0$ is zero. That is,

$$\frac{\int_\omega \mathrm{sgn}(b_i) d_b \tilde{\gamma}_b dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)} + \frac{\int_{-c_i(\omega)\geq 0} \mathrm{sgn}(c_i) d_c \tilde{\gamma}_c dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)} = \frac{\int_\omega \mathrm{sgn}(b_i) d_b \tilde{\gamma}_b dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)} > 0.$$

Since the term $\frac{\int_{-c_i(\omega)<0} \mathrm{sgn}(c_i) d_c \tilde{\gamma}_c dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)}$ is also strictly positive and $q + p - q_f = 0$, we have that

$$\tilde{\psi} = q + p - q_f + \frac{\int_\omega \mathrm{sgn}(b_i) d_b \tilde{\gamma}_b dF(\omega)}{\int_\omega M_b(n_b, d_b)|b_i| dF(\omega)} + \frac{\int_{-c_i(\omega)<0} \mathrm{sgn}(c_i) d_c \tilde{\gamma}_c dF(\omega)}{\int_\omega M_c(n_c, d_c)|c_i| dF(\omega)} > 0,$$

which concludes the proof. □

Proof of Proposition 5. With a naked CDS ban and unlimited bond shorting, the only way to obtain negative exposure is by choosing $b < 0$. Because the support of ω is unlimited, some agents will do this (the gains from trade can be arbitrarily large, so for any price q some investors will short sell the bond). This gives $\mathrm{sgn}(b_i) > 0$ for some investors, as well as $d_b > 0, n_b > 0$.

Investors wanting positive exposure have a choice of $b > 0$ or $c < 0$. Suppose first the the basis holds in inter-dealer prices. Then the markets offer the same gains from trade, so the market choice is over which technology is superior. Because the CDS search technology is better (lower entry costs, more matching) than the bond search technology, any investor preferring positive exposure chooses to go to the CDS market. Consequently, any investor with positive exposure obtains that through $c < 0$. And because the support of ω is unlimited, at any price some investors will be willing to sell that protection. This gives $\operatorname{sgn}(c_i) < 0$ for some investors, as well as $d_c > 0, n_c > 0$. This further shows $\operatorname{sgn}(b_i) \geq 0$ for all investors. The naked CDS ban ensures $c_i \leq 0$, so we have $\operatorname{sgn}(c_i) \leq 0$ (but strictly for some). (Dealers are the counterparties to those trades, so they have $c > 0$. Dealers must hold all the bonds in an equilibrium of this type, as investors buy none.)

Now suppose for contradiction the basis were not to hold. Then $c < 0$ is even more preferable, because $q > q_f - p$ (the bond is expensive). But in this case, no investor wants to own bonds, so market clearing requires the dealers must. But if dealers hold bonds, the inter-dealer basis must hold.

We have established $d_b, n_b, d_c, n_c > 0$, $\operatorname{sgn}(b_i) \geq 0$ and strictly for some, and $\operatorname{sgn}(c_i) \leq 0$ and strictly for some. So, equations (21) and (22) give $\mathbb{E}[\tilde{q}] < q$ and $\mathbb{E}[\tilde{p}] < p$, respectively. Consequently, $\tilde{\psi}$ in (23) has $\tilde{\psi} < 0$.

□

B.4 Proofs for Section 3.4

Proof of Proposition 6. We begin by characterizing the gains from trade for investors. In this example, we constructed an equilibrium where $\mu_b < 0$ and large enough in absolute value such that investors hold all the bonds in equilibrium. That is, market clearing dictates $\alpha_b b = B'/I$. From the investor Euler equation we have that $q = X'(b+\omega) = \beta(1-\bar{\delta})(1-\sigma\bar{\delta}(b+\omega))$, where the second equality follows from the functional form the the utility function in assumption 2.

The gain in utility terms of moving from the initial exposure μ_ω to a new desired exposure x_i^* (note $|x_i^* - \omega|$ is the change in exposure due to activity in market i, so $|x_i^* - \omega| = |i|$.) is given by

$$\begin{aligned}
\Gamma_i(x_i^*, \omega) &\equiv X(x_i^*) - X(\omega) \\
&= \int_\omega^{x_i^*} \beta(1-\bar{\delta})(1-\sigma\bar{\delta}x)dx \\
&= \beta(1-\bar{\delta})(x - \sigma\bar{\delta}x^2/2)|_\omega^{x_i^*} \\
&= \beta(1-\bar{\delta})(x_i^* - \omega - \sigma\bar{\delta}((x_i^*)^2 - \omega^2)/2) \\
&= \beta(1-\bar{\delta})(x_i^* - \omega - \sigma\bar{\delta}((x_i^* - \omega)(x_i^* + \omega)/2) \\
&= (x_i^* - \omega)\beta(1-\bar{\delta})(1 - \sigma\bar{\delta}((x_i^* + \omega)/2) \\
&= (x_i^* - \omega)X'((x_i^* - \omega)/2 + \omega)
\end{aligned}$$

Since by assumption investors trade in the bond market, factoring in the cost of acquiring the position in the bond market, we have the actual gains from trade:

$$\begin{aligned}
\Delta_b &= \Gamma_b(x_b^*, \omega) - qb \\
&= bX'(b/2 + \omega) - qb \\
&= b(X'(b/2 + \omega) - X'(b + \omega)) \\
&= b\beta(1-\bar{\delta})(1 - (1 - \sigma\bar{\delta}(b/2)) \\
&= \sigma\beta\mathbb{V}[\delta]b^2/2 \\
&= \sigma\beta\mathbb{V}[\delta](B'/(I\alpha_b))^2/2,
\end{aligned}$$

where the last equality follows from market clearing. From the first order condition for θ and noting that $\gamma_i = \tilde{\gamma}_i$ because dealers hold $b_d = c_d = 0$, we have that $\gamma_i = \bar{\alpha}_i \xi_i \theta_i^{\xi_i - 1}\Delta_i$, which implies an equilibrium market tightness of $\theta_i = \left(\frac{\gamma_i}{\bar{\alpha}_i \xi_i \Delta_i}\right)^{1/(\xi_i - 1)}$. Plugging into the matching technology of assumption 3 we have that

$$\alpha_i = \bar{\alpha}_i^{1/(1-\xi_i)} \left(\frac{\xi_i}{\gamma_i}\right)^{\xi_i/(1-\xi_i)} (\Delta_i)^{\xi_i/(1-\xi_i)}$$

Then focusing on $i = b$ and plugging in Δ_b into the probability of trading we have that

$$\alpha_b = \left(\bar{\alpha}_b^{1/\xi_b} \frac{\xi_b}{\gamma_b} \sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{\xi_b/(1+\xi_b)} \left(\frac{B'}{I}\right)^{2\xi_b/(1+\xi_b)}$$

Replacing α_b into our expression for λ_b we obtain

$$\Delta_b = \left[\left(\frac{B'}{I}\right)^{2(1-\xi_b)} \left(\frac{\sigma\beta\mathbb{V}[\delta]}{2}\right)^{1-\xi_b} \left(\frac{\gamma_b}{\bar{\alpha}_b^{1/\xi_b}\xi_b}\right)^{2\xi_b}\right]^{1/(1+\xi_b)},$$

which is the result of proposition 6a. Having found α_b, we can calculate the equilibrium intermediation fees in the bond market. Since $\alpha_b f_b = \tilde{\gamma}_b \theta_b$, one has $f_b/b = \gamma_b\theta_b/(\alpha_b b)$. From bond market clearing, $\alpha_b b$ must equal B'/I. Thus, $f_b/b = \gamma_b\theta_b I/B$. Therefore, using the expression for α_b and doing some algebra we obtain that

$$\begin{aligned}\frac{f_b}{b} &= (\gamma_b I/B')\left(\frac{\alpha_b}{\bar{\alpha}_i}\right)^{1/\xi_b} = \gamma_b \frac{I}{B'}\left(\frac{\left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{2\xi_b/(1+\xi_b)}}{\bar{\alpha}_i}\right)^{1/\xi_b} \\ &= \left(\gamma_b^{\xi_b}(\frac{B'}{I})^{1-\xi_b}\bar{\alpha}_b^{1/\xi_b - 1 - \xi_b}\xi_b\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{1/(1+\xi_b)},\end{aligned}$$

as show in proposition 6b. In addition, using the expression for α_b we can pin down the equilibrium allocation of each investor $b = B'/(I\alpha_b)$, given by

$$\begin{aligned}1/b = \left(\frac{B'}{I}\right)^{-1}\alpha_b &= \left(\frac{B'}{I}\right)^{-1}\left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{2\xi_b/(1+\xi_b)} \\ \implies b &= \left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{1-2\xi_b/(1+\xi_b)},\end{aligned}$$

which is the result in proposition 6c. Next, evaluating the marginal gain for investors we can determine the equilibrium bond price at the inter-dealer market from investors first order condition,

$$\begin{aligned}q &= \beta(1-\bar{\delta})\left(1 - \sigma\bar{\delta}\left(\omega + b\right)\right) \\ &= \beta(1-\bar{\delta})\left(1 - \sigma\bar{\delta}\left(\omega + \left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{1-2\xi_b/(1+\xi_b)}\right)\right)\end{aligned}$$

Expressed in risk premium $\lambda_b = 1 - q/(\beta(1-\bar{\delta}))$,

$$\lambda_b = \sigma\bar{\delta}\left(\omega + \left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-\xi_b/(1+\xi_b)}\left(\frac{B'}{I}\right)^{1-2\xi_b/(1+\xi_b)}\right),$$

the expression in proposition 6d. A negative risk premium is needed to support the positive CDS-bond basis, so we need μ_ω sufficiently less than zero. $\lambda_c = 0$ follows from CDS being traded at risk neutral prices $p = \beta\bar{\delta}$ because dealers are unconstrained in CDS trading and their allocation is $b_d = c_d = 0$ by construction.

Finally, since $q = \beta(1-\bar{\delta})(1-\lambda_b), p = \beta\bar{\delta}$ and $q_f = \beta$, the CDS-bond basis in the inter-dealer market is given by $q + p - q_f = -\lambda_b\beta(1-\bar{\delta}) = -\sigma\beta\mathbb{V}[\delta](\mu_\omega + b)$, as stated in proposition 6e. Therefore, the change of the CDS-bond basis in the inter-dealer market after changes in the default probability is given by

$$-\sigma\beta\frac{d\mathbb{V}[\delta]}{d\bar{\delta}}(\mu_\omega + b) - \sigma\beta\mathbb{V}[\delta]\frac{db}{d\bar{\delta}}.$$

For all $\bar{\delta} \in (0, 1/2)$ an increase the probability of default increases the $\mathbb{V}[\delta]$ and since $\mu_\omega + b < 0$ the first term is positive. We are left to show that the second term is positive as well, which is true if $\frac{db}{d\mathbb{V}[\delta]}\frac{d\mathbb{V}[\delta]}{d\bar{\delta}} < 0$. Since for all $\bar{\delta} \in (0, 1/2)$, $\frac{d\mathbb{V}[\delta]}{d\bar{\delta}} > 0$, we can use the expression for b in proposition 6c. and take derivative with respect to $\mathbb{V}[\delta]$. We then have that

$$\frac{db}{d\mathbb{V}[\delta]} = -\frac{\xi_b}{1+\xi_b}\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\frac{1}{2}\left(\bar{\alpha}_b^{1/\xi_b}\frac{\xi_b}{\gamma_b}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-(1+2\xi_b)/(1+\xi_b)}\left(\frac{B'}{I}\right)^{1-2\xi_b/(1+\xi_b)} < 0,$$

where the minus sign of the expression is preserved because all parameters in the equation are positive. This concludes the proof. □

Proof of Proposition 7. To derive the expressions in items a. and b. we proceed in the same way as in proposition 6, except that investors' positions are $c_i = -\frac{B'}{\alpha_c I}$ and dealers' positions are $b_d = B'/D, c_d = B'/D$, by construction, where D is the total mass of dealers. The CDS bond basis in the inter-dealer market holds because by proposition 2 we know that $q + p = q_f$ whenever $b_d > \underline{b}$. Following the same steps to derive λ_c that we used in proposition 6 to derive λ_b, we can find that

$$\lambda_c = \sigma\bar{\delta}\left[\omega + \left(\bar{\alpha}_c^{1/\xi_c}\frac{\xi_c}{\gamma_c}\sigma\beta\mathbb{V}[\delta]\frac{1}{2}\right)^{-\xi_c/(1+\xi_c)}\left(\frac{B'}{I}\right)^{1-2\xi_c/(1+\xi_c)}\right] \neq 0.$$

The fact that $\lambda_c = \lambda_b$ follows directly from $q = q_f - p$ and the definitions $\lambda_b \equiv 1 - \frac{q}{\beta(1-\bar{\delta})}$ and $\lambda_c \equiv 1 - \frac{q_f - p}{\beta(1-\bar{\delta})}$. Finally, following the same steps as in proposition 6 we find that

$$\frac{f_c}{c} = -\left[\frac{1}{2}\left(\frac{B'}{I}\right)^{(1-\xi_c)}\gamma_c^{\xi_c}\bar{\alpha}_c^{-1}\xi_c\sigma\beta\mathbb{V}[\delta]\right]^{1/(1+\xi_c)} < 0.$$

Since, all parameters are positive $f_c/c < 0$ and since the variance of default risk, $\mathbb{V}[V]$, is increasing in default risk for all $\bar{\delta} \in (0, 1/2)$, increases in $\bar{\delta}$ result in more negative values for f_c/c, which concludes the proof. □

C Additional quantitative results

In this section, we present additional quantitative results from our model. In the first subsection, we explore the relationship between debt issuance and the OTC block. In the second subsection, we simulate a default event and analyze its impact on the market. In the third subsection, we conduct additional welfare analysis to assess the impact of our model on the sovereign and investors.

C.1 CDS-bond basis, trading frictions and exposure

Figure C.1, top-left panel, displays the distribution of CDS-bond basis we obtain from a predictive prior exercise. In the exercise, we fix a hypercube of OTC parameters, draw random parameter vectors from that space, and compute the resulting CDS-bond basis deviation. That gives an unconditional distributions in black, while the blue bars give the distribution conditional on a negative effective basis $\tilde{\psi} < 0$. The CDS-bond basis tend to be centered around zero in our simulations. However, it can go as high as 1%, and as low as -1% (in fact, it we have truncated at $\pm 1\%$ readability). The other panels in Figure C.1 depict the marginal distribution of different parameter that we used both, unconditional (black) and conditional (red) on having a CDS-bond basis smaller than -25 basis points. The basis tend to be negative when the dealers entry cost into the CDS market is not too high, the distribution of exposures ω is fairly concentrated in negative values (negative average and low dispersion), CDS matching is reasonably efficient, and the CDS matching elasticity is not too low.

Figure C.2 graphs the CDS-bond basis deviations both in inter-dealer prices $q + p - q_f$ and effective prices $\mathbb{E}[\tilde{q}] + \mathbb{E}[\tilde{p}] - q_f$ as parameters are varied from benchmark values one at a time. (Deviations have been capped as indicated in the table.) One can see $\bar{\alpha}_c$ has a monotonic effect on the basis, with lower $\bar{\alpha}_c$ driving up the basis as it increases sorting. Entry costs have little effect. The elasticity parameters can have large effects and be non-monotone. Usually the inter-dealer prices and effective prices are close to one another, but they decouple when matching elasticities parameters governing ω are at extreme.

C.2 The OTC block as debt issuance varies

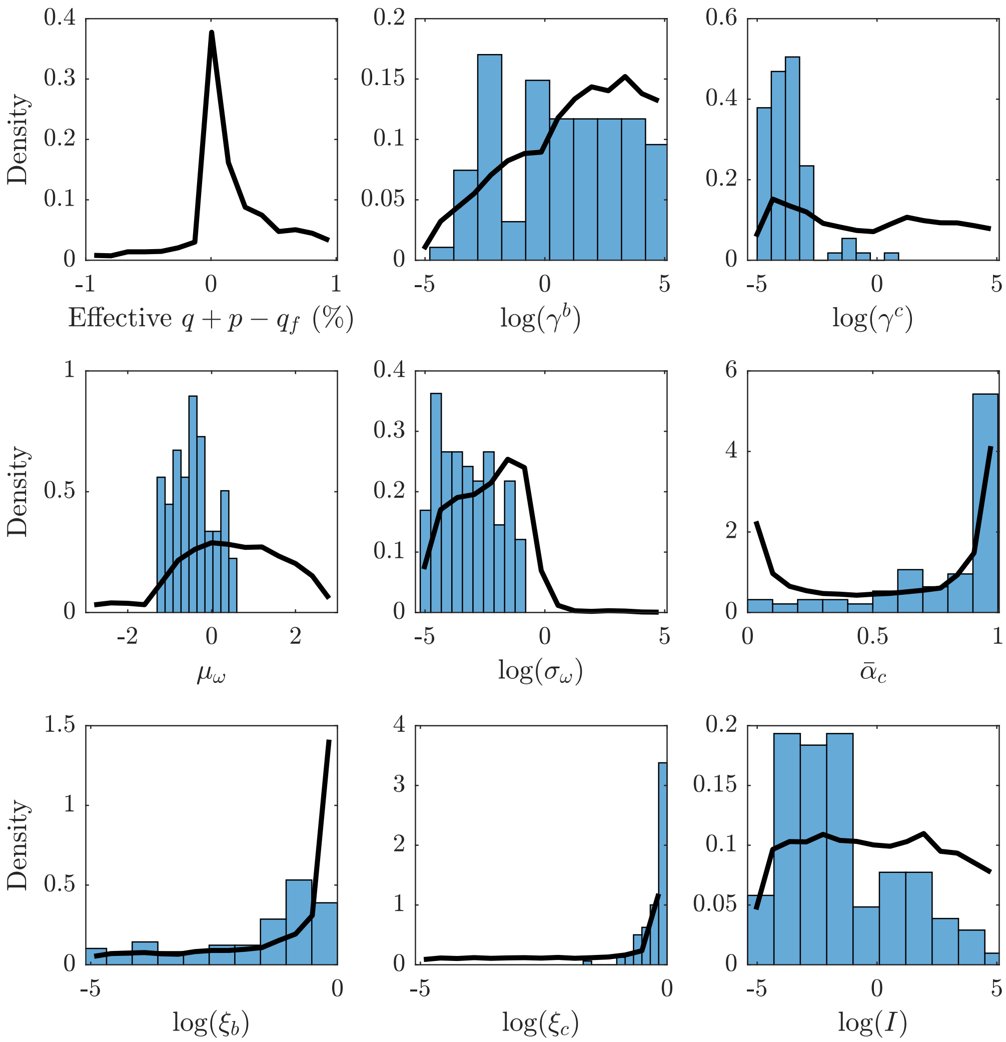
Figure C.3 shows the relationship between debt issuance and the OTC block as debt insurance varies. The graphs are plotted for a default rate of 8.7% and apply to both the benchmark and the case with liquid sovereign policies but frictional OTC markets since the default risk is fixed and bond issuance is on the horizontal axis.

The first row of panels shows the CDS-bond basis deviation, running spread CDS, and Z-spread bonds as debt issuance varies. The CDS-bond basis deviation decreases as debt issuance increases. This is because the spread on bonds have to increase more than that of CDS to make it more attractive to investors with lower ω to switch from selling CDS protection to buying bonds.

The second row of panels shows the bond choice, probability of matching in the bond market and expected bond purchase as debt issuance varies. Investors with low-ω buy more bonds and choose higher matching probabilities since they are negatively exposed to risk, while medium-ω investors only enter the bond market when the supply is sufficiently high (and the spread has risen enough), and high-ω investors are never active in the bond market.

The second row of panels shows the CDS choice, probability of matching in the CDS market, and expected CDS purchase as debt issuance varies. Investors with high-ω buy more CDS and choose higher matching probabilities since they are negatively exposed to risk, while medium-ω and high-ω investors enter the CDS market selling protection. But notice that only medium-ω are actually active in this market. Since the bond market is more efficient, investor with high-ω, which have higher gains from trade, prefer to get exposure in the bond market.

Figure C.1: Parameter distribution of negative CDS-bond basis

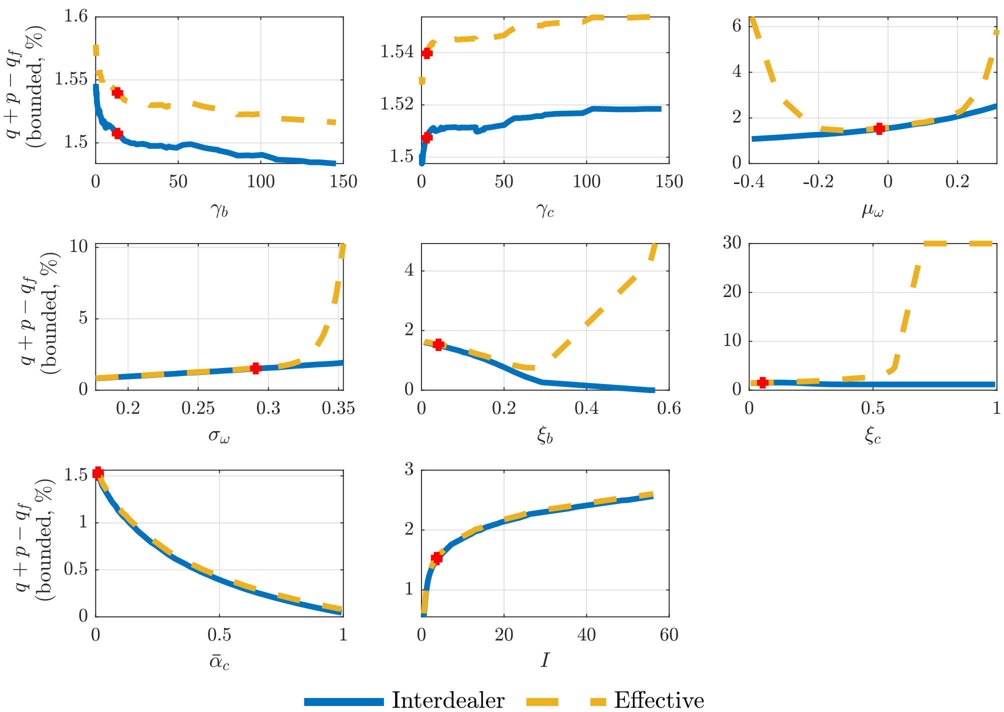


Note: "$q + p - q_f$ (eff.)" is the CDS-bond basis deviation measured in effective prices; the blue histogram and black line both represent a density estimate for a sample of statistics corresponding to pseudo-random draws from a hypercube of OTC parameters; the "unconditional" distribution in the figure requires (i) computed market clearing errors be sufficiently small, (ii) the effective basis be less than 1% in absolute value (to make the graph readable) and (iii) aggregate CDS volume be at least 10% relative to GDP; the conditional distribution labeled "$q + p - q_f$ (eff.) < -25 bps" additionally requires the CDS-bond basis deviation measured in effective prices to be less than -25 basis points; the parameter lower bounds are γ_b = 0.01, γ_c = 0.01, $\bar{\alpha}_c$ = 0.01, ξ_b = 0.01, ξ_c = 0.01, μ_ω = -2.9, σ_ω = 0.01, and I = 0.01; the parameter upper bounds are γ_b = 148, γ_c = 148, $\bar{\alpha}_c$ = 0.99, ξ_b = 0.99, ξ_c = 0.99, μ_ω = 2.9, σ_ω = 148, and I = 148; parameters not listed are fixed at benchmark levels; the draws are i.i.d. across parameters but uniform in a transformed parameter space; this figure is constructed using a lower precision solution to reduce runtimes.

C.3 A default event on the simulated path

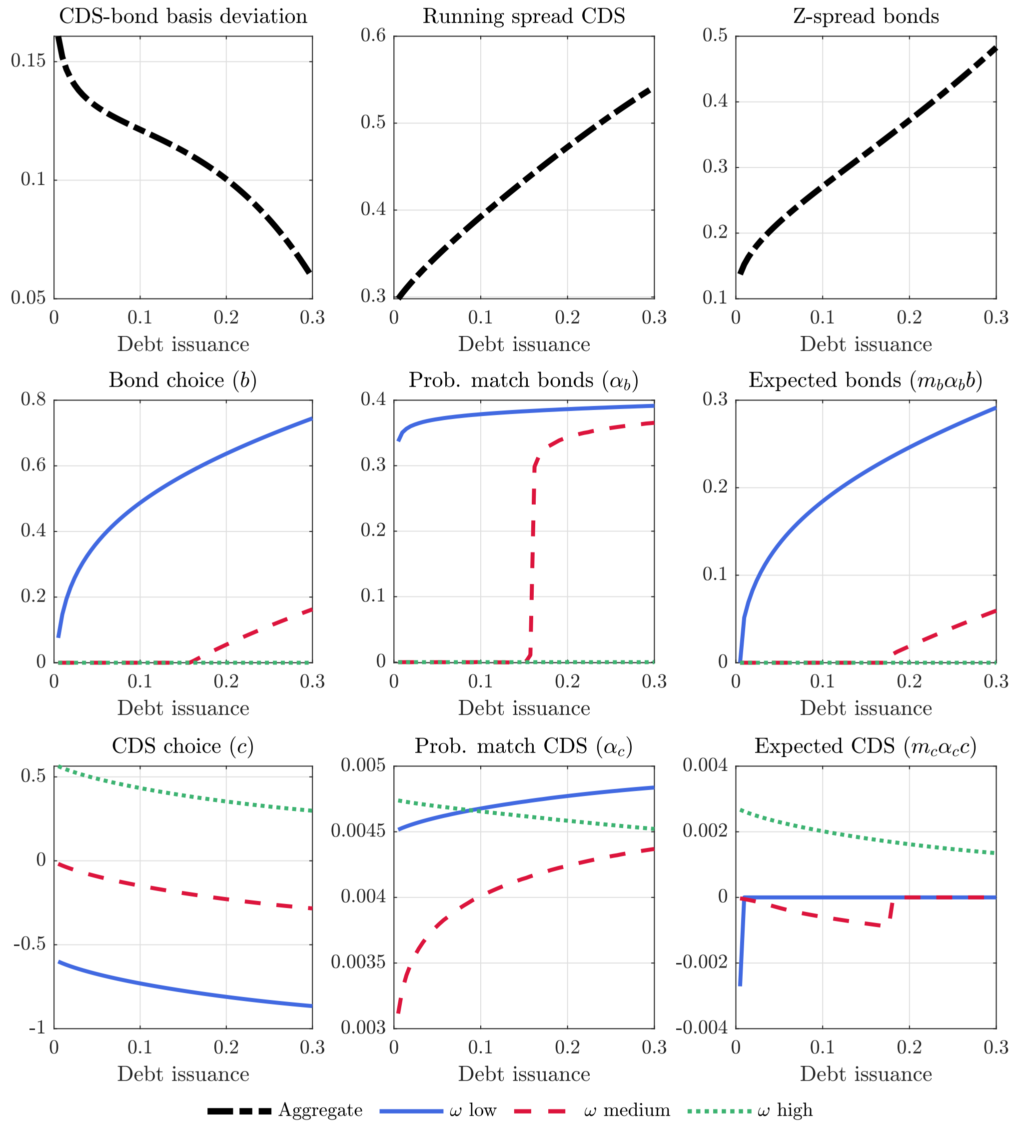
Figure C.4 shows one episode in the simulation where the sovereign transitions from default (at $t - 1$) to default (at $t + 19$). In response to a GDP boom from $t + 5$ to $t + 7$, spreads decline (and the CDS-bond basis with them) and the sovereign leverages up. But beginning in $t + 8$ GDP begins a multiyear decline. In response the sovereign deleverages, but spreads and the basis continue to rise until a final large drop at $t + 19$ induces default. As argued previously, the CDS-bond

Figure C.2: Comparative statics: Response of the CDS-bond basis deviations measured in prices



Note: The figure shows the simulation average of the inter-dealer CDS-bond basis deviation in prices $q + p - q_f$ and the deviation in effective prices; the effective price series has been bounded at $\pm 30\%$ and is not well-defined when there are no investors engaged in CDS or bond trading, which can occur if all investors arriving in a market prefer not to trade; the red plus signs indicate the benchmark parameter values; values only plotted if the market clearing error was sufficiently small; this figure is constructed using a lower precision solution to reduce runtimes, see the appendix for details.

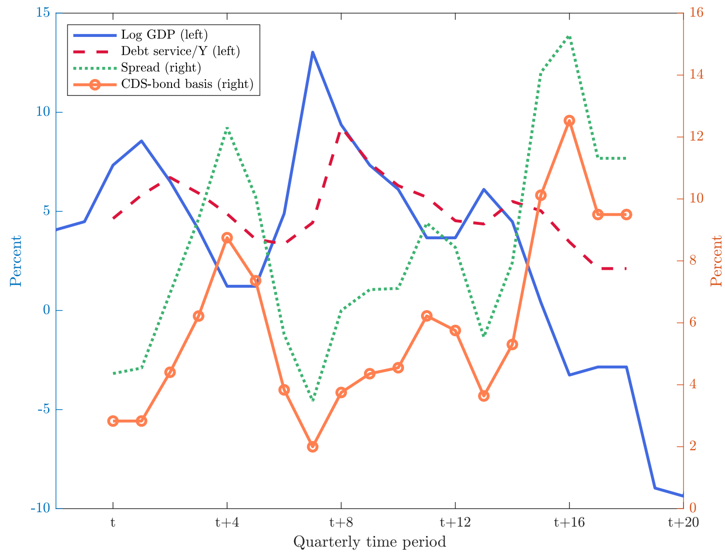
Figure C.3: The OTC block as debt issuance varies



Note: plotted for a default rate of 8.7%; the bond choice and probability of matching bonds are conditional on selecting the bond market, and similarly for CDS; this graph applies to both the benchmark and the case with liquid sovereign policies but frictional OTC markets since the default risk is fixed and bond issuance is on the horizontal axis.

basis deviations are increasing in default risk and decreasing in debt issuance, the CDS-bond basis deviations increase in crises.

Figure C.4: Simulated key variables along the simulated path



Note: the figure gives a model-based simulation of four key series; the gaps in the series occur when the sovereign defaults or is in default; the left-axis applies to log GDP (relative to trend/its mean) and debt-service to GDP; the right-axis applies to the bond spread and CDS-bond basis deviations.

C.4 Dealer behavior

Table C.1 reports how dealer variables respond to different counterfactuals. As discussed in the main text, dealers in the benchmark obtain exposure through CDS, which is cheaper than buying bonds because of the positive CDS-bond basis deviations. This gets undone in the short-bonds case, when the CDS-bond basis necessarily holds in inter-dealer prices.

Table C.1: Dealer behavior along the simulated path

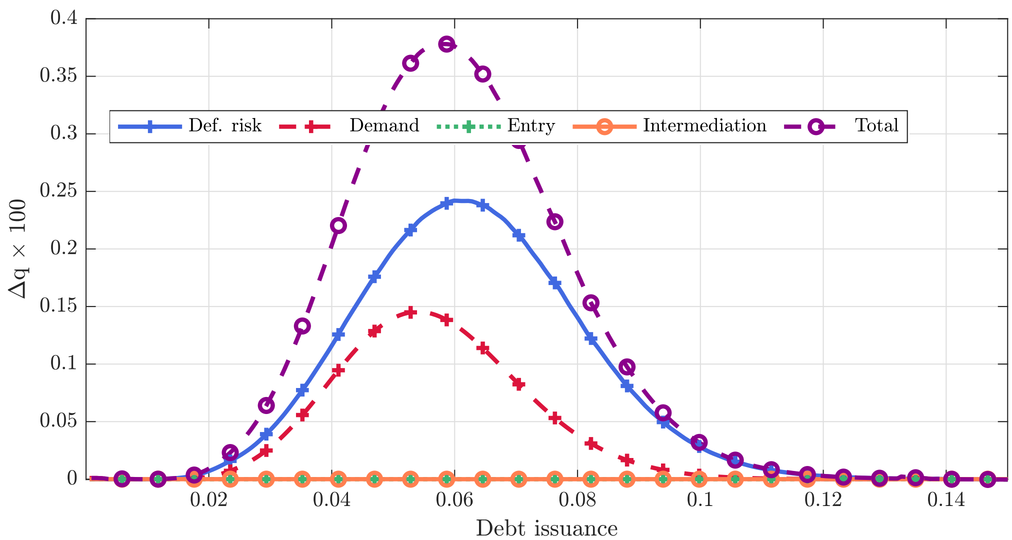
Statistic	Benchmark	No Naked CDS	No CDS	Short Bonds
Measure of dealers $\times$ 100	0.10	0.10	0.10	0.10
Individual dealer bond $\times$ 100	0.00	0.00	0.05	1.17
Individual dealer CDS $\times$ 100	-8.01	-4.92	0.00	0.01
Individual dealer exposure $\times$ 100	8.01	4.92	0.05	1.16
Individual dealer profit $\times$ 100	0.02	0.01	0.00	0.00
Individual dealer buy vol $\times$ 100	0.03	0.02	0.00	0.00
Individual dealer sell vol $\times$ 100	0.03	0.02	0.00	0.00

C.5 Closing the CDS market

Credit default swaps are a relatively recent financial innovation that are widely used by dealers and investors. In this section, we consider the consequences of shutting down the CDS market.

Despite their evident usefulness, the benchmark model predicts almost no spillover from the CDS market to the bond market in terms of prices (see Figure 8). This is not a general result, and we will show in Sections C.7 and C.8 that some parameterizations of the model predict large spillovers. To understand why spillovers are estimated to be small in the benchmark model, we decompose the total effect into its four components in Figure C.5.

Figure C.5: No CDS decomposition



Note: Each line represents a decomposition of $q(Y^{median}, B')$ for various B' values and Y^{median} the midpoint of the discretized Y process; positive values indicate better pricing for the sovereign.

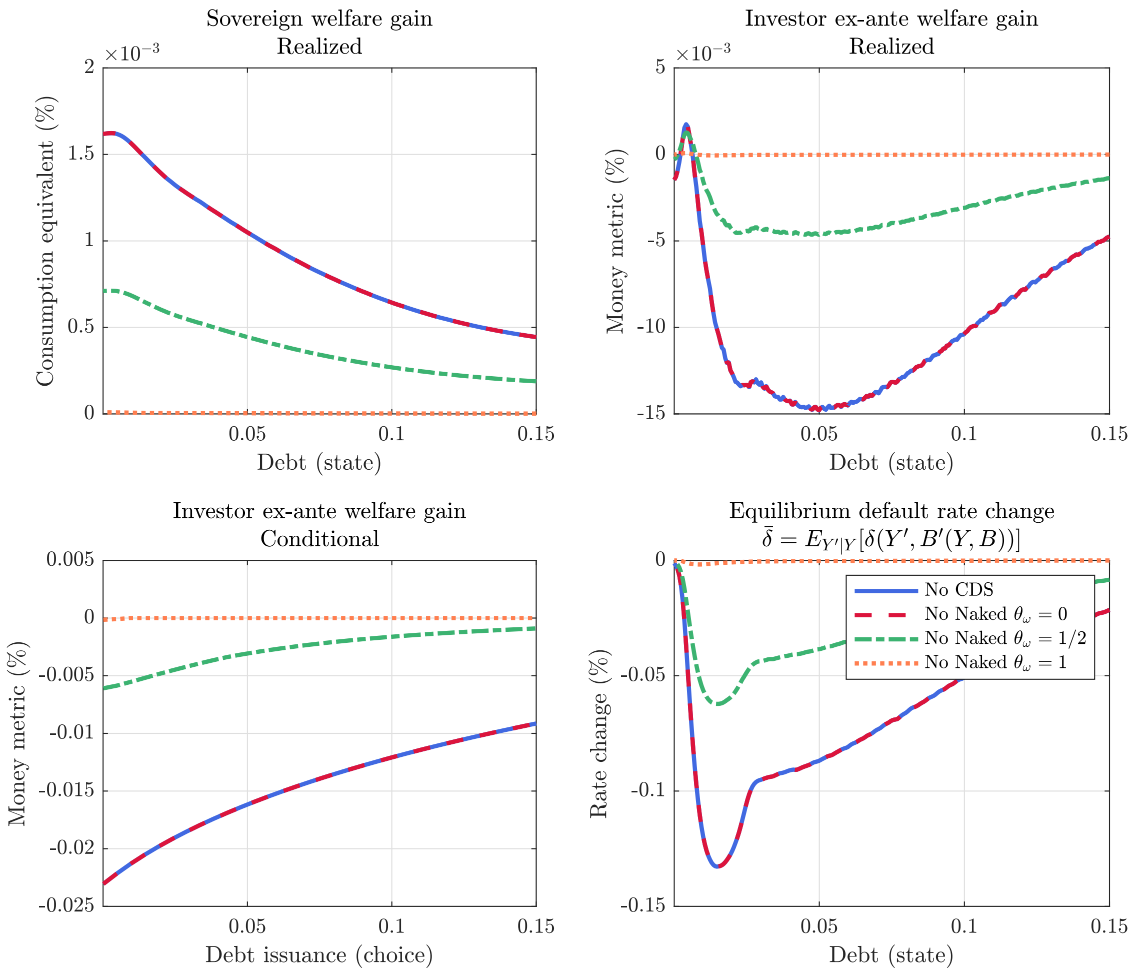
The figure reveals that the positive demand effect and the positive default risk effect, arising from a higher bond price overall, combine to generate a positive total effect (as in the main text, entry and intermediation effects are basically zero).

The demand effect is signed as theory would predict: By closing an opportunity to take positions in synthetic bonds, the demand curve for bonds shifts out and prices rise. However, the magnitude of the demand effect is much smaller than in a qualitatively similar bond-shorting-ban counterfactual (the reverse of allowing bond shorting). There are two main reasons for this. The CDS market is estimated to be much more frictional than the bond market since $\bar{\alpha}_c$ is so much smaller than $\bar{\alpha}_b$. This means that when the CDS market is shut down, the demand boost from CDS-protection-selling investors substituting into bond-buying is only a small percentage of overall bond demand. As a result of these two factors, the demand effect of shutting down the CDS market is relatively small.

C.6 Additional welfare analysis with naked CDS robustness

Figure C.6 reports welfare just for the naked CDS and CDS bans, as well as robustness to different values of θ_ω. The case of $\theta_\omega = 0$ is indistinguishable from the no-CDS case, proving an upper bound on the welfare effects. Conversely, $\theta_\omega = 1$ has essentially zero effect. So this robustness bounds the naked CDS ban case between no effect and the small effect of shutting down the CDS market.

Figure C.6: Welfare analysis (naked CDS for differing θ_ω and no CDS)

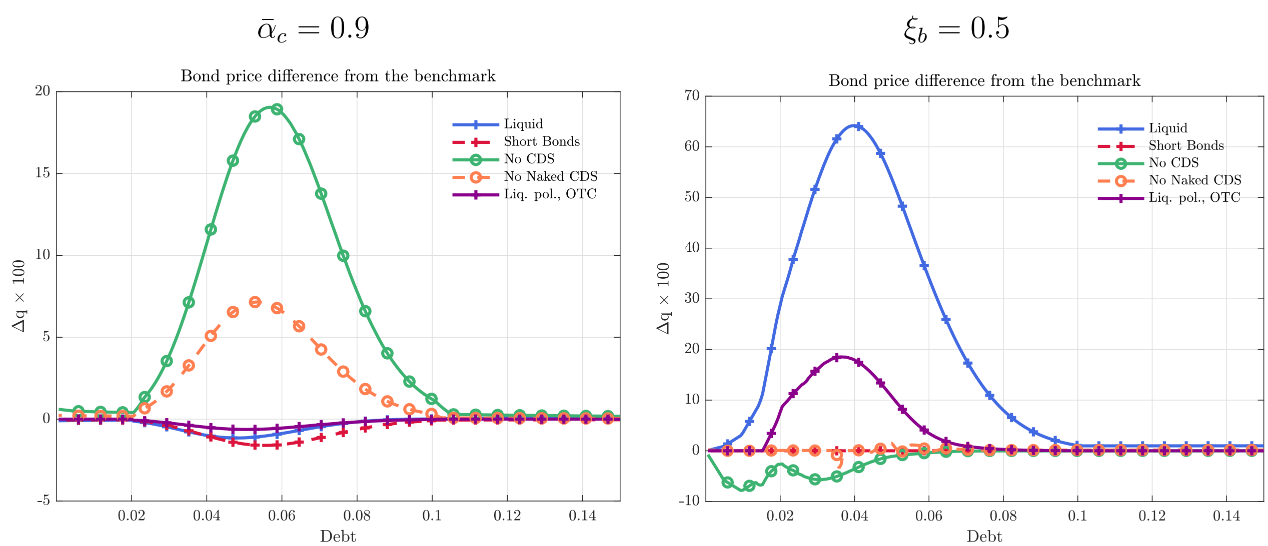


Note: The top panels and the bottom right panel are functions of (Y, B) and have been averaged using the invariant distribution of Y; "Realized" indicates the usual general equilibrium welfare measures where the sovereign's bond policies and default rates are allowed to vary; the panel labeled "Conditional" is a partial equilibrium concept that holds $(B', \bar{\delta})$ fixed but takes a numerical average across the $\bar{\delta}$ grid.

C.7 Naked CDS and CDS ban large effects, bond shorting small effects

A ban on naked CDS trading or a total ban on CDS trading has a positive, but very small, impact on prices and welfare. This is not a generic result coming from the model but rather a result of the calibration. To illustrate this point, we show how variation in even a single parameter can significantly affect the counterfactuals. The left panel of Figure C.7 shows how increasing $\bar{\alpha}_c$ to 0.9 (close to $\bar{\alpha}_b = 1$) sharply changes the policy conclusions. In that case, a naked CDS ban or eliminating CDS drastically increases bond prices. In contrast, the effects of eliminating search frictions or allowing bond shorting has small effects. In the same spirit, the right panel increases ξ_b from its benchmark value to 0.5. Now liquidity boosts bond prices tremendously, bond-shorting has almost no effect, and eliminating CDS sharply worsens bond prices. These results highlight the necessity of a careful identification strategy, like the one we espoused in Section 4.3.

Figure C.7: Price schedule differences with only one parameter different



Note: Each line represents $q(Y^{median}, B')$ for various B' values and Y^{median} the midpoint of the discretized Y process minus the benchmark model's value; positive values indicate better pricing for the sovereign.

C.8 Policy experiments under alternative parameterizations

Building on Section C.7, we now consider more general variation of the key matching and risk-sharing parameters and reassess the counterfactuals' impacts on bond prices.

Table C.2 provides a summary of the decompositions from Figure 8 for a number of parameter specifications. (Select statistics corresponding to these parameters are reported in Table C.3.) The maximum change in q relative to the benchmark appears in the first of the five numeric columns, while the minimum change in q appears in the last five. A finding that holds in virtually every parameter specification is that allowing for bond shorting reduces bond prices, often by a substantial amount but sometimes only weakly.

The effects of naked CDS bans and eliminating CDS are usually close to zero, but can be large and negative for some matching elasticities and sometimes large and positive. Eliminating OTC frictions does not necessarily improve prices, like in the benchmark, but can sometimes reduce them by a large amount, particularly when there are at least some agents with negative exogenous exposure. The key there is who the marginal investor is. That is why the average exposure μ_ω and the measure of investors I play such a crucial role for that counterfactual.

Table C.3 provides select moments corresponding to the parameter values in Table C.2 (all moments are evaluated for the benchmark assumptions $\underline{b} = 0$ and $\underline{x} = -\infty$). By comparing the two tables, one can see that whenever CDS bans or naked CDS bans have large effects, the volume of CDS tends to be large. But this is not always the case, such as when ξ_b or I is varied.

Table C.2: Robustness of counterfactuals to various parameters

	Maximum $\Delta q \times 100$					Minimum $\Delta q \times 100$				
	Liquid	Short	No CDS	No Naked	Liq. pol., OTC	Liquid	Short	No CDS	No Naked	Liq. pol., OTC
Benchmark	0.25	0.00	0.18	0.08	0.00	-7.2	-7.0	0.00	0.00	-3.0
$\xi_b = 0.5$	33	0.00	0.00	0.32	6.5	0.00	0.00	-9.1	-3.9	0.00
$\xi_b = 0.05, \xi_c = 0.05$	0.18	0.00	0.25	0.11	0.00	-7.4	-7.1	0.00	0.00	-3.0
$\xi_b = 0.01, \xi_c = 0.05$	0.13	0.00	0.17	0.08	0.00	-7.7	-7.5	0.00	0.00	-3.2
$\xi_b = 0.05, \xi_c = 0.01$	0.28	0.00	0.33	0.15	0.00	-6.9	-6.7	0.00	0.00	-2.8
$\bar{\alpha}_c = 0.9$	0.46	0.00	6.6	2.4	0.04	-0.72	-0.54	0.00	0.00	-0.28
$\bar{\alpha}_b = 0.5, \bar{\alpha}_c = 0.5$	1.2	0.00	5.8	2.1	0.18	-0.33	-0.25	0.00	0.00	-0.13
$\bar{\alpha}_b = 0.1, \bar{\alpha}_c = 0.9$	0.77	0.00	2.2	2.4	0.12	-0.21	0.00	-5.4	0.00	-0.08
$\bar{\alpha}_b = 0.9, \bar{\alpha}_c = 0.1$	0.29	0.00	1.6	0.66	0.00	-5.6	-5.4	0.00	-0.54	-2.2
$\gamma_b = 10, \gamma_c = 10$	0.24	0.00	0.16	0.07	0.00	-7.2	-7.0	0.00	0.00	-3.0
$\gamma_b = 5, \gamma_c = 15$	0.23	0.00	0.16	0.07	0.00	-7.2	-7.0	0.00	-0.45	-3.0
$\gamma_b = 15, \gamma_c = 5$	0.25	0.00	0.17	0.08	0.00	-7.2	-7.0	0.00	0.00	-3.0
$\mu_\omega = 0.2$	1.2	0.00	0.21	0.02	0.09	-3.4	-7.8	0.00	0.00	-1.6
$\mu_\omega = 0.0$	0.29	0.00	0.18	0.07	0.00	-6.7	-7.0	0.00	0.00	-2.8
$\mu_\omega = -0.2$	0.00	0.00	0.16	0.12	0.00	-11	-6.8	0.00	0.00	-4.1
$\sigma_\omega = 0.05$	8.7	0.00	0.49	0.00	2.9	0.00	-0.09	0.00	0.00	0.00
$\sigma_\omega = 0.25$	8.7	0.00	0.32	0.00	2.8	0.00	-0.27	0.00	0.00	0.00
$\sigma_\omega = 0.45$	0.66	0.00	0.01	0.00	0.07	-0.86	-0.62	0.00	-0.55	-0.42
$I = 0.1$	32	0.00	0.02	0.56	4.0	-0.55	-1.3	0.00	-5.3	0.00
$I = 1$	2.4	0.00	0.07	0.03	0.27	-4.0	-4.4	0.00	0.00	-1.8
$I = 10$	0.00	0.00	0.30	0.14	0.00	-9.9	-9.6	0.00	0.00	-3.8

Note: each row indicates a different set of parameters (parameters not listed are held constant at the benchmark); $\mathbb{E}_Y[q(Y, B')]$ is computed for each experiment and then the benchmark version's subtracted off; the first five numeric columns report the maximum of that difference over B' values for each of the five counterfactual experiments, and the second five report the minimum.

D Computation

In this section, we describe the computational methods used to solve the model. The model is divided into two blocks: the OTC block and the sovereign block. The OTC block models the interactions between investors and dealers, while the sovereign block models the government debt issuance and default decisions. The two blocks are linked through debt issuance, default rates, and bond prices.

D.1 OTC block

To represent functions of ω, such as the investor value functions, we project them onto the Chebyshev polynomials ($f(\omega) \approx \sum a_i T_i(\phi(\omega))$ for ϕ a linear map from $[\mu_\omega - 5\sigma_\omega, \mu_\omega + 5\sigma_\omega]$ to $[-1, 1]$). Unless otherwise stated, we use 16,384 (2^{14}) polynomials to obtain an extremely accurate approximation. The lower quality approximation uses 4,096 polynomials. We integrate functions of ω by constructing the Chebyshev approximation and integrating that (which is a simple function of the

Table C.3: Selected statistics at different parameters

	E[spread]	σ(spread)	Debt/Y	$q(Y^m, .06)$	Basis dev.	Sov. gain	Inv. gain	CDS vol.
Benchmark	8.1	3.8	5.5	0.54	6.0	0.00	0.00	0.19
$\xi_b = 0.5$	19	6.4	0.80	0.00	1.7	-0.27	0.41	0.80
$\xi_b = 0.05, \xi_c = 0.05$	8.1	3.9	5.6	0.55	5.9	0.00	-0.01	0.33
$\xi_b = 0.01, \xi_c = 0.05$	8.1	3.9	5.7	0.56	6.4	0.01	-0.04	0.22
$\xi_b = 0.05, \xi_c = 0.01$	8.2	3.8	5.4	0.53	5.4	0.00	0.01	0.42
$\bar{\alpha}_c = 0.9$	8.9	4.7	4.3	0.35	-0.21	-0.05	0.44	22
$\bar{\alpha}_b = 0.5, \bar{\alpha}_c = 0.5$	9.0	4.7	4.2	0.33	-0.43	-0.06	0.37	13
$\bar{\alpha}_b = 0.1, \bar{\alpha}_c = 0.9$	9.1	4.9	4.2	0.34	-0.76	-0.05	0.45	27
$\bar{\alpha}_b = 0.9, \bar{\alpha}_c = 0.1$	8.4	3.9	5.2	0.49	4.0	-0.01	0.08	2.6
$\gamma_b = 10, \gamma_c = 10$	8.2	3.8	5.5	0.54	6.0	0.00	0.00	0.18
$\gamma_b = 5, \gamma_c = 15$	8.1	3.8	5.5	0.54	6.0	0.00	0.00	0.18
$\gamma_b = 15, \gamma_c = 5$	8.2	3.8	5.5	0.54	6.0	0.00	0.00	0.19
$\mu_\omega = 0.2$	8.4	3.3	4.6	0.37	8.8	-0.04	78	0.22
$\mu_\omega = 0.0$	8.2	3.8	5.4	0.52	6.2	0.00	8.8	0.20
$\mu_\omega = -0.2$	8.3	4.4	6.3	0.66	5.1	0.03	-56	0.18
$\sigma_\omega = 0.05$	11	3.3	5.0	0.59	-0.93	-0.09	54	0.00
$\sigma_\omega = 0.25$	11	3.3	5.0	0.59	-0.90	-0.09	54	0.00
$\sigma_\omega = 0.45$	8.8	4.4	4.3	0.34	-0.21	-0.05	0.85	0.01
$I = 0.1$	15	4.3	1.5	0.03	-0.65	-0.22	0.04	0.00
$I = 1$	8.7	3.1	4.6	0.35	3.2	-0.04	0.07	0.03
$I = 10$	8.3	4.3	6.1	0.64	7.7	0.02	-0.41	0.68

Note: $E(\text{spread})$ and $\sigma(\text{spread})$ are the mean and standard deviation of the sovereign spread over the simulated path, respectively; Y^m is the midpoint of the discretized Y process; Basis dev. is the CDS-bond basis deviations (running spread minus Z-spread) in percent; Sov. gain is ex-ante utility ($\mathbb{E}[W(Y,0)]$) relative to the benchmark measured in consumption equivalent variation (%); Inv. gain is the welfare gain measured in money-metric comparing the ex-ante utility (i.e., $V(\omega; \bar{\delta}, B')$ averaged over ω) averaged over $\bar{\delta}, B'$ arising in the simulation; CDS vol. is CDS volume relative to mean GDP.

projection coefficients).[4]

To solve, we first guess that the CDS-bond basis in prices holds, in which case the general equilibrium solution reduces to finding q (one-dimensional root finding). We then solve allowing for the basis to not hold, looking for an equilibrium q, p with $q + p \geq q_f$, using a combination of the Levenberg-Marquardt algorithm and Controlled Random Search. Whichever has lower sum of squared residuals error in market clearing is the one we choose.

The solution of the OTC block is precomputed on a grid of $\bar{\delta}$ and B'. Unless otherwise stated, half the points for the $\bar{\delta}$ grid are equally spaced on 0.01 to 0.49 and the other half are equally spaced on 0.51 to 0.99 with 32 points total, and the debt grid is equally spaced from 0.0001 to 0.3 with 64 points. The lower quality approximation uses an unequally spaced grid for $\bar{\delta}$ with 16 points at 0.99, 0.75, 0.5, and the rest equally spaced from 0.34 to 0.01; the lower quality grid for B' is linearly spaced from 0 to 0.3 with 16 points.

D.2 Sovereign block

The bond choice and debt state is discretized into equally-spaced points from 0 to 0.3. The GDP process is discretized using the Tauchen (1986) method with a coverage of 3 unconditional standard deviations. The bond grid has 512 points and the GDP grid has 127 points (these are not reduced for the lower quality approximations). To aid in making the price schedules smooth, we assume a small taste shock that influences the choice between default and repayment (this is not for convergence but for graphing). The scale parameter for this is 5×10^3, chosen to be small while still making the price schedules visually smooth. With taste shocks, we found the sovereign would occasionally

[4] The code uses Takuya Ooura's Fast Fourier Transform code, and we thank him.

issue a tremendous amount of debt at very low prices. So we impose a minimum threshold for issuing debt such that $q(Y, B')$ must exceed a threshold 0.01 (which is roughly a 400% annualized spread).

D.3 Linking the blocks

In solving the sovereign's problem or simulating, we stay on the sovereign block grids and linearly interpolate on the $\{\bar{\delta}, B'\}$ OTC grids. We prevent any extrapolation by using the nearest-neighbor value when necessary, which only occurs at very small debt levels or very small/large default rates.